

Tutorial 9

Magnetic Field

Practice Questions:

Chapter 28

Questions: 1, *1, *11, *18, **46, **49, **61

1

1 Figure 28-22 shows three situations in which a positively charged particle moves at velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$ and experiences a magnetic force $\vec{F}_B$. In each situation, determine whether the orientations of the vectors are physically reasonable.

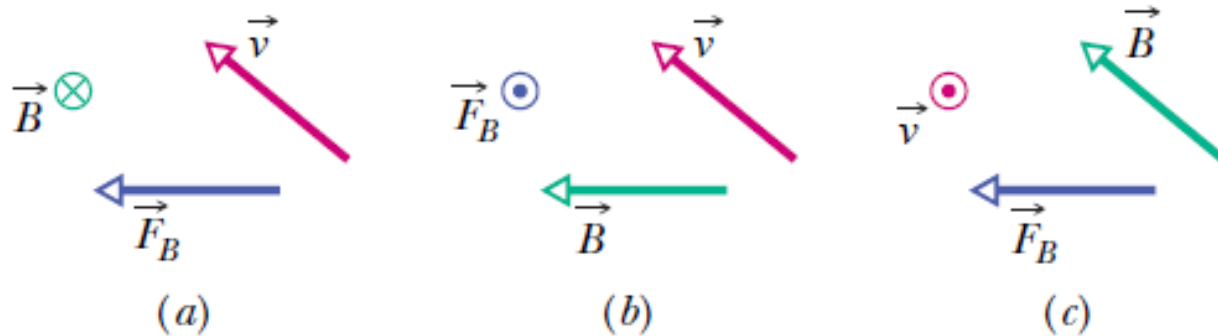


Fig. 28-22 Question 1.

- (a) not possible ($\vec{v}$ cannot be in the same plane as $\vec{F}$)
- (b) possible ($\vec{v}$ and $\vec{B}$ are in the same plane and $\vec{F}$ perpendicular)
- (c) not possible ($\vec{F}$ cannot be in the same plane as $\vec{B}$)

***1**

sec. 28-3 The Definition of $\vec{B}$

•1 **SSM** **ILW** A proton traveling at 23.0° with respect to the direction of a magnetic field of strength 2.60 mT experiences a magnetic force of $6.50 \times 10^{-17} \text{ N}$. Calculate (a) the proton's speed and (b) its kinetic energy in electron-volts.

1. (a) Equation 28-3 leads to

$$v = \frac{F_B}{eB \sin \phi} = \frac{6.50 \times 10^{-17} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(2.60 \times 10^{-3} \text{ T}) \sin 23.0^\circ} = 4.00 \times 10^5 \text{ m/s}.$$

(b) The kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(4.00 \times 10^5 \text{ m/s})^2 = 1.34 \times 10^{-16} \text{ J}$$

which is equivalent to $K = (1.34 \times 10^{-16} \text{ J}) / (1.60 \times 10^{-19} \text{ J/eV}) = 835 \text{ eV}$.

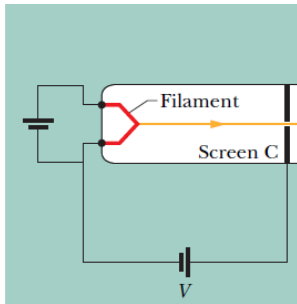


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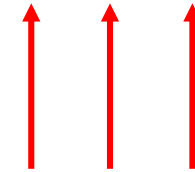
*11

•11 An ion source is producing ${}^6\text{Li}$ ions, which have charge $+e$ and mass 9.99×10^{-27} kg. The ions are accelerated by a potential difference of 10 kV and pass horizontally into a region in which there is a uniform vertical magnetic field of magnitude $B = 1.2$ T. Calculate the strength of the smallest electric field, to be set up over the same region, that will allow the ${}^6\text{Li}$ ions to pass through undeflected.



${}^6\text{Li}$
 $+e$

$B = 1.2$ T



With B field alone: Ion would be deviated out of the paper plane

$\Rightarrow$ **countering** E field must point into the paper plane

$= 0$ (meaning that the net force is zero)

11. Since the total force given by $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$ vanishes, the electric field $\vec{E}$ must be perpendicular to both the particle velocity $\vec{v}$ and the magnetic field $\vec{B}$. The magnetic field is perpendicular to the velocity, so $\vec{v} \times \vec{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V , $mv^2 / 2 = eV$ and $v = \sqrt{2eV/m}$.

Thus,

$$E = B \sqrt{\frac{2eV}{m}} = (1.2 \text{ T}) \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(10 \times 10^3 \text{ V})}{(9.99 \times 10^{-27} \text{ kg})}} = 6.8 \times 10^5 \text{ V/m}.$$

•18 In Fig. 28-35, a particle moves along a circle in a region of uniform magnetic field of magnitude $B = 4.00 \text{ mT}$. The particle is either a proton or an electron (you must decide which). It experiences a magnetic force of magnitude $3.20 \times 10^{-15} \text{ N}$. What are (a) the particle's speed, (b) the radius of the circle, and (c) the period of the motion?

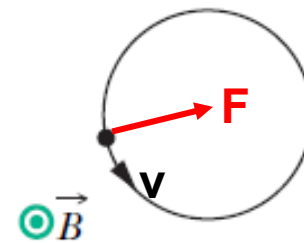


Fig. 28-35
Problem 18.

***18**

18. With the $\vec{B}$

pointing “out of the page,” we evaluate the force (using the right-hand rule) at the particle (black dot) for the situation of a positive charged particle. For this case, the force at the black dot would be toward the left. However, the particle's path is shown to be counterclockwise, which requires the force to be towards the right.

Therefore, the particle is negatively charged; it is an electron.

(a) Using Eq. 28-3 (with angle ϕ equal to 90°), we obtain $v = \frac{|\vec{F}|}{e|\vec{B}|} = 5.00 \times 10^6 \text{ m/s}.$

(b) Using either Eq. 28-14 or Eq. 28-16, we find $r = 0.0071 \text{ m} = 7.1 \text{ mm}.$ ($F = m v^2/r$) (28-14)
 $m_e = 9.11 \times 10^{-31} \text{ kg}$

(c) Using Eq. 28-17 (in either its first or last form) readily yields $T = 8.9 \times 10^{-9} \text{ s}.$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \quad (\text{period}).$$

(28-17)



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••46 In Fig. 28-43, a metal wire of mass $m = 24.1$ mg can slide with negligible friction on two horizontal parallel rails separated by distance $d = 2.56$ cm. The track lies in a vertical uniform magnetic field of magnitude 56.3 mT. At time $t = 0$, device G is connected to the rails, producing a constant current $i = 9.13$ mA in the wire and rails (even as the wire moves). At $t = 61.1$ ms, what are the wire's (a) speed and (b) direction of motion (left or right)?

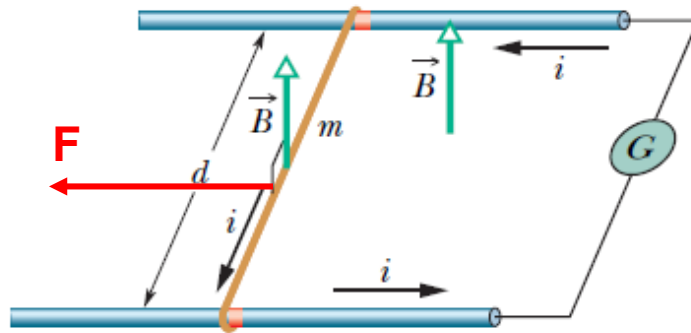
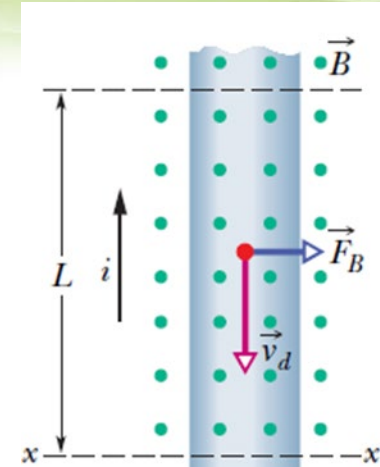


Fig. 28-43 Problem 46.



$$\vec{F}_B = i\vec{L} \times \vec{B} \quad (\text{force on a current}).$$

46. (a) The magnetic force on the wire is $F_B = idB$, pointing to the left. Thus

$$v = at = \frac{F_B t}{m} = \frac{idBt}{m} = \frac{(9.13 \times 10^{-3} \text{ A})(2.56 \times 10^{-2} \text{ m})(5.63 \times 10^{-2} \text{ T})(0.0611 \text{ s})}{2.41 \times 10^{-5} \text{ kg}} \\ = 3.34 \times 10^{-2} \text{ m/s}.$$

(b) The direction is to the left (away from the generator).

sec. 28-9 Torque on a Current Loop

•49 SSM Figure 28-44 shows a rectangular 20-turn coil of wire, of dimensions 10 cm by 5.0 cm. It carries a current of 0.10 A and is hinged along one long side. It is mounted in the xy plane, at angle $\theta = 30^\circ$ to the direction of a uniform magnetic field of magnitude 0.50 T. In unit-vector notation, what is the torque acting on the coil about the hinge line?

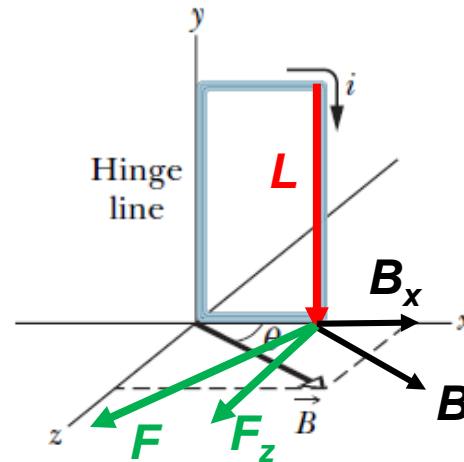


Fig. 28-44 Problem 49.

****49**

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

(force on a current).

$$\text{Torque: } \vec{\tau} = \vec{r} \times \vec{F},$$

- A force leading to a torque is only produced only along z-direction
- Torque about the hinge line (y-axis) is zero (forces acting at the axis of rotation produce no torque about that axis).
- The top and bottom straight segments experience forces due to Eq. 28-26 (caused by the B_z component) are unable to produce a torque about the y axis.
- Consequently, the torque derives completely from the force $\vec{F}$ (marked green) exerted on the straight segment located at $x = 0.050$ m, which has length $L = 0.10$ m (marked red)
- $\vec{F}$ has two components: $\vec{F}_x$ does not produce a torque about the hinge
only $\vec{F}_z$ produces torque about the hinge ($\vec{F}_z$ is solely produced by B_x):

$$\tau = (NiLB_x)(x) = NiLxB \cos \theta = (20)(0.10 \text{ A})(0.10 \text{ m})(0.050 \text{ m})(0.50 \text{ T}) \cos 30^\circ = 0.0043 \text{ N} \cdot \text{m}.$$

Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is $-y$. In unit-vector notation, the torque is

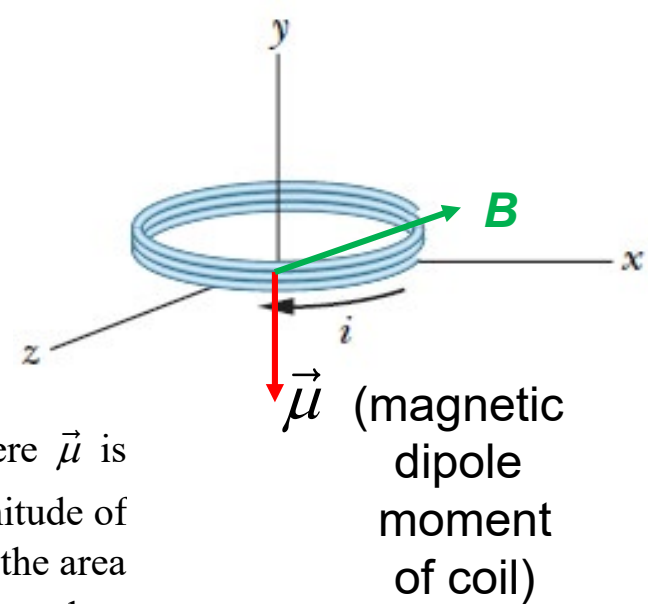
$$\vec{\tau} = (-4.3 \times 10^{-3} \text{ N} \cdot \text{m}) \hat{j}$$



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•61 SSM The coil in Fig. 28-49 carries current $i = 2.00$ A in the direction indicated, is parallel to an xz plane, has 3.00 turns and an area of $4.00 \times 10^{-3} \text{ m}^2$, and lies in a uniform magnetic field $\vec{B} = (2.00\hat{i} - 3.00\hat{j} - 4.00\hat{k}) \text{ mT}$. What are (a) the orientation energy of the coil in the magnetic field and (b) the torque (in unit-vector notation) on the coil due to the magnetic field?



61. The orientation energy of the magnetic dipole is given by $U = -\vec{\mu} \cdot \vec{B}$, where $\vec{\mu}$ is the magnetic dipole moment of the coil and $\vec{B}$ is the magnetic field. The magnitude of $\vec{\mu}$ is $\mu = NiA$, where i is the current in the coil, N is the number of turns, A is the area of the coil. On the other hand, the torque on the coil is given by the vector product $\vec{\tau} = \vec{\mu} \times \vec{B}$.

(a) By using the right-hand rule, we see that $\vec{\mu}$ is in the $-y$ direction. Thus, we have

$$\vec{\mu} = (NiA)(-\hat{j}) = -(3)(2.00 \text{ A})(4.00 \times 10^{-3} \text{ m}^2)\hat{j} = -(0.0240 \text{ A} \cdot \text{m}^2)\hat{j}$$

(μ has value only in y -direction!)

The corresponding orientation energy is $U = -\vec{\mu} \cdot \vec{B} = -\mu_y B_y = -(-0.0240 \text{ A} \cdot \text{m}^2)(-3.00 \times 10^{-3} \text{ T}) = -7.20 \times 10^{-5} \text{ J}$

(b) Using the fact that $\hat{j} \cdot \hat{i} = 0$, $\hat{j} \times \hat{j} = 0$, and $\hat{j} \times \hat{k} = \hat{i}$,

the torque on the coil is

$$\begin{aligned} \vec{\tau} &= \vec{\mu} \times \vec{B} = \mu_y B_z \hat{i} - \mu_y B_x \hat{k} \\ &= (-0.0240 \text{ A} \cdot \text{m}^2)(-4.00 \times 10^{-3} \text{ T})\hat{i} - (-0.0240 \text{ A} \cdot \text{m}^2)(2.00 \times 10^{-3} \text{ T})\hat{k} \\ &= (9.60 \times 10^{-5} \text{ N} \cdot \text{m})\hat{i} + (4.80 \times 10^{-5} \text{ N} \cdot \text{m})\hat{k}. \end{aligned}$$

Vector cross product in Cartesian coordinates

1 Consider two general three-dimensional vectors defined in Cartesian coordinates.

- $\mathbf{a} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$

- $\mathbf{b} = D\mathbf{i} + E\mathbf{j} + F\mathbf{k}$

- Here, $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors, and A, B, C, D, E, F are constants.

2 **Set up the matrix.** One of the easiest ways to compute a cross product is to set up the unit vectors with the two vectors in a matrix.

- $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A & B & C \\ D & E & F \end{vmatrix}$

3 **Calculate the determinant of the matrix.** Below, we use cofactor expansion (expansion by minors).

- $\mathbf{a} \times \mathbf{b} = (BF - EC)\mathbf{i} - (AF - DC)\mathbf{j} + (AE - DB)\mathbf{k}$

- This vector is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.