

CITY UNIVERSITY OF HONG KONG

Course code and title : MA1201 Calculus and Basic Linear Algebra II

Session : Semester A, 2013/2014

Time allowed : Two hours

This paper has **THREE** pages (including this cover page and the attached table).

Instructions to candidates:

1. This paper has two sections.
 2. Each section has three questions and carries 50 marks.
 3. Attempt **ALL** questions in Section A and B.
 4. Start each question on a new page.
 5. Show all steps clearly.
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*This is a **closed-book** examination.*

Candidates are allowed to use the following materials/aids:

Non-programmable calculators

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorised materials or aids are found on them.

NOT TO BE TAKEN AWAY

**NOT TO BE TAKEN AWAY
BUT FORWARDED TO LIB**

Section A (50%)

Question 1

Evaluate the following indefinite integrals:

- (a) $\int \sin 5x \sin 2x dx$ [4]
- (b) $\int \frac{2x^2 + 21x + 37}{(x-1)(x^2 + 6x + 13)} dx$ [9]
- (c) $\int \sec^3 x dx$ [4]
- (d) $\int \sqrt{4+9x^2} dx$ [6]

Question 2

Compute the following definite integrals:

- (a) $\int_0^4 \frac{dx}{3\sqrt{x+2}}$ [6]
- (b) $\int_{-1}^1 \frac{\sin^3 x \ln(1+x^2)}{2+\cos x} dx$ (Hint: Check whether the integrand is an odd or even function.) [3]

Question 3

- (a) Find the area of the region bounded by the parabola $y=3-x^2$ and the straight line $y=x-3$. [9]
- (b) Calculate the volume of the solid generated by revolving the region bounded above by the ellipse $\frac{x^2}{4} + y^2 = 1$ and bounded below by the parabola $y = \frac{\sqrt{3}}{2}x^2$ about the x -axis. [9]

Section B (50%)

Question 4

Let $A(-1,2,1)$, $B(2,1,3)$ and $C(0,-1,2)$ be three points on a plane π .

- (a) Find the angle $\angle BAC$. [6]
- (b) Determine a unit vector perpendicular to the plane π . [6]
- (c) Evaluate the shortest distance from a point $D(1,-2,-3)$ to the plane π . [5]

Question 5

- (a) Solve $z^5 + 1 = -\sqrt{3}i$ and express your answer in Euler's form with principal argument. [6]
- (b) Given $A(2\sqrt{3},1)$ in R^2 , rotate OA by 30° in clockwise direction and stretch the resulting vector by a factor of 6 to OB . Determine the coordinates of B in surd form using complex number technique. [5]

Question 6

- (a) Use the cofactor method to determine the inverse of the matrix $A = \begin{pmatrix} 2 & 3 & -4 \\ -1 & 1 & 2 \\ 1 & -3 & 2 \end{pmatrix}$. [11]

- (b) Given a system of linear equations as follows.

$$\begin{aligned} x - 2y + 3z - 4w &= 1 \\ -2x + 3y - 4z + 10w &= 2 \\ x - y + 2z - 3w &= 3 \end{aligned}$$

- (i) Solve the above linear system by Gaussian elimination and express the solution in vector form. [8]
- (ii) Write down the corresponding homogeneous system and determine a non-trivial solution without resolving it. [3]

Brief Table of Derivatives and Integrals

Standard Derivatives	Standard Integrals
$\frac{d}{dx}(x^p) = px^{p-1}$	$\int x^p dx = \frac{x^{p+1}}{p+1} + C, \quad p \neq -1$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x dx = -\cot x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x dx = -\csc x + C$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$	
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$