

EE2104 Introduction to Electromagnetics
Assignment 3

Deadline (for uploading): 7:00 pm, 7 Nov 2020

Please put down your name and university ID.

SI units are used unless otherwise specified.

1. Given a field $\mathbf{G} = y\hat{\mathbf{x}} - x\hat{\mathbf{y}}$. There are points $A(1,0)$ and $B(0,1)$.
(a) Find

$$\int_{\text{Path } S} \mathbf{G} \cdot d\mathbf{l}$$

where the path S is a straight line from point A to point B .

- (b) Find

$$\int_{\text{Path } C} \mathbf{G} \cdot d\mathbf{l}$$

where the path C is the circular arc of radius 1 connecting point A and point B .

- (c) Is $\mathbf{G}$ conservative? Why?

2. Consider a copper wire with a cross-sectional area $S = 1 \text{ mm}^2$. Copper has $n = 10^{29}$ electrons in every 1 m^3 . It carries a current of $I = 4 \text{ A}$ for driving a light bulb $L = 50 \text{ m}$ away from a voltage source. The conductivity is $\sigma = 10^8 (\Omega^{-1}\text{m}^{-1})$.
(a) Find the time needed for an electron to drift from the source to the light bulb.
(b) Ohm's law in the macroscopic form gives the voltage $V = IR$. Find the resistance R of the copper wire.
(c) Find the power dissipated in every 1 m of the wire.

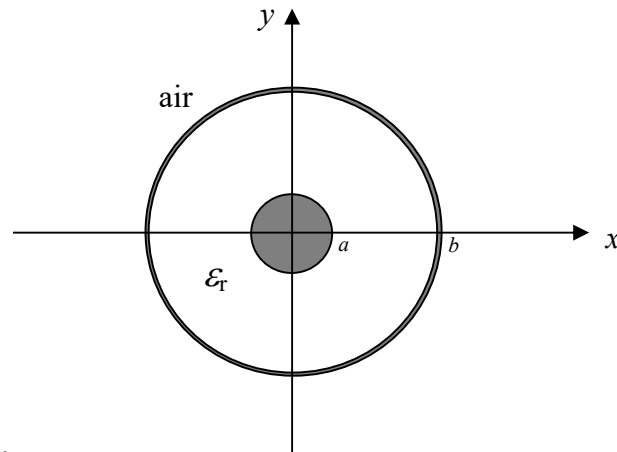
3. In a dielectric of permittivity ϵ , the potential field is given in spherical coordinates by

$$V(r, \theta, \phi) = V_0 \frac{r}{a} \cos\theta$$

where V_0 is a constant potential and a is a constant length.

- (a) Find the electric field.
(b) Find the total charge contained within the region $r < a$.

4. Consider a coaxial cable below consisting of cylinders of perfect conductors of length $l = 7$ m. In the cylindrical coordinate system, the inner conductor is at $0 < \rho < a$ and the outer conductor is at $\rho = b$, where $a = 3$ mm and $b = 8$ mm. The outer conductor is grounded. The region between the conductors is filled with a dielectric with relative permittivity $\epsilon_r = 4$. The inner and outer conductors hold charges of $+Q$ and $-Q$ with $Q = 10$ μC .



- (a) Find the maximal $|\mathbf{E}|$ in the structure.
 (b) Find the capacitance of the whole cable.
5. The figure below shows a planar structure of infinite length. The structure is along z -axis. The different layers contain the following materials:
- (i) Lower conductor for $y < 0$,
 - (ii) Dielectric with relative permittivity ϵ_r for $0 < y < a$,
 - (iii) Air gap for $a < y < 3a$,
 - (iv) Upper conductor for $y = 3a$.

The potential of the upper conductor is maintained at $+V_0$ while the potential of the lower conductor is grounded to zero. Perform the following tasks to solve for the potential V in or between the two conductors.

- (a) Using the Laplace's equation, find the expressions for the potential V for $0 < y < a$ and $a < y < 3a$. There should be a total of 4 unknown constants.
- (b) Using the boundary condition of V at the air-dielectric boundary, reduce the number of unknown constants to 3.
- (c) Using the boundary condition for the electric field at the air-dielectric boundary, reduce the number of unknown constants to 2.
- (d) Using the boundary conditions for V at the conductors, solve for V .

