

L10-11: Smith Charts

EE3109 Applied Electromagnetics

Lecture Notes

Dr. Alex M. H. Wong

Jan – Apr 2023

Outline

- Course Matters
- Smith Charts
- Matching with a Smith Chart
- L-Section Matching
- Single Stub Tuning

Announcement (Week 10)

- Lab 1 is ongoing. Reports are due 1 week after the lab session
- Lab 2 Schedule (Also posted on Announcements)
 - Session 01: Wed Apr 12th, 9-12 (Yeung P1615)
 - Session 02: Wed Mar 29th, 9-12 (Yeung P1615)
 - Session 03: Wed Mar 31st, 7-10 (pm) (Yeung P1615)
- Test 2 (Thursday 6th April):
 - Similar format as Test 1
 - Venue: LT-9
 - Test material will be up to and including Week 10's Lecture

Announcement (Week 11)

- Assignment 3 Posted; Due on Week 13
- Lab 2 is ongoing – Check announcement for lab times
- Test 2 (Thursday 6th April):
 - Similar format as Test 1
 - Venue: LT-9
 - Time: 3pm – 4pm; Class will resume at 4:30pm in B4302
 - Test material will be up to and including Week 10's Lecture
 - Reminder: 2 Aid Sheets allowed for Test 2 and Exam
 - Please bring S.C. plotting utensils. The S.C.'s if needed, will be provided
 - Make-up test not allowed as per department regulation
 - Extra office hour: Tues 4th April, 5:30-7pm, G6302

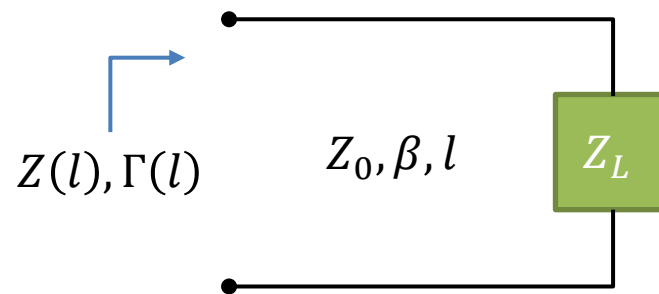
Tentative Course Schedule

Week #	Week of	Course Coverage
1	Jan 10	Course Introduction, Math Review
2	Jan 17	Introduction to Maxwell Equations, Materials and Boundaries
3	Jan 24	Wave Equations, Plane Waves, Propagation in Lossy Media
	Jan 31	<i>Happy Chinese New Year!</i>
4	Feb 7	Polarizations, EM Potential <i>Asst. 1a Due</i>
5	Feb 14	Reflection and Transmission <i>Asst. 1b Due</i>
6	Feb 21	Test 1 , Reflection and Transmission Cont'd
7	Feb 28	Poynting Vector, Power and Velocity;
8	Mar 7	Introduction to Waveguides and Transmission Lines (TL)
9	Mar 14	VIP in a TL, Terminated and Lossy TLs, <i>Asst. 2 Due</i>
10	Mar 21	Impedance in a TL, Smith Charts, <i>Lab 1</i>
11	Mar 28	Smith Charts Cont'd, Impedance Matching, Quarter-wave Transformers
12	Apr 4	Test 2 , Introduction to Antennas, <i>Lab 2</i>
13	Apr 11	Dipole Antennas, <i>Asst. 3 Due</i>

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Impedance vs. Reflection



- Impedance transformation:

$$Z(l) = Z_0 \frac{Z_L + j \tan \beta l}{1 + j Z_L \tan \beta l} \quad (\text{Ulaby 2.79})$$

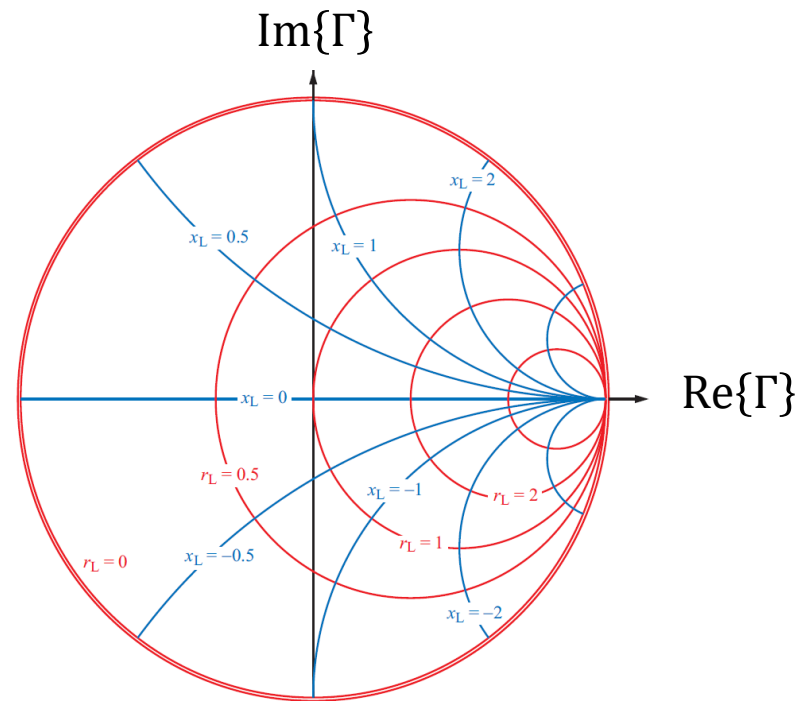
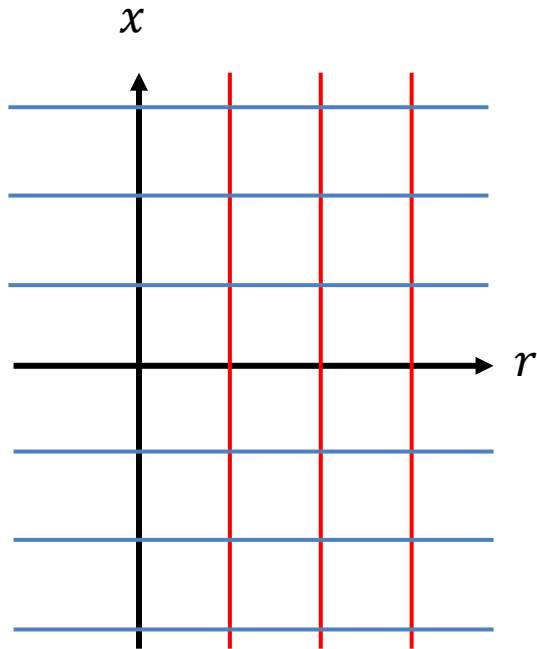
- “ Γ -transformation”:

$$\Gamma(l) = \Gamma_L e^{-2j\beta l} \quad (\text{Ulaby 2.75})$$

(It is much simpler to work with Γ !!)

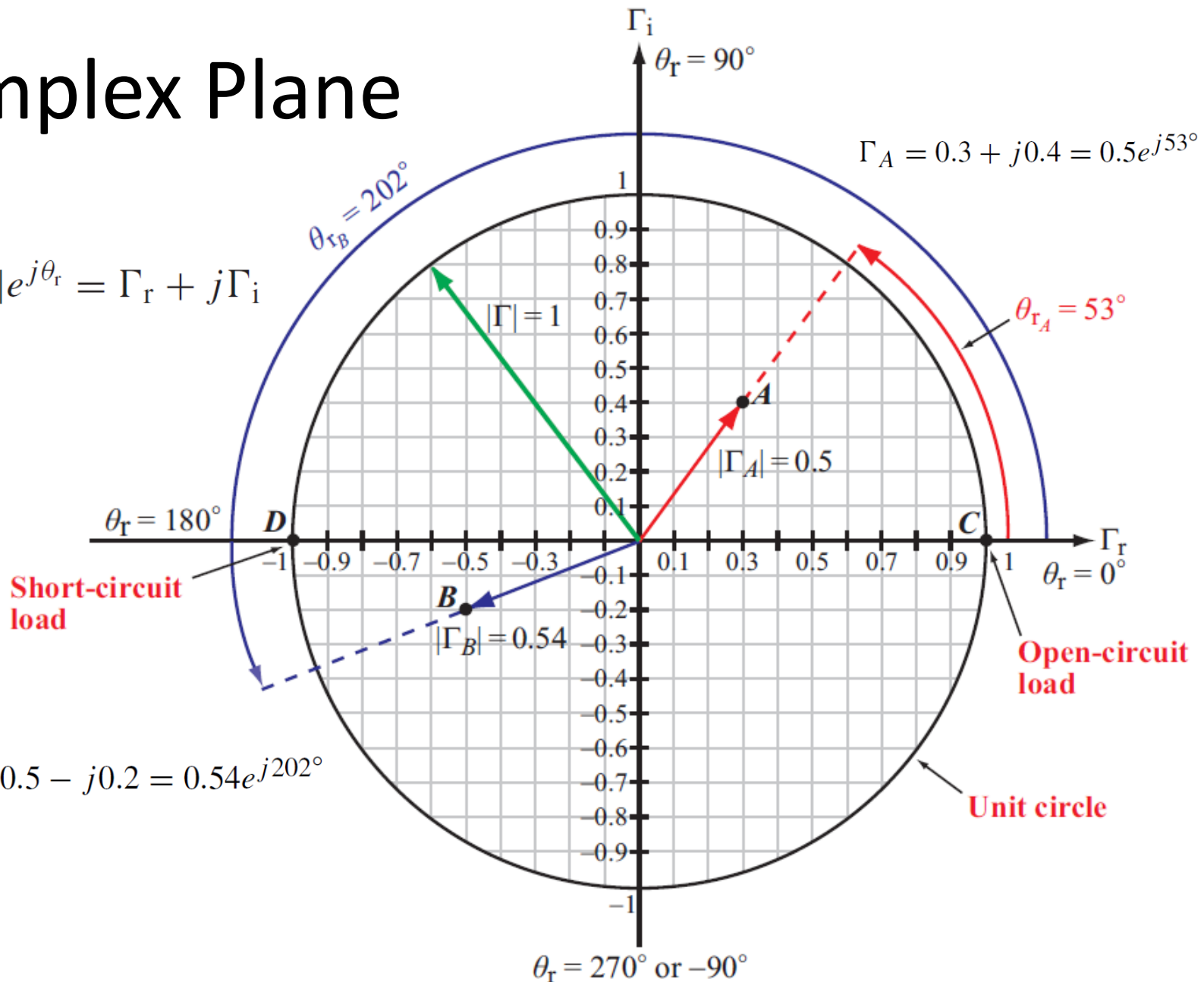
Smith Chart

- A plane of $\{\Gamma_r, \Gamma_i\}$, with transformations from $\{r, x\}$.
- Graphical tool for performing impedance transformations on a TL system.



Complex Plane

$$\Gamma = |\Gamma|e^{j\theta_r} = \Gamma_r + j\Gamma_i$$



Smith Chart Parametric Equations

$$\Gamma = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1 + \Gamma}{1 - \Gamma} \quad (2.112)$$

$$Z_L = r_L + jx_L.$$

$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

$$x_L = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

Equation for a circle

$$\left(\Gamma_r - \frac{r_L}{1 + r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + r_L}\right)^2 \quad (2.116)$$

The standard equation for a circle in the x - y plane with center at (x_0, y_0) and radius a is

$$(x - x_0)^2 + (y - y_0)^2 = a^2. \quad (2.117)$$

A similar manipulation of the expression for x_L given by Eq. (2.115b) leads to

$$r_L + jx_L = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i}$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2, \quad (2.118)$$

Smith Chart Parametric Equations

$$\left(\Gamma_r - \frac{r_L}{1 + r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + r_L}\right)^2. \quad (2.116)$$

r_L circles

r_L circles are contained inside the unit circle

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2, \quad (2.118)$$

x_L circles

Only parts of the x_L circles are contained within the unit circle

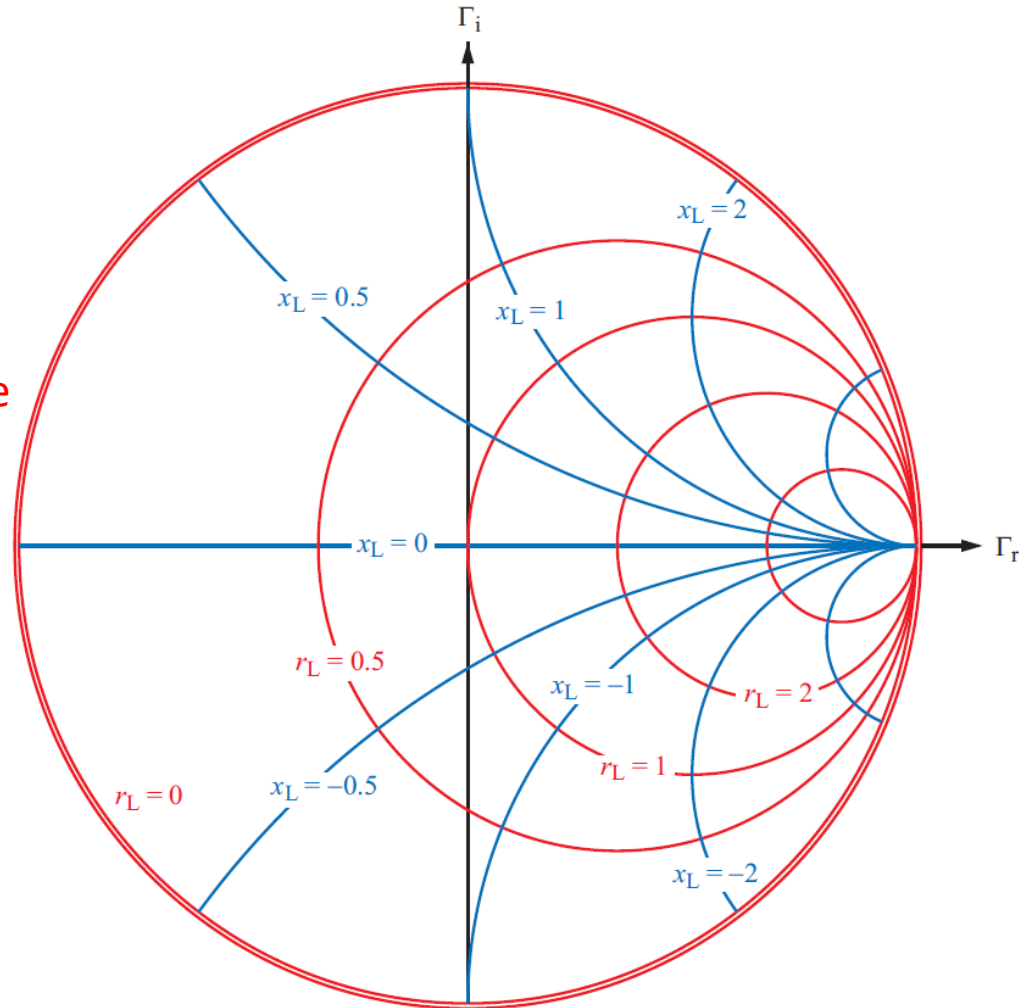
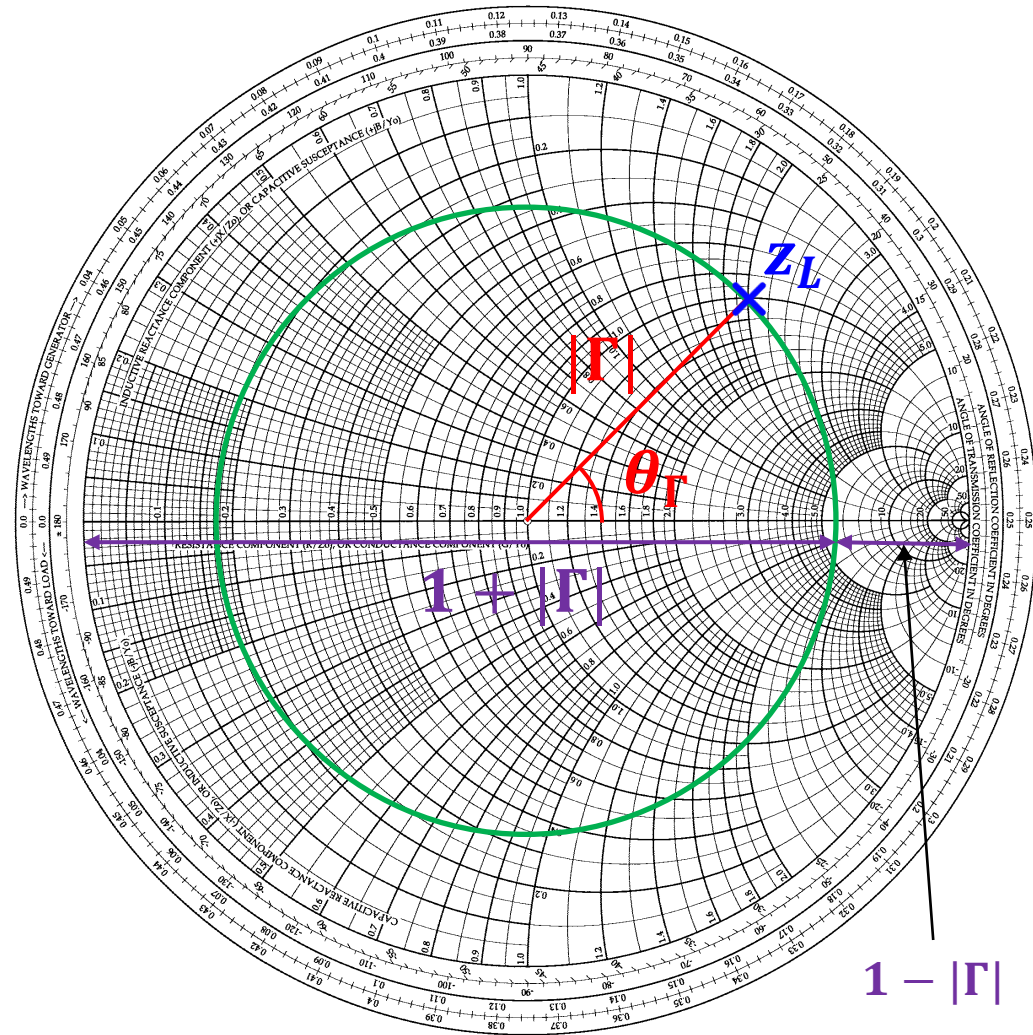


Figure 2-25: Families of r_L and x_L circles within the domain $|\Gamma| \leq 1$.

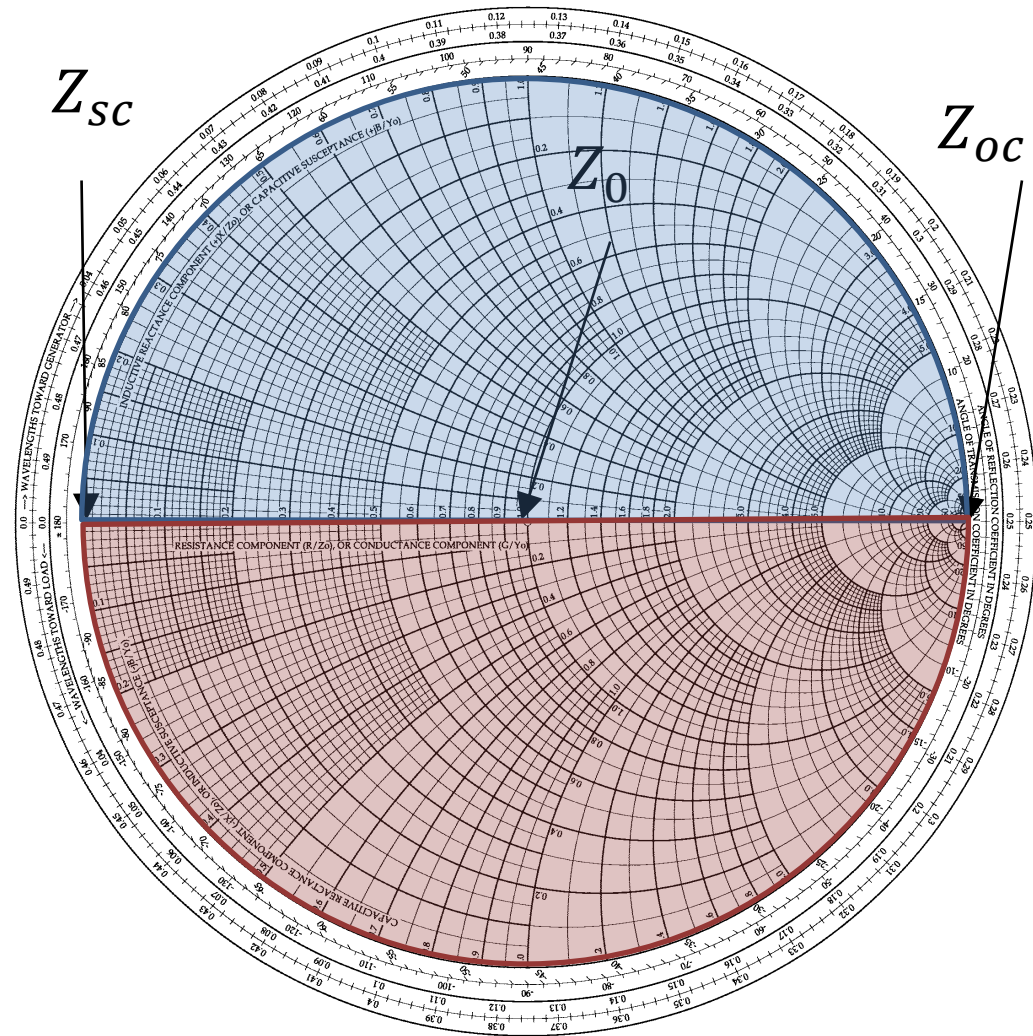
The Smith Chart (2)

- Locate z_L :
 $Z_L = 50 + j100 [\Omega]$
 $Z_0 = 50 [\Omega]$
 $\Rightarrow z'_L = 1 + j2$
- Constant Γ -circle
- Complex Γ :
 $\Gamma = |\Gamma|e^{j\theta_\Gamma}$
- Standing wave ratio:
 $S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$



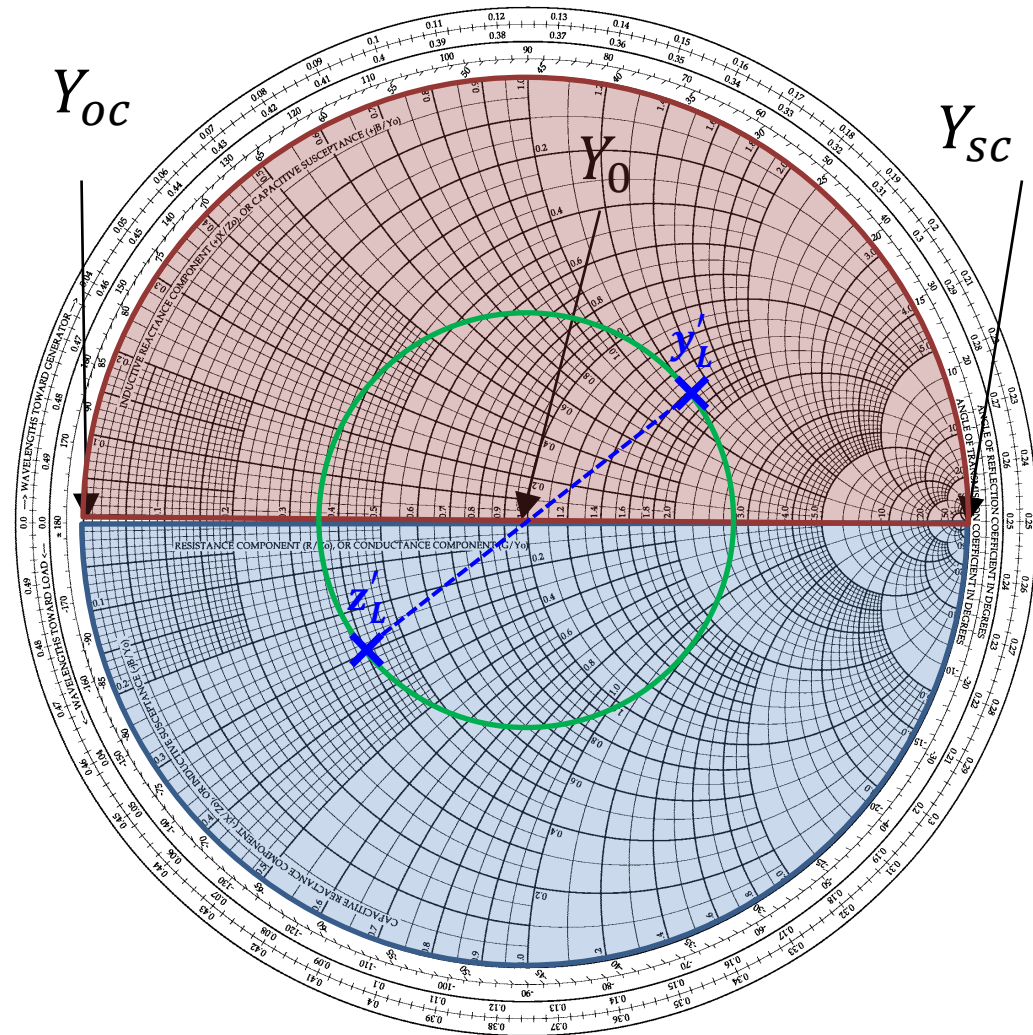
The Smith Chart (3)

- Important points
 - Open Circuit (Z_{oc})
 - Short Circuit (Z_{sc})
 - Matched ($Z = Z_0$)
- Important regions
 - Inductive
 - Capacitive

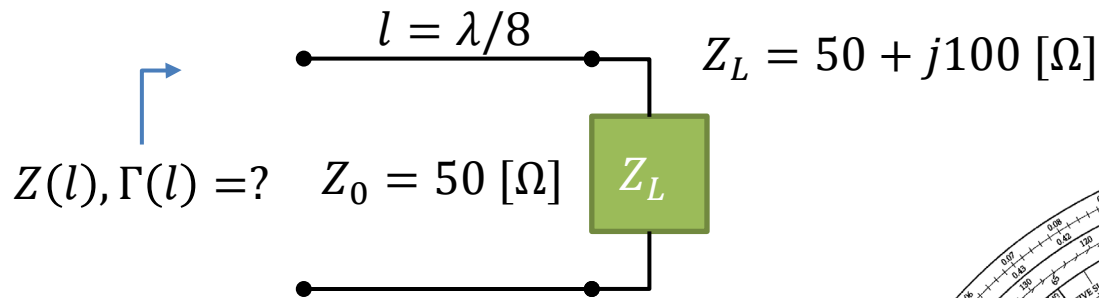


Smith Chart in Admittance Mode

- By transforming z'_L to its diametrically opposite point y'_L , we can use the Smith Chart in admittance mode
- Changes
 - Const. $r \Rightarrow$ Const. g
 - Const. $x \Rightarrow$ Const. b
 - $Z_0 \Rightarrow Y_0$ normalization
 - Capacitive / inductive regions switch
 - OC and SC points switch
 - Travel toward generator is **still clockwise**



Example 1

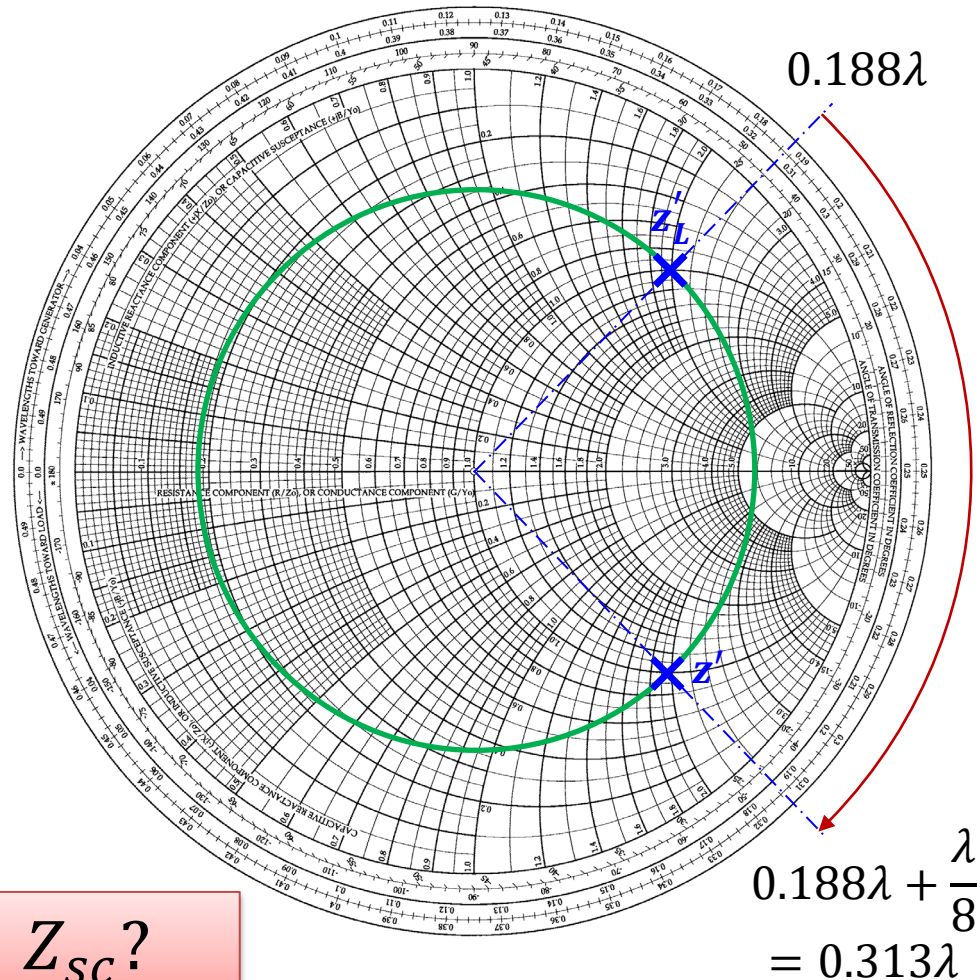


1. Find and plot z'_L
2. Const. $|\Gamma|$ -circle
3. 'Move' $\lambda/8$ **toward generator**.

4. Find $Z(l)$ from z' :

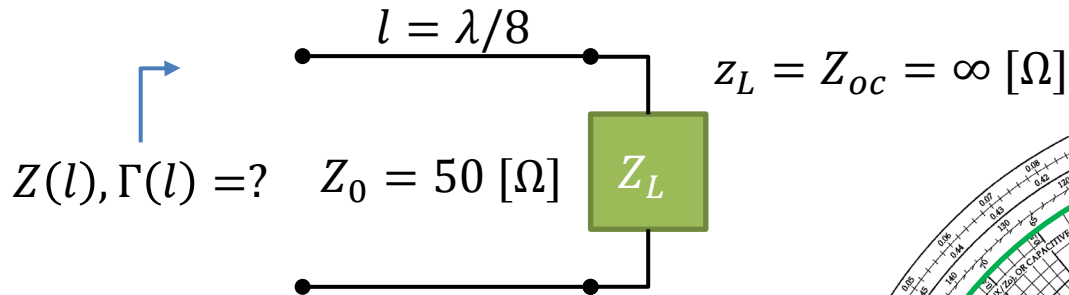
$$Z(l) = 50[\Omega] \times (1 - j2)$$

$$= 50 - j100[\Omega]$$



Q: What if $Z_L = Z_{oc}$? $Z_L = Z_{sc}$?

Example 2



$$\begin{aligned}
 Z(l) &= 50[\Omega] \times (0 - j1) \\
 &= 0 - j50[\Omega]
 \end{aligned}$$

Remarks:

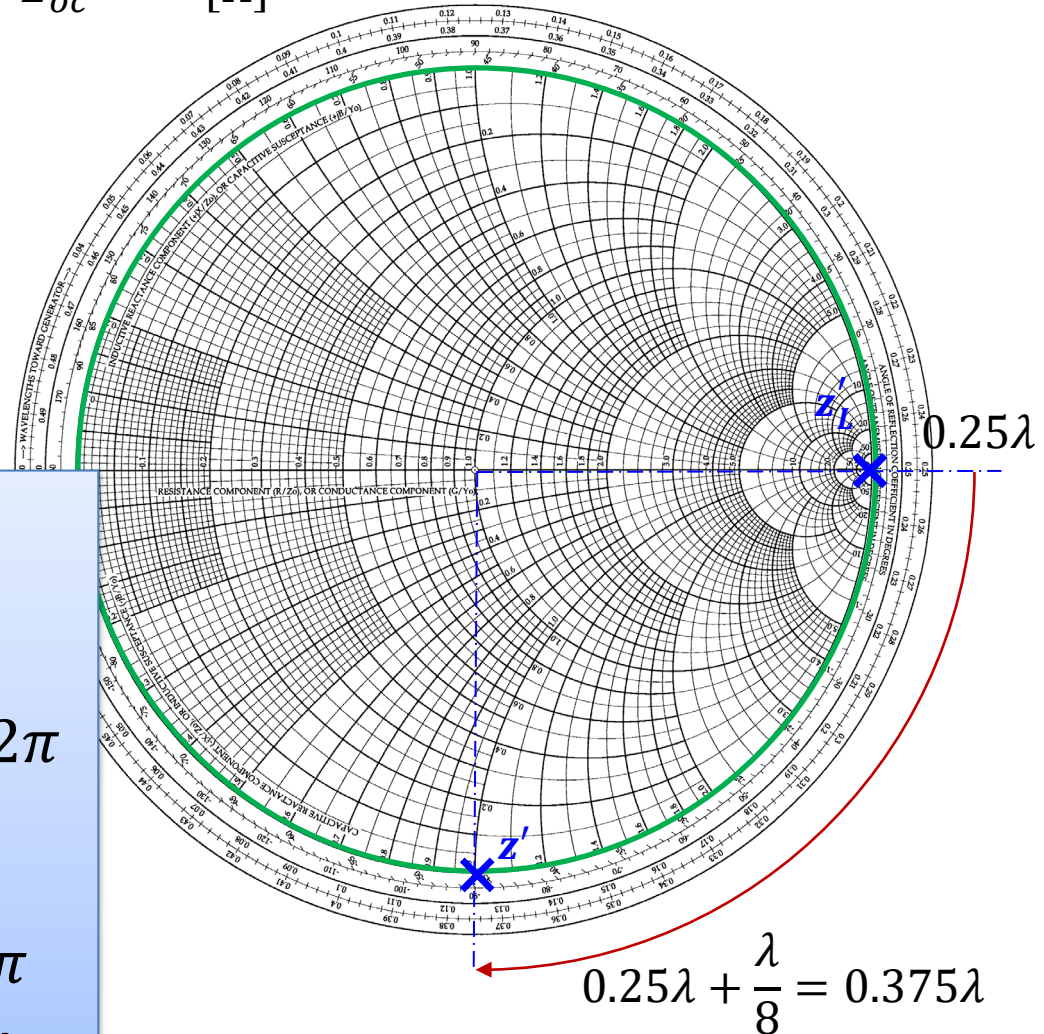
1. If $l = \lambda/2$:

$$\Delta\theta_\Gamma = 2\beta l = 2 \left(\frac{2\pi}{\lambda} \right) \left(\frac{\lambda}{2} \right) = 2\pi$$

2. If $l = \lambda/4$:

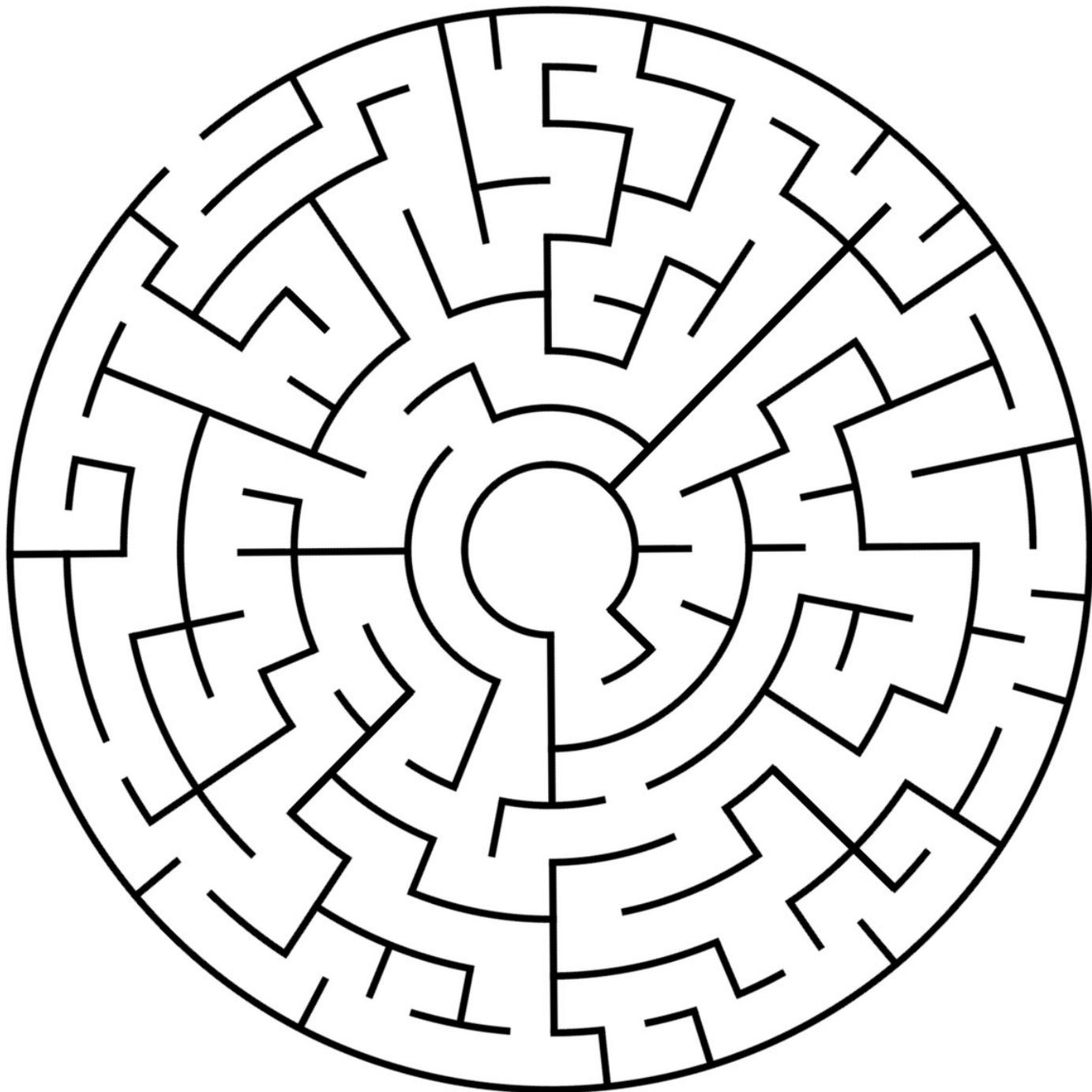
$$\Delta\theta_\Gamma = 2\beta l = 2 \left(\frac{2\pi}{\lambda} \right) \left(\frac{\lambda}{4} \right) = \pi$$

(Quarter Wave Transformer)

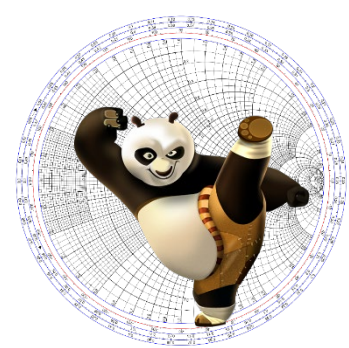


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- Single Stub Tuning



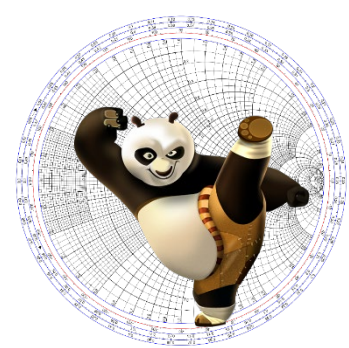




Smith Chart Checklist

Do I know how to....

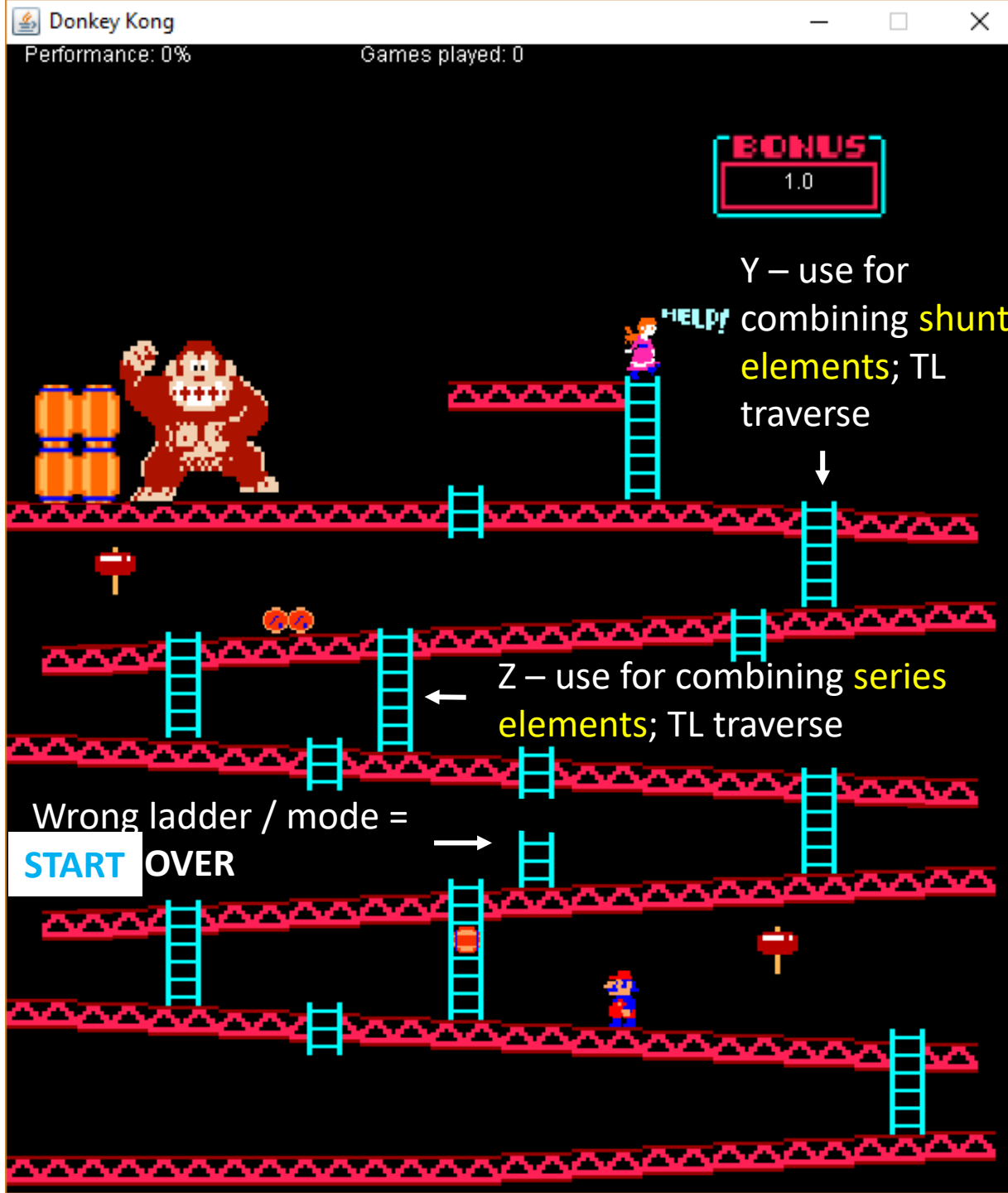
0. Convert between the normalized and unnormalized impedance / admittance?
1. Locate an impedance / admittance / reflection coefficient (Z , Y or Γ) onto the Smith Chart?
2. Locate the special points (OC, SC, match) on the Smith Chart, in impedance and admittance modes?
3. Read, from a point of the Smith Chart, the corresponding Z , Y , Γ or S (VSWR)?
4. Locate the constant Γ , constant r and constant x circles for an impedance point?
5. Locate the constant g and constant b circles for an admittance point?



SC Checklist (Cont'd)

Do I know how to....

6. Transform the Smith Chart between the impedance and admittance modes?
7. Combine Z / Y in series / in parallel on a Smith chart?
8. Transform the $Z / Y / \Gamma$ along a transmission line (toward the source and/or toward the generator)?
9. Transform a group (usually a circle) of $Z / Y / \Gamma$'s along a transmission line (toward the source and/or toward the generator)?
10. Find the electrical length of the TL that will transform a Z / Y from one point to another on the constant Γ circle?
11. Convert between the electrical and physical lengths of the TL?



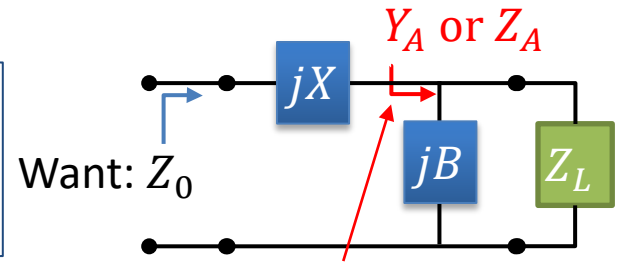
https://www.researchgate.net/figure/Recreation-of-Donkey-Kong_fig1_320987776

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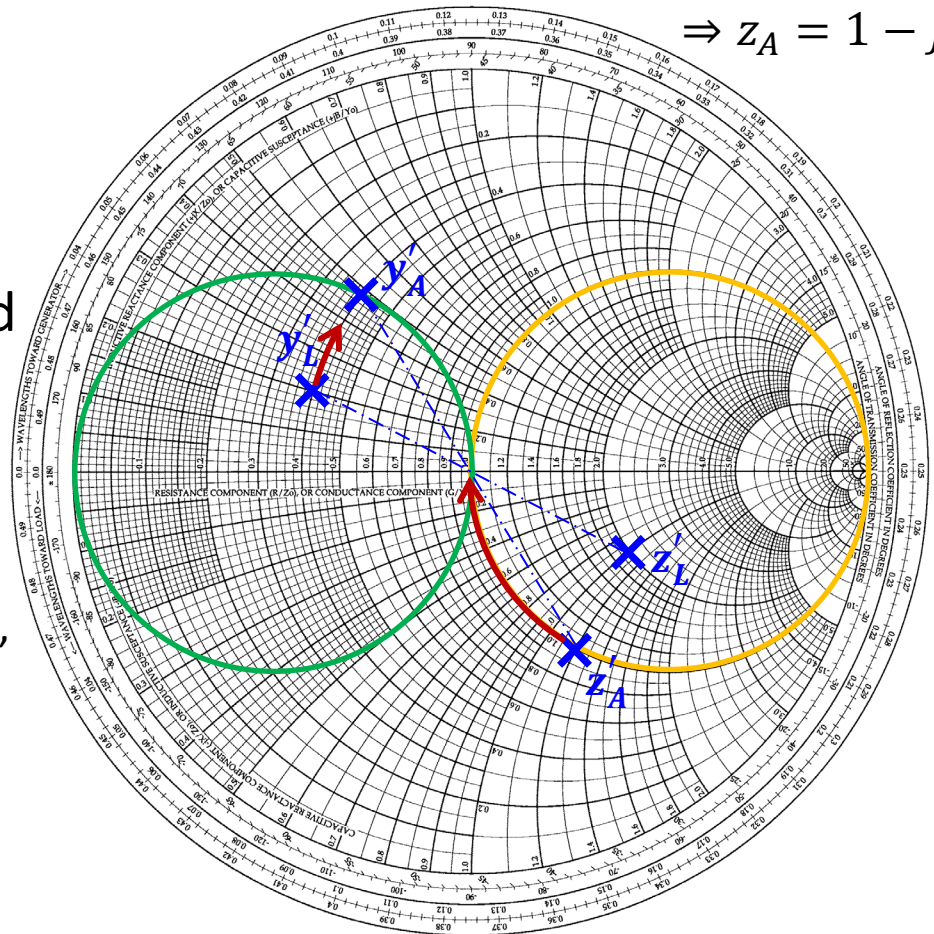
Example 3

$$\begin{aligned} Z_L &= 200 - j100 \text{ } [\Omega] \\ Z_0 &= 100 \text{ } [\Omega] \\ f &= 500 \text{ } [\text{MHz}] \end{aligned}$$



Note: if matched:
 $1 = z_0 = z_A + jx$
 $\Rightarrow z_A = 1 - jx$

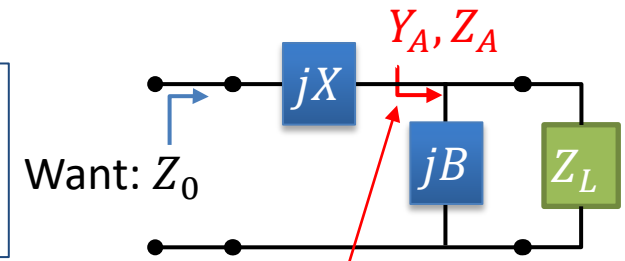
1. Plot $z'_L = 2 - j1$
2. Find y'_L
3. $r=1$ circle (in impedance mode)
4. $r=1$ circle (in admittance mode)
5. Plot y'_A at the intersection of the constant g-circle of y'_L , and the $r=1$ circle (2 sol'ns).
6. Find $y'_A = y'_L + jb \Rightarrow b = 0.29, B = bY_0 = 0.0029 \text{ } [\text{S}]$
7. Find z'_A (Change to impedance mode, yellow circle becomes $r=1$ circle).
8. Read: $z'_A = 1 - j1.22 \Rightarrow x = 1.22, X = xZ_0 = 122 \text{ } [\Omega]$



Example 3

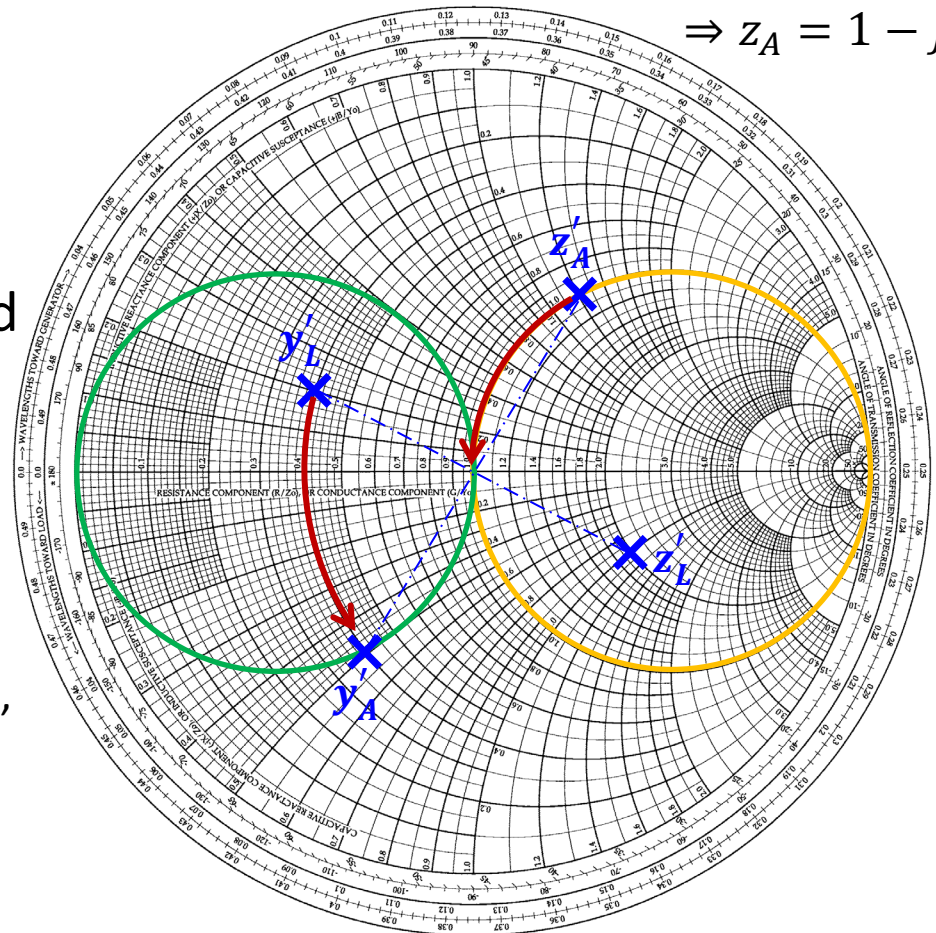
(other solution)

$$\begin{aligned} Z_L &= 200 - j100 \text{ } [\Omega] \\ Z_0 &= 100 \text{ } [\Omega] \\ f &= 500 \text{ } [\text{MHz}] \end{aligned}$$



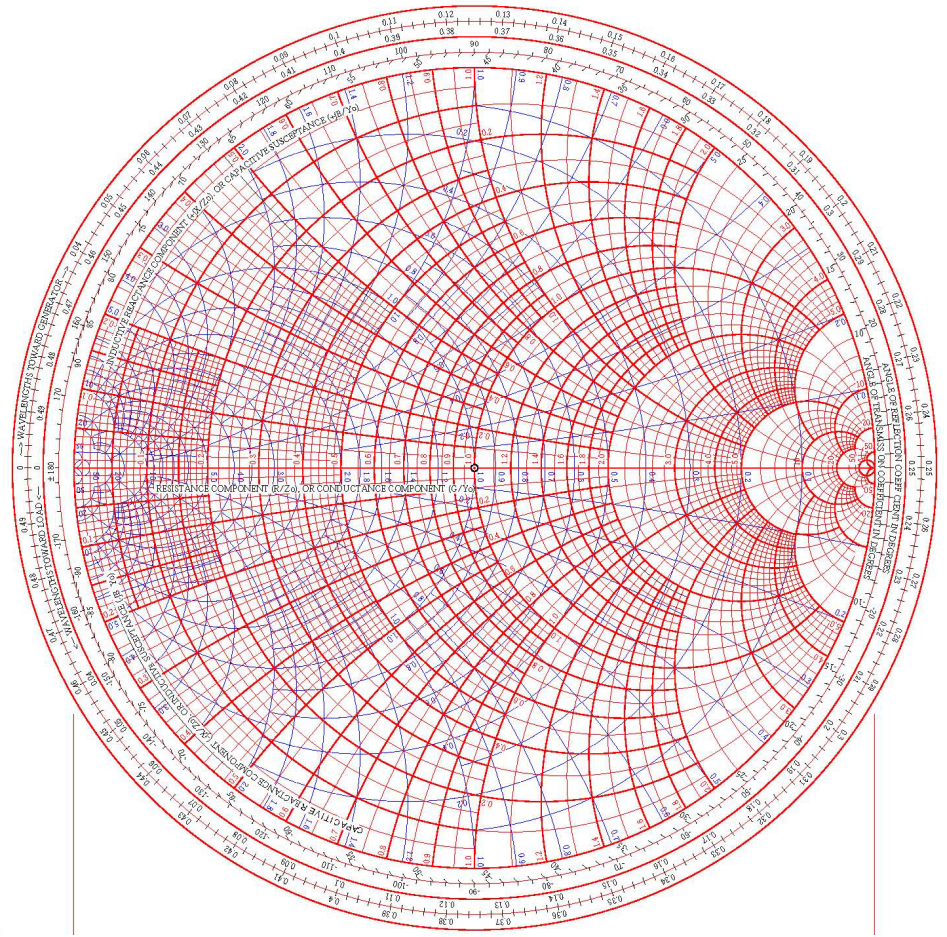
Note: if matched:
 $1 = z_0 = z_A + jx$
 $\Rightarrow z_A = 1 - jx$

1. Plot $z'_L = 2 - j1$
2. Find y'_L
3. $r=1$ circle (in impedance mode)
4. $r=1$ circle (in admittance mode)
5. Plot y'_A at the intersection of the constant g -circle of y'_L , and the $r=1$ circle (2 sol'ns).
6. Find $y'_A = y'_L + jb \Rightarrow b = -0.69, B = bY_0 = -0.0069 \text{ } [\text{S}]$
7. Find z'_A (Change to impedance mode, yellow circle becomes $r=1$ circle).
8. Read: $z'_A = 1 + j1.22 \Rightarrow x = -1.22, X = xZ_0 = -122 \text{ } [\Omega]$



ZY-Smith Chart

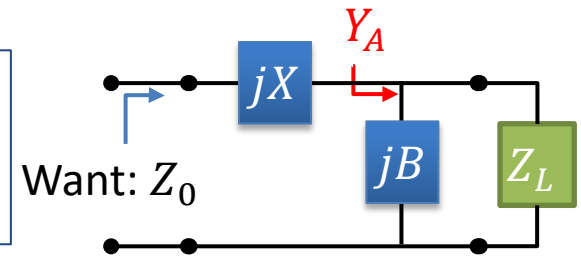
- Has impedance (red) and admittance (blue) lines overlaid.
- Can do series and shunt operations without “flipping” the chart!



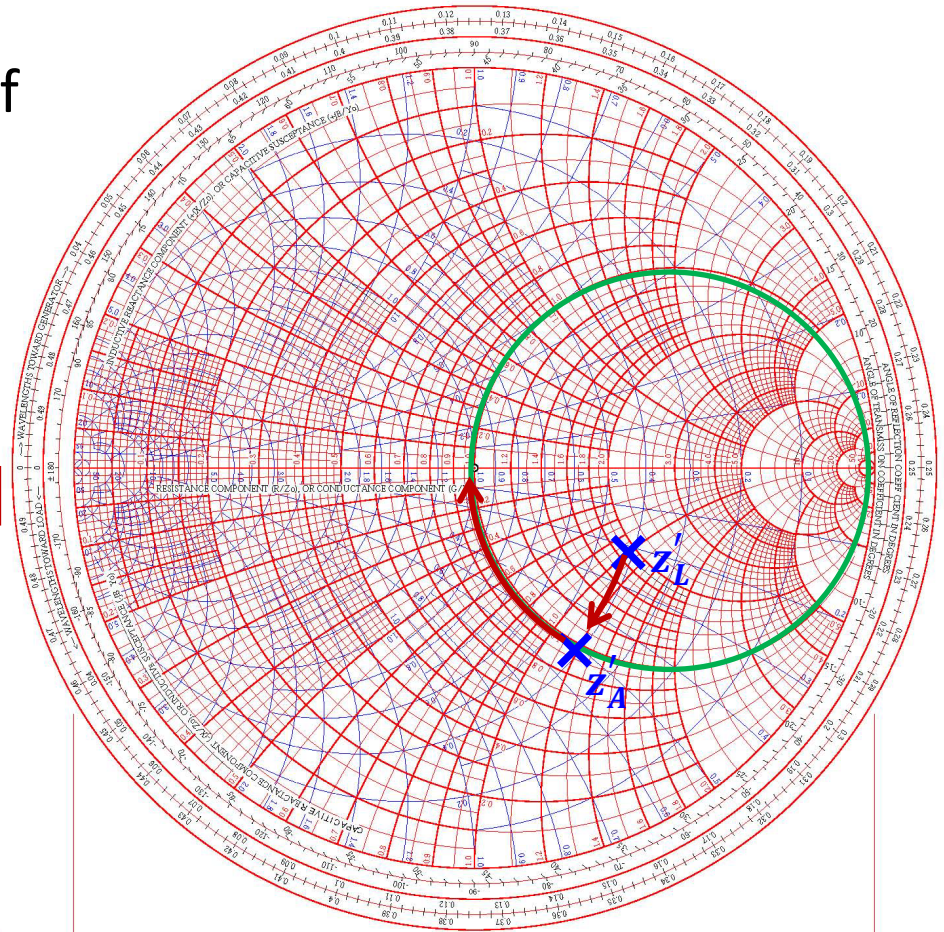
Example 3

(with a ZY-Smith Chart)

$$\begin{aligned} Z_L &= 200 - j100 [\Omega] \\ Z_0 &= 100 [\Omega] \\ f &= 500 [\text{MHz}] \end{aligned}$$



1. Plot $z'_L = 2 - j1$
2. $r=1$ circle
3. Plot z'_A at the intersection of the constant **g-circle** of z'_L (blue lines!), and the $r=1$ circle (2 sol'ns).
4. Find $y'_A = y'_L + jb \Rightarrow b = 0.29, B = bY_0 = 0.0029[\Omega]$
5. Read: $z'_A = 1 - j1.22 \Rightarrow x = 1.22, X = xZ_0 = 122[\Omega]$



Implementing jB and jX with L or C

Implementing jX :

$$X > 0 \Rightarrow \text{use L: } jx'Z_0 = j\omega L \Rightarrow L = \frac{x'Z_0}{\omega}$$

$$X < 0 \Rightarrow \text{use C: } jx'Z_0 = \frac{1}{j\omega C} \Rightarrow C = -\frac{1}{\omega x'Z_0}$$

Implementing jB :

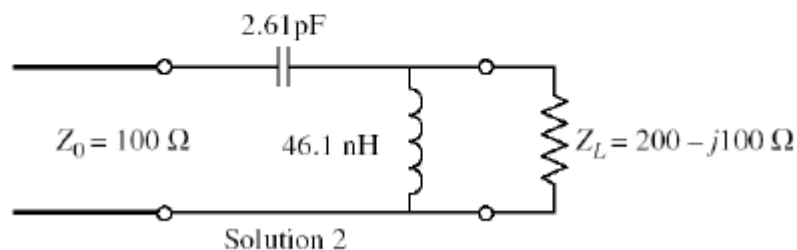
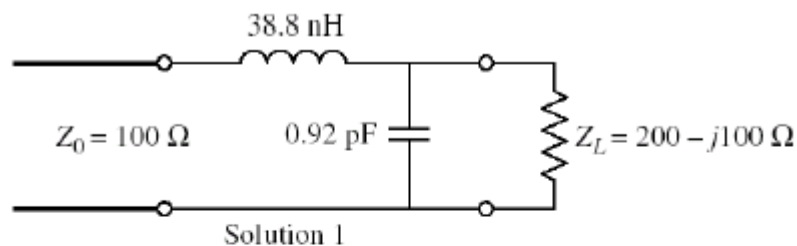
$$B > 0 \Rightarrow \text{use C: } jb'Y_0 = j\omega C \Rightarrow C = \frac{b'}{\omega Z_0}$$

$$B < 0 \Rightarrow \text{use L: } jb'Y_0 = \frac{1}{j\omega L} \Rightarrow L = -\frac{Z_0}{\omega b'}$$

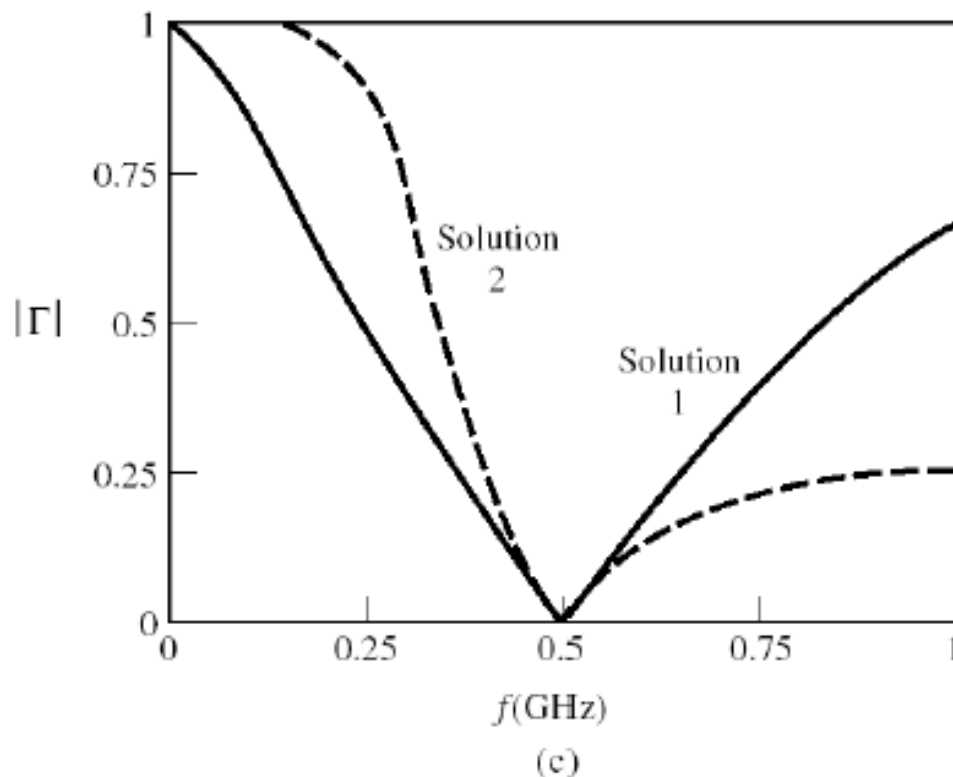
Solution Bandwidths

Figure 5.3b (p. 227)

(b) The two possible L -section matching circuits. (c) Reflection coefficient magnitudes versus frequency for the matching circuits of (b).

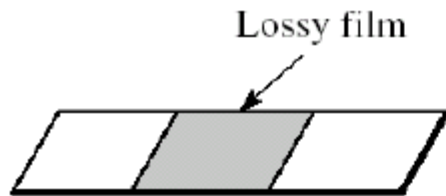


(b)



Microwave Engineering, 3rd Edition by David M. Pozar
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Inductors and Capacitors

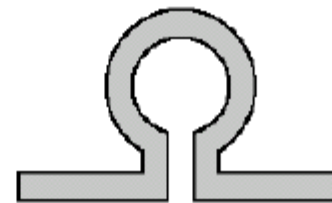


Planar resistor

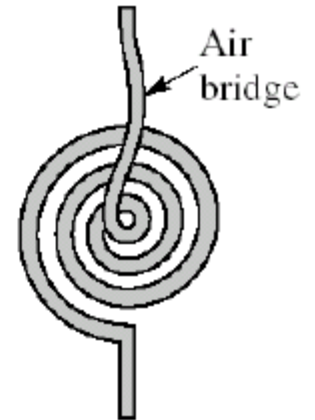
Lossy film



Chip resistor



Loop inductor

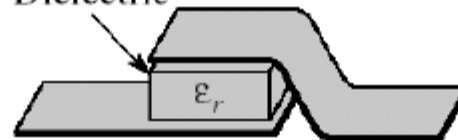


Spiral inductor

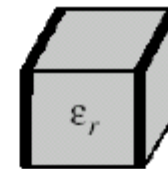


Interdigital
gap capacitor

Dielectric



Metal-insulator-
metal capacitor

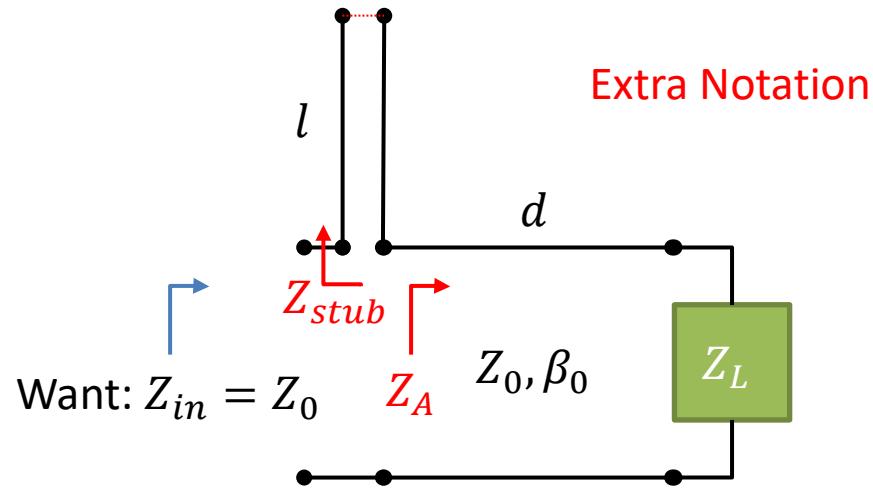


Chip capacitor

Outline

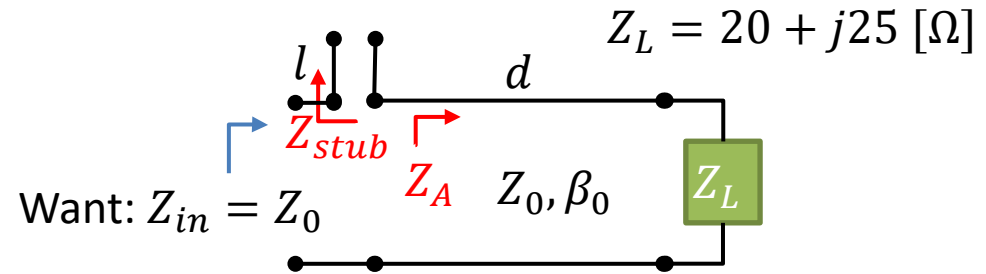
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Conceptual Introduction

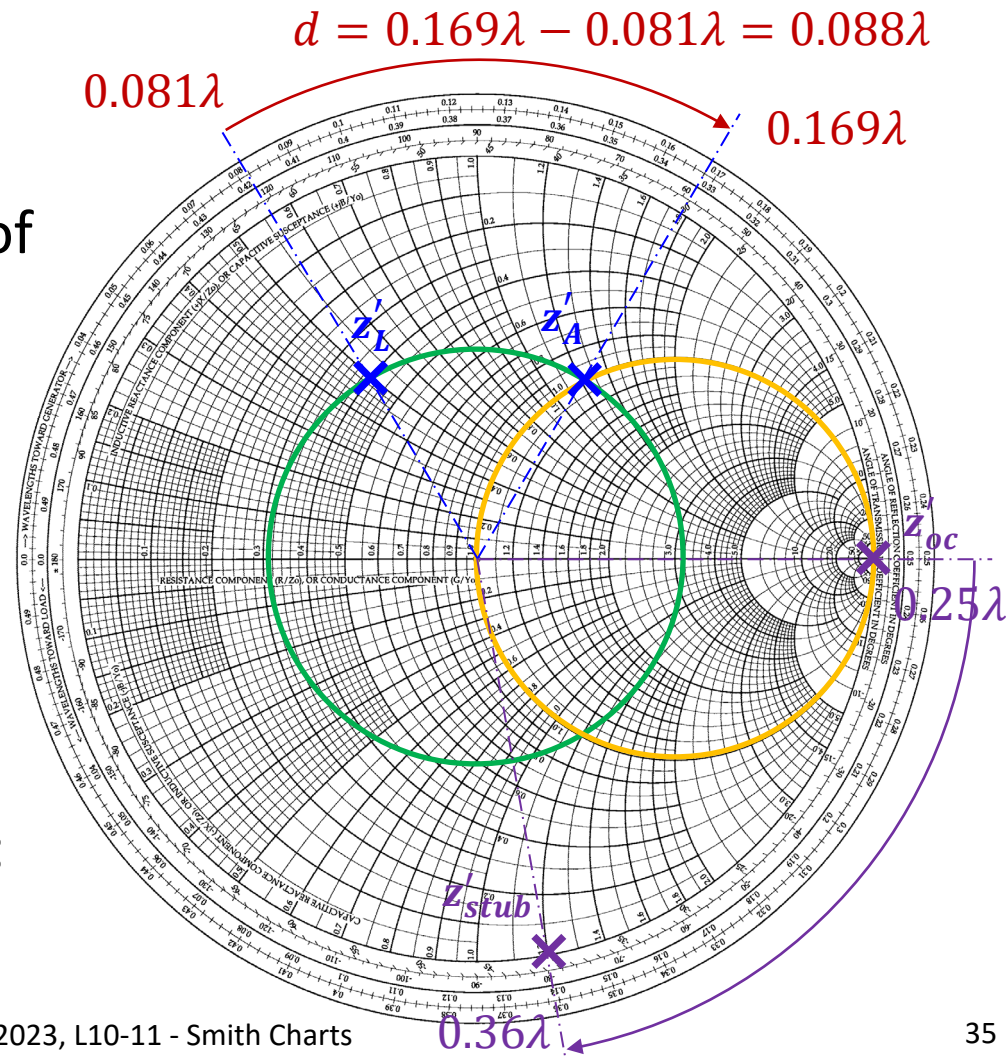


- Introduce a main line of length d , and one stub of length l (usually open- or short-circuited), such that $Z_{in} = Z_0$
- Idea: Choose d s.t. $z'_A = 1 + jx$; choose l s.t. $z'_{in} = 1$.

Example 4



1. Plot $z'_L = 0.4 + j0.5$
2. Constant Γ -circle
3. $r=1$ circle
4. Plot z'_A at the intersection of both circles.
5. Find $d = 0.088\lambda$ from distance travelled.
6. Read: $z'_A = 1 + j1.2$
7. $z'_{in} = z'_A + z'_{stub}$
 $\Rightarrow x'_{stub} = -x'_A = -1.2$
8. Find l s.t. $z'_{stub} = 0 - j1.2$:
 $l = 0.36\lambda - 0.25\lambda = 0.11\lambda$



Single Stub Tuning Remarks

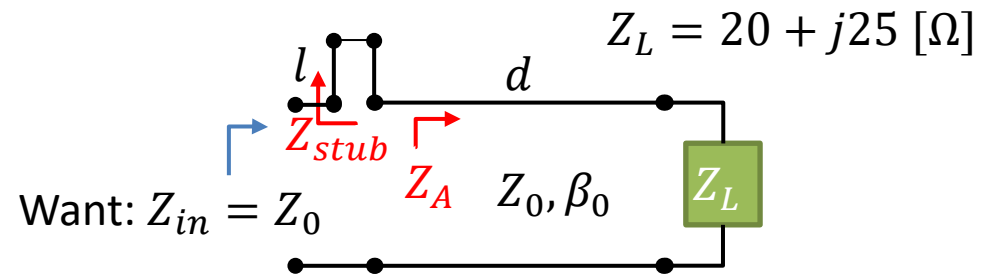
1. One can also calculate the needed length for the stub:

$$1.2 = \cot \beta l \Rightarrow \beta l = 0.7 \Rightarrow l = 0.7 \left(\frac{\lambda}{2\pi} \right) = 0.11\lambda$$

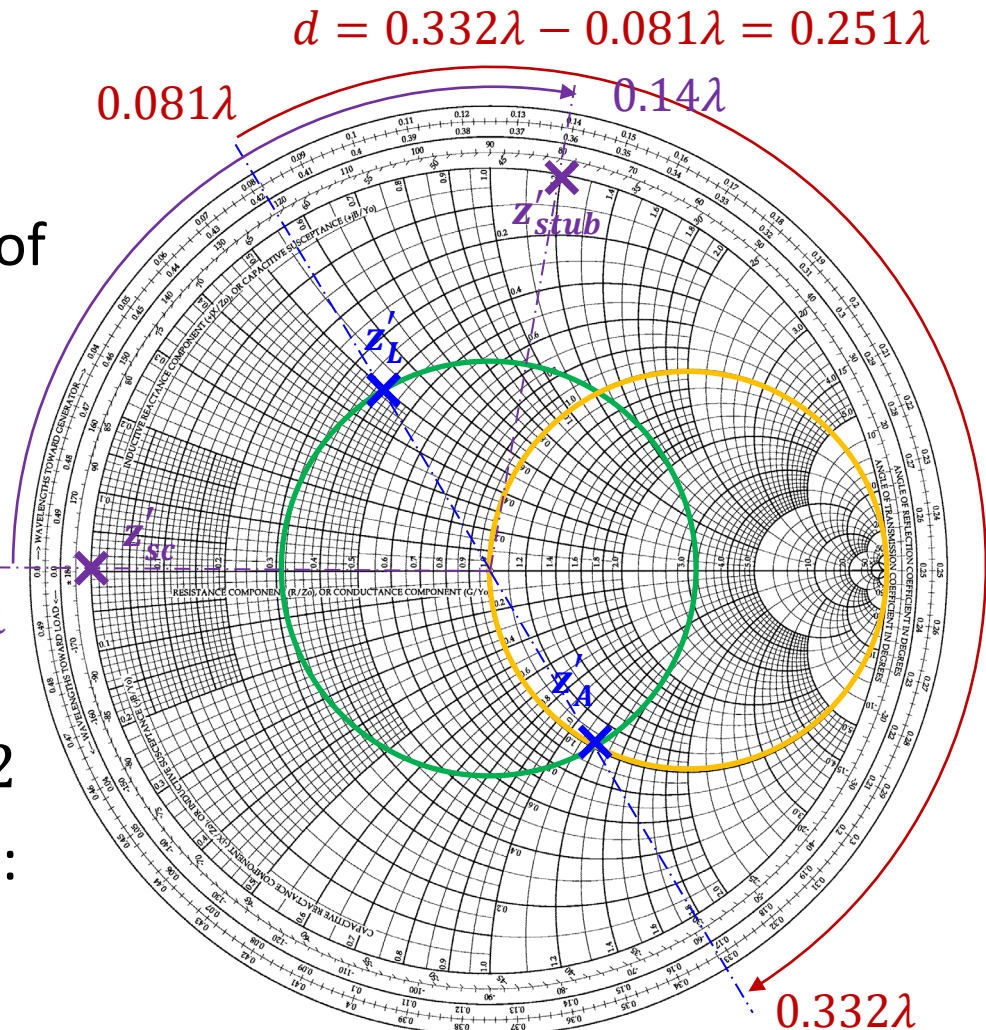
2. There are two intersection points between the Γ -circle and the $r=1$ circle. Either intersection would suffice, but we usually choose the one which leads to less rotation (hence a smaller d).
3. The open-circuited stub can be replaced by a short-circuited stub of length $l + \lambda/4$.
4. l and d can both be increased by integral multiples of $\lambda/2$. (i.e. $l = 0.61\lambda, 1.11\lambda$, etc., would all suffice).
5. In general, the stub:
 - Can be terminated by a reactive load (a capacitor or an inductor)
 - Can be of a different impedance ($Z_{stub} \neq Z_0$)
 - Can be replaced by a reactive load (a capacitor or an inductor)

Example 4

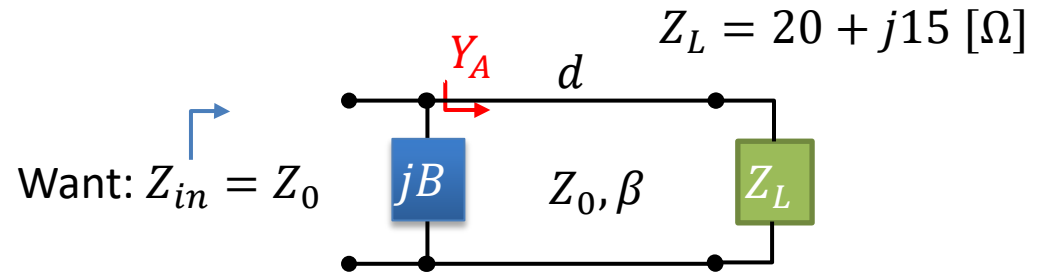
(other solution)



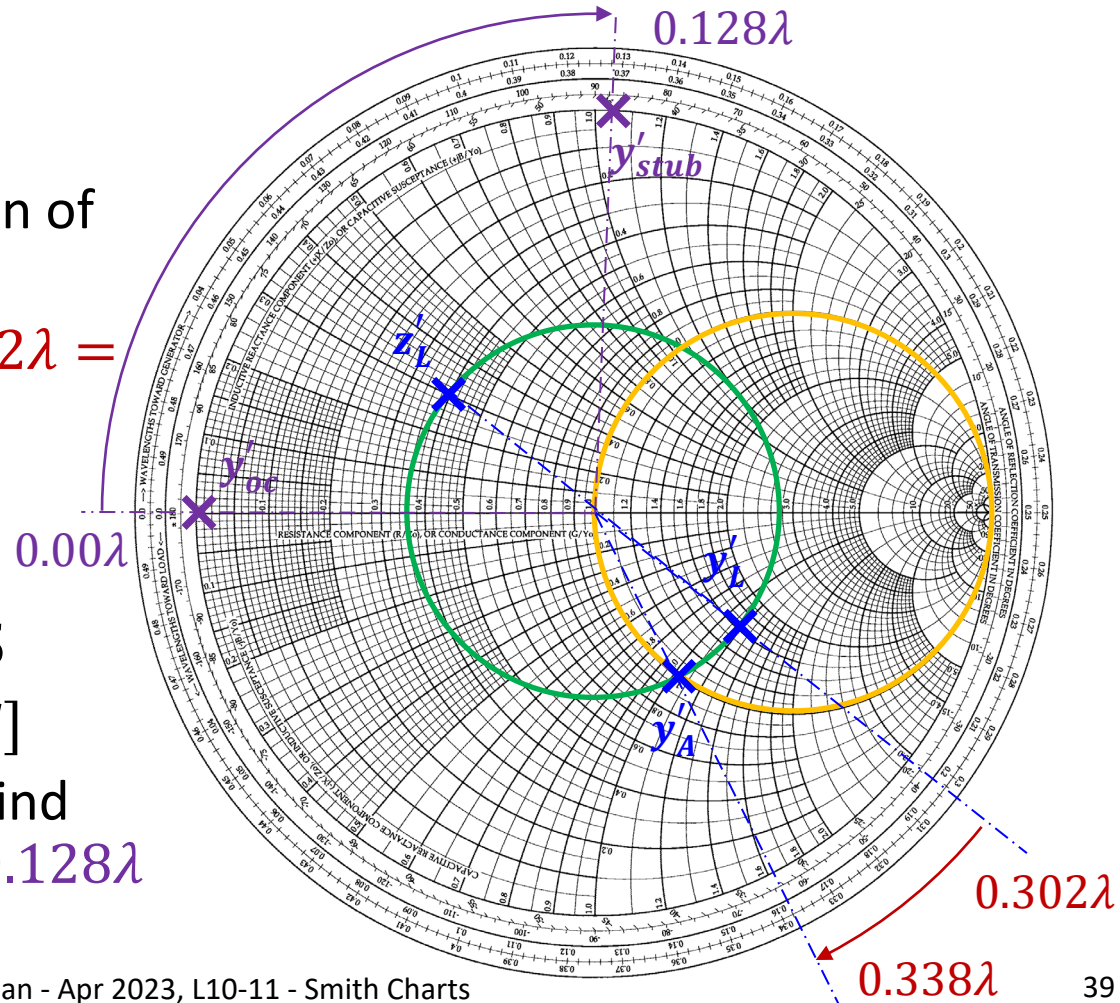
1. Plot $z'_L = 0.4 + j0.5$
2. Constant Γ -circle
3. $r=1$ circle
4. Plot z'_A at the intersection of both circles.
5. Find $d = 0.251\lambda$ from distance travelled.
6. Read: $z'_A = 1 - j1.2$
7. $z'_{in} = z'_A + z'_{stub}$
 $\Rightarrow x'_{stub} = -x'_A = +1.2$
8. Find l s.t. $z'_{stub} = 0 + j1.2$:
 $l = 0.14\lambda - 0.00\lambda = 0.14\lambda$



Example 5



1. Plot $z'_L = 0.4 + j0.3$
2. Constant Γ -circle
3. Find y'_L
4. $g=1$ circle
5. Plot y'_A at the intersection of both circles.
6. Find $d = 0.338\lambda - 0.302\lambda = 0.036\lambda$.
7. Read: $y'_A = 1 - j1.05$
8. $y'_{in} = y'_A + jb$
 $\Rightarrow b = -b'_A = +1.05$
9. Find $B = bY_0 = 0.021[S]$
10. If an open stub is used, find
 $l = 0.128\lambda - 0.00\lambda = 0.128\lambda$



Summary

- In this class, we
 - Introduced the Smith chart
 - Looked simple approaches to impedance matching, with help from the Smith Chart.
 - 2 Matching Methods:
 - L-section
 - Single stub tuning