

EE2104 Introduction to Electromagnetics
Assignment 1 Solution

Deadline: 21 Sep 2023 at 1:00pm Collection: YEUNG LT-4
Please put down your name and university ID.

1. Given the position vectors $\mathbf{P} = 20\hat{x} + 23\hat{y}$ and $\mathbf{Q} = 2\hat{x} + \hat{y}$. Draw the two vectors and then find:
- (a) the distance between $\mathbf{P}$ and $\mathbf{Q}$
 - (b) the unit vector pointing from $\mathbf{P}$ to $\mathbf{Q}$
 - (c) the dot product $\mathbf{P} \cdot \mathbf{Q}$
 - (d) the cross product $\mathbf{P} \times \mathbf{Q}$
 - (e) the angle between $\mathbf{P}$ and $\mathbf{Q}$

Solution:

(a)

$$\mathbf{R} = \mathbf{Q} - \mathbf{P} = -18\hat{x} - 22\hat{y}$$

The distance between $\mathbf{P}$ and $\mathbf{Q}$ is $|\mathbf{R}| = \sqrt{18^2 + 22^2} = 28$

(b) The unit vector pointing from $\mathbf{P}$ to $\mathbf{Q}$ is

$$\frac{\mathbf{R}}{|\mathbf{R}|} = \frac{-18\hat{x} - 22\hat{y}}{28} = -0.64\hat{x} - 0.79\hat{y}$$

(c) The dot product is

$$\mathbf{P} \cdot \mathbf{Q} = (20\hat{x} + 23\hat{y}) \cdot (2\hat{x} + 1\hat{y}) = 40 + 23 = 63$$

(d) The cross product is

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 20 & 23 & 0 \\ 2 & 1 & 0 \end{vmatrix} = -26\hat{z}$$

(e) The angle between $\mathbf{P}$ and $\mathbf{Q}$ is

$$\theta = \cos^{-1} \left(\frac{\mathbf{P} \cdot \mathbf{Q}}{|\mathbf{P}||\mathbf{Q}|} \right) = \cos^{-1} \left(\frac{63}{\sqrt{20^2 + 23^2} \times \sqrt{2^2 + 1^2}} \right) = \cos^{-1}(0.92) = 23^\circ$$

2. Given two points $A = (3, 1)$ and $B = (5, 0)$. Find

(a) the cylindrical unit vectors $\hat{\rho}$ and $\hat{\phi}$ at A

(b) the cylindrical unit vectors $\hat{\rho}$ and $\hat{\phi}$ at B

Then, draw the four vectors on the xy -plane with points A and B labeled.

Solution:

$$\rho = \sqrt{x^2 + y^2}, \quad \varphi = \tan^{-1} \frac{y}{x}, \quad \hat{\rho} = \cos\varphi \hat{x} + \sin\varphi \hat{y}, \quad \hat{\phi} = -\sin\varphi \hat{x} + \cos\varphi \hat{y},$$

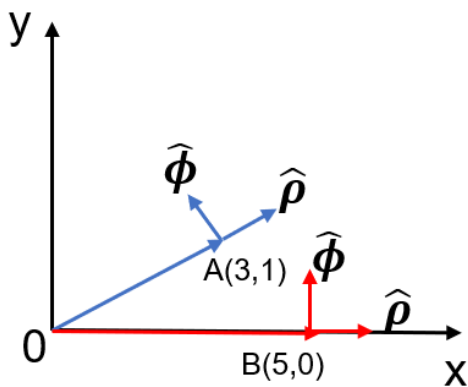
(a) For $A = (3, 1)$, $\varphi = \tan^{-1} \frac{y}{x} = 18^\circ$.

$$\hat{\rho} = 0.95 \hat{x} + 0.31 \hat{y}, \quad \hat{\phi} = -0.31 \hat{x} + 0.95 \hat{y}$$

(b) For $B = (5, 0)$, $\varphi = \tan^{-1} \frac{y}{x} = 0^\circ$.

$$\hat{\rho} = \hat{x}, \quad \hat{\phi} = \hat{y}$$

(c)



3. A field is given as

$$\mathbf{G}(x, y) = \frac{5y\hat{\mathbf{x}} + 2x\hat{\mathbf{y}}}{x^2 + y^2}$$

- (a) What is the x -component of $\mathbf{G}$?
- (b) What is the y -component of $\mathbf{G}$?
- (c) At point $P(2, 2)$, find G .

Solution:

(a) The x -component of $\mathbf{G}$ is $\frac{5y}{x^2+y^2}$.

(b) The y -component of $\mathbf{G}$ is $\frac{2x}{x^2+y^2}$.

(c) To find G at point $P(2, 2)$, we substitute $x = 2$ and $y = 2$ into the vector field

$$\mathbf{G}(x, y) = \frac{5 \cdot 2\hat{\mathbf{x}} + 2 \cdot 2\hat{\mathbf{y}}}{2^2 + 2^2} = 1.25\hat{\mathbf{x}} + 0.5\hat{\mathbf{y}}$$

Then the magnitude G is $\sqrt{1.25^2 + 0.5^2} = 1.35$

4. Consider a line at $y = L$ to $2L$. It contains electric charge of density $\rho_L = \rho_{L0}(y/L)^2$ for some constant ρ_{L0} in C/m. Find the total charge of this line.

Solution:

The total charge is

$$\begin{aligned} Q &= \int_L^{2L} \rho_L dy \\ &= \int_L^{2L} \rho_{L0} (y/L)^2 dy \\ &= \rho_{L0} \cdot L^{-2} \int_L^{2L} y^2 dy \\ &= \rho_{L0} \cdot L^{-2} \left[\frac{L^3(8-1)}{3} \right] \\ &= \frac{7L \cdot \rho_{L0}}{3} \text{ (C)} \end{aligned}$$

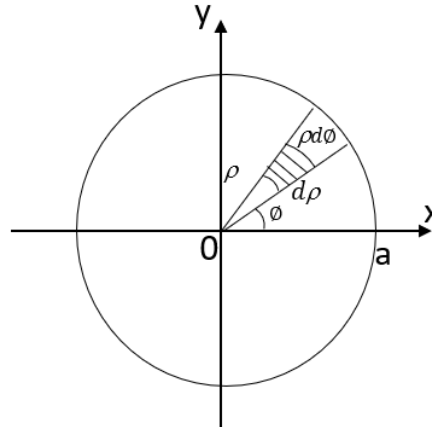
5. In the polar coordinate system (ρ, ϕ) , a disc of radius a centered at the origin holds charges of density

$$\rho_s = Q(\rho/a)^2 \sin \frac{\phi}{2}$$

where $Q = 5 \text{ nC/m}^2$. Find the total charge contained by the disc.

Solution:

Consider the charged disc:



The differential arc length = $\rho d\phi$

The differential area = $\rho d\rho d\phi$

Total charge = sum of (charge density $\times$ differential area)

$$\begin{aligned} &= \int_{Disc} Q(\rho/a)^2 \sin(\phi/2) \rho d\rho d\phi \\ &= (Q/a^2) \int_{\rho=0}^a \rho^3 d\rho \int_{\phi=0}^{2\pi} \sin(\phi/2) d\phi \\ &= (Q/a^2) \left(\frac{1}{4} a^4 \right) (-2 \cos(\phi/2)) \Big|_{\phi=0}^{2\pi} \\ &= Q a^2 \\ &= 5 a^2 \text{ (nC)} \end{aligned}$$