

EE1001 - Assignment 4 (Answer)

Q1. Use Boolean algebraic theorems to simplify the following Boolean functions:

(i) $f(X, Y, Z) = (\overline{X}Y + XZ)(X + \overline{Y})$

(ii) $f(x, y, z) = \overline{x}y(z + \overline{y}x) + \overline{y}z$

Ans.

(i)

$$\begin{aligned} f(X, Y, Z) &= (\overline{X}Y + XZ)X + (\overline{X}Y + XZ)\overline{Y} \\ &= \overline{X}YX + XZX + \overline{X}Y\overline{Y} + XZ\overline{Y} \\ &= ZX + XZ\overline{Y} \\ &= XZ(1 + \overline{Y}) \\ &= XZ \end{aligned}$$

(ii)

$$\begin{aligned} f(x, y, z) &= \overline{x}y(z + \overline{y}x) + \overline{y}z \\ &= \overline{x}yz + \overline{x}y\overline{y}x + \overline{y}z \\ &= \overline{x}yz + \overline{y}z \\ &= z(\overline{x}y + \overline{y}) \\ &= z(\overline{x} + \overline{y}) \\ &= \overline{x}z + \overline{y}z \end{aligned}$$

Q2. Find the minterms and maxterms of the following functions with K-map:

(i) $f(A, B, C) = \overline{B}C + A\overline{B}$

(ii) $f(A, B, C, D) = \overline{B}\overline{D} + \overline{B}CD + A\overline{B}\overline{C}$

(iii) $f(A, B, C, D) = (A + B + D)(A + \overline{B} + C)(B + C + D)(\overline{A} + \overline{C} + \overline{D})$

Ans.

(i) $f(A, B, C) = \overline{B}C + A\overline{B}$

	AB			
C	00	01	11	10
0				1
1	1			1

$$\begin{aligned} f(A, B, C) &= \sum m(1, 4, 5) \\ &= \overline{A}\overline{B}C + A\overline{B}\overline{C} + A\overline{B}C \\ &= \prod M(0, 2, 3, 6, 7) \\ &= (A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C}) \end{aligned}$$

(ii) $f(A, B, C, D) = \overline{B}\overline{D} + \overline{B}CD + \overline{A}BC$

	AB			
CD	00	01	11	10
00	1			1
01		1	1	1
11				
10	1			1

$$\begin{aligned}
 f(A, B, C, D) &= \sum m(0, 2, 5, 8, 9, 10, 13) \\
 &= \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}BC\overline{D} + \overline{A}BCD + \overline{A}\overline{B}CD + \overline{A}BCD + \overline{A}BCD \\
 &= \prod M(1, 3, 4, 6, 7, 11, 12, 14, 15) \\
 &= (A + B + C + \overline{D}) \cdot (A + B + \overline{C} + \overline{D}) \cdot (A + \overline{B} + C + D) \cdot (A + \overline{B} + \overline{C} + D) \cdot (A + \overline{B} + \overline{C} + \overline{D}) \cdot \\
 &\quad (\overline{A} + B + \overline{C} + \overline{D}) \cdot (\overline{A} + \overline{B} + C + D) \cdot (\overline{A} + \overline{B} + \overline{C} + D) \cdot (\overline{A} + \overline{B} + \overline{C} + \overline{D})
 \end{aligned}$$

(iii) $f(A, B, C, D) = (A + B + D)(A + \overline{B} + C)(B + C + D)(\overline{A} + \overline{C} + \overline{D})$

	AB			
CD	00	01	11	10
00	0	0		0
01		0		
11			0	0
10	0			

$$\begin{aligned}
 f(A, B, C, D) &= \prod M(0, 2, 4, 5, 8, 11, 15) \\
 &= (A + B + C + D) \cdot (A + B + \overline{C} + D) \cdot (A + \overline{B} + C + D) \cdot (A + \overline{B} + C + \overline{D}) \cdot \\
 &\quad (\overline{A} + B + C + D) \cdot (\overline{A} + B + \overline{C} + \overline{D}) \cdot (\overline{A} + \overline{B} + \overline{C} + \overline{D})
 \end{aligned}$$

$$\begin{aligned}
 f(A, B, C, D) &= \sum m(1, 3, 6, 7, 9, 10, 12, 13, 14) \\
 &= \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}BC\overline{D} + \overline{A}BCD + \overline{A}\overline{B}CD + \overline{A}BCD + \overline{A}BCD + \overline{A}BCD + \overline{A}BCD
 \end{aligned}$$

Q3 Find the simplified Boolean functions of the following functions with K-map:

(c) $f(a, b, c, d) = \sum m(0, 1, 4, 6, 7, 8, 11, 12, 15)$

	ab			
cd	00	01	11	10
00	1	1	1	1
01	1			
11		1	1	1
10		1		

$$f(a, b, c, d) = \overline{c}\overline{d} + \overline{a}\overline{b}\overline{c} + \overline{a}bc + acd$$

(d) $f(a,b,c,d) = \Sigma m(0,2,4,7,9,13,14,15)$

	ab			
cd	00	01	11	10
00	1	1		
01			1	1
11		1	1	
10	1		1	

$$f(a,b,c,d) = \overline{a}\overline{b}\overline{d} + \overline{a}\overline{c}\overline{d} + bcd + \overline{a}cd + abc$$

(c) $f(a,b,c,d) = \Sigma m(3,7,8,9,11,13) + \Sigma d(2,10,14)$

	ab			
cd	00	01	11	10
00				1
01			1	1
11	1	1		1
10	x		x	x

$$f(a,b,c,d) = a\overline{b} + \overline{a}\overline{c}d + \overline{a}cd$$

(d) $f(a,b,c,d) = \prod M(1,4,5,6,9,10,11) \cdot \prod d(12,13)$

	ab			
cd	00	01	11	10
00		0	X	
01	0	0	X	0
11				0
10		0		0

$$f(a,b,c,d) = (c + \overline{d}) \cdot (a + \overline{b} + d) \cdot (\overline{a} + b + \overline{c})$$

- Q4. Determine the Boolean function of the circuit in Figure Q4, and simplify it. With the aid of DeMorgan's Theorem, implement the circuit with (i) NAND gates, and (ii) NOR gates, only. Draw the circuit diagrams.

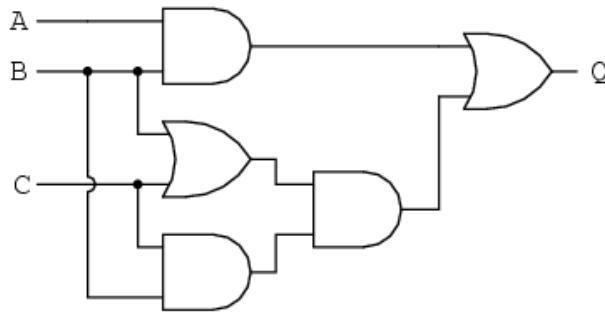
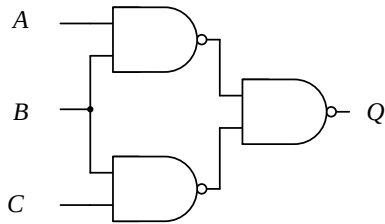


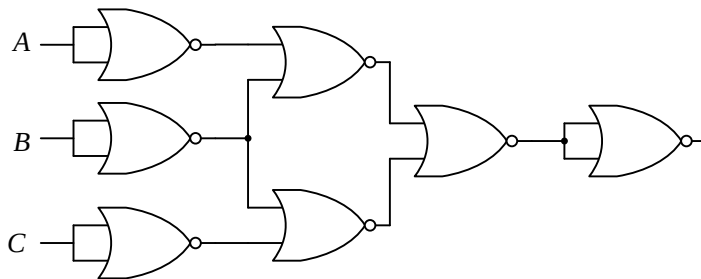
Figure Q4

$$Q(A, B, C) = AB + BC(B + C) = AB + BC$$

$$Q(A, B, C) = \overline{\overline{AB} + \overline{BC}} = \overline{\overline{AB}} \cdot \overline{\overline{BC}}$$



$$Q(A, B, C) = \overline{\overline{AB} + \overline{BC}} = \overline{\overline{A} + \overline{B}} + \overline{\overline{B} + \overline{C}} = \overline{\overline{\overline{A} + \overline{B}}} + \overline{\overline{\overline{B} + \overline{C}}} = \overline{\overline{\overline{A} + \overline{B}} + \overline{\overline{B} + \overline{C}}}$$



Q5. Figure Q5 shows the block diagram of a circuit which has four inputs A, B, C, D and one output Y. It operates according to the following rules:

- (i) If the inputs A, B, C and D have more 0s than 1s, the circuit will generate an odd parity bit for the inputs.
- (ii) If the number of 0s and 1s of the inputs are equal, the machine will generate an even parity for the inputs.
- (iii) If there is only one 0, the output is equal to 1.
- (iv) The output of the remaining case(s) is (are) 1.

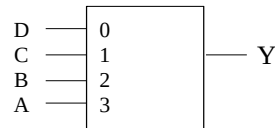


Figure Q9. Block diagram of a 4 inputs to 1 output circuit.

- (a) Tabulate the truth table to describe the function of the circuit.
- (b) Obtain the minimum SOP expression of the circuit from the truth table and design the circuit.

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$$Y(A,B,C,D) = \sum m(7,11,13,14,15)$$

	AB			
CD	00	01	11	10
00				
01			1	
11		1	1	1
10			1	

$$Y(A,B,C,D) = BCD + ACD + ABC + ABD$$