

EE2302 Foundations of Information and Data Engineering

Assignment 2 (Solution)

1.

- a) No, because the elements c and d map to the same element in the co-domain of F .
- b) Yes, because all the elements in Y are images of some element in X .
- c) $\{e, f, g\}$.
- d) No, because the elements a and b map to the same element in the co-domain of G .
- e) No, because the element g is not an image of any element in the domain of G .
- f) $\{e, f\}$.

2.

- a) Yes. Suppose that $g(n_1) = g(n_2)$ for some elements n_1 and n_2 in $\mathbf{Z}$. By definition of g , $4n_1 - 5 = 4n_2 - 5$, which implies that $n_1 = n_2$. Therefore, g is injective.
- b) No. One possible counter example: Choose $0 \in \mathbf{Z}$. There does not exist $n \in \mathbf{Z}$ such that $g(n) = 4n - 5 = 0$, since the only root is $5/4$, which is not in $\mathbf{Z}$. Therefore, g is not surjective.

3.

- a) $x^3 - 1$
- b) $(x - 1)^3$

4.

- a) 2
- b) 0

5. Let x_1 and x_2 be elements in X such that $g \circ f(x_1) = g \circ f(x_2)$. That is equivalent to $g(f(x_1)) = g(f(x_2))$. Since g is injective, $f(x_1) = f(x_2)$. Since f is also injective, $x_1 = x_2$. Hence, $g \circ f$ is injective.

6. Yes. Define a function $f: (0,1) \rightarrow (2,5)$ such that $f(x) = 3x + 2$. Suppose $f(x_1) = f(x_2)$. Then $3x_1 + 2 = 3x_2 + 2$, which implies that $x_1 = x_2$. Hence, f is one to one. Given any $y \in (2,5)$, there exists $x = \frac{y-2}{3} \in (0,1)$ such that $f(x) = y$. Hence, f is onto. Therefore, f is a one to one correspondence. Hence, the two sets have the same cardinality.