

Homework 1 Solutions

Question 1: Consider the following demand and supply functions for a commodity

$$q^D(p) = 8 - 4p \quad q^S(p) = -4 + 12p$$

- (a) (**4 points**) What is the equilibrium price in the market? Derive it.

The equilibrium price is obtained by equating q^D and q^S :

$$(2 \text{ points}) \quad 8 - 4p = -4 + 12p$$

$$(2 \text{ points}) \quad \implies p = \frac{3}{4}$$

- (b) (**16 points**) If the government imposes an excise tax of T per unit, find the selling price and quantity sold in equilibrium. Note that the government collects the tax from the suppliers.

The new supply function is given by (selling price is p)

$$(3 \text{ points}) \quad q^{S_T}(p) = -4 + 12(p - T)$$

The demand function remains the same.

In the new equilibrium, we must have

$$(3 \text{ points}) \quad 8 - 4p^T = -4 + 12(p^T - T)$$

$$\implies 12 = 12p^T - 12T + 4p^T$$

$$\implies 16p^T = 12 + 12T$$

$$(5 \text{ points}) \quad \implies p^T = \frac{3}{4} + \frac{3}{4}T$$

And, equilibrium quantity q^T is

$$8 - 4p^T = 8 - 4\left(\frac{3}{4} + \frac{3}{4}T\right)$$

$$(5 \text{ points}) \quad \implies q^T = 5 - 3T$$

Question 2: Suppose we have the following equation

$$x^2 + \alpha x + 1 = 0$$

- (a) (**5 points**) Write down the expression for the solutions (or roots) of the equation in terms of the discriminant.

$$\begin{aligned} & \text{(5 points)} \quad x = \frac{-\alpha \pm \sqrt{\alpha^2 - 4}}{2} \\ \text{or,} \quad & x_1 = \frac{-\alpha + \sqrt{\alpha^2 - 4}}{2} \quad x_2 = \frac{-\alpha - \sqrt{\alpha^2 - 4}}{2} \end{aligned}$$

(Note: You may write the answer as $x = \frac{-\alpha \pm \sqrt{D}}{2}$, where $D = \alpha^2 - 4$ is the discriminant.)

- (b) (**10 points**) For which values of α does the above equation has no solutions? Why?

The equation has no solutions when the discriminant is less than zero or

$$\text{(4 points)} \quad \alpha^2 - 4 < 0$$

$$\text{(4 points)} \quad \text{i.e., } -2 < \alpha < 2$$

(2 points) This is because there is no real number solution to the square root of a negative discriminant.

- (c) (**5 points**) If $\alpha = -2$, what is the solution of the equation? How many solutions are there in this case?

(3 points) If $\alpha = -2$, then $x = 1$.

(2 points) There is only one solution in this case.

Question 3: The new Euro-commissioner for agriculture has inherited a surplus ‘grain mountain’ of 30,000 tonnes, held in a warehouse near Strasbourg. Each year 5% of the grain in the warehouse at the start of the year is eaten by Euro-mice. The commissioner is obliged to add N tonnes to the mountain each year, where N is to be fixed in advance by negotiation between governments.

- (a) (**5 points**) Write down the recurrence equation for y_t , the amount of grain (in tonnes) in the warehouse at the end of year t .

(5 points) $y_t = 0.95y_{t-1} + N$

- (b) (5 points) What is the time-independent solution, y^* ?

Comparing with the standard form, $y_t = ay_{t-1} + b$, the time-independent solution is given by

(1 point) $y^* = \frac{b}{1-a}$

Here, $a = 0.95, b = N$. Therefore,

(4 points) $y^* = \frac{N}{1-0.95} = 20N$

- (c) (5 points) Write down the general solution of the recurrence equation.

The general solution of the recurrence equation is

(1 point) $y_t = y^* + (y_0 - y^*)a^t$

Here, $y_0 = 30,000$. Therefore, we have

(4 points) $y_t = 20N + (30000 - 20N)(0.95)^t$

- (d) (5 points) If $N = 1300$, show that the condition required for the grain mountain to shrink to 27,500 tonnes is given by $(0.95)^t \leq 0.375$.

If $N = 1300$, we have

$$y_t = 26000 + 4000(0.95)^t$$

Then the condition “grain mountain to shrink to 27,500 tonnes” requires

(3 points) $y_t = 26000 + 4000(0.95)^t \leq 27500$

(2 points) $\implies (0.95)^t \leq 0.375$

Question 4: Suppose that you have won a competition in a national newspaper and you can choose either to receive a lump sum of \$100,000 now or a payment of \$20,000 at the end of each year for the next 7 years. Assume that the interest rate r is 7% per annum.

- (a) (**15 points**) What is the present value of the payment of \$20,000 at the end of each year for the next 7 years? (*You may use the formula directly*)

Using the formula with $r = 0.07$, $N = 7$, $I = 20,000$, we have

(15 points)
$$P = \frac{I}{r} \left(1 - \frac{1}{(1+r)^N} \right) = \frac{20000}{0.07} \left(1 - \frac{1}{(1.07)^7} \right) = 107,785.79$$

- (b) (**5 points**) Comparing the present value of the income stream obtained in part (a) and the lump-sum of \$100,000, which one should you choose?

(5 points) As the present value of the income stream in (a) is higher than the lump-sum of \$100,000, you should choose the income stream (or annuity).

Question 5: Suppose that the supply and demand functions for a certain good are

$$q^S(p) = -4 + \frac{2}{3}p \quad p^D(q) = \frac{20 - q}{2}$$

Suppliers operate according to the cobweb model so that if p_t and q_t are (respectively) the price and quantity in year t , then $p_t = p^D(q_t)$ and $q_t = q^S(p_{t-1})$. Suppose also that the initial price is $p_0 = 10$.

- (a) (**10 points**) Derive the recurrence equation for p_t .

We have

(2 points)
$$q_t = q^S(p_{t-1}) = -4 + \frac{2}{3}p_{t-1}$$

(3 points)
$$p_t = p^D(q_t) = \frac{20 - q_t}{2} = \frac{20 - \left(-4 + \frac{2}{3}p_{t-1}\right)}{2}$$

Then

(5 points)
$$p_t = 12 - \frac{1}{3}p_{t-1} = \left(-\frac{1}{3}\right)p_{t-1} + 12$$

- (b) (**5 points**) What is the general solution for the recurrence equation?

The time-independent solution is given by

$$(\mathbf{2\ points}) \quad p^* = \frac{12}{1 - (-\frac{1}{3})} = 9$$

The general solution, therefore, is given by

$$(\mathbf{3\ points}) \quad p_t = p^* + (p_0 - p^*) \left(-\frac{1}{3}\right)^t = 9 + \left(-\frac{1}{3}\right)^t$$

- (c) (**5 points**) How does p_t behave as $t \rightarrow \infty$?

$$(\mathbf{5\ points}) \quad p_t \rightarrow 9 \quad \text{as } t \rightarrow \infty$$