

- Exam Time: 10:00 - 11:59, March 8, 2024
- Once you finish solving the exam, please submit your answer sheet to the TAs.
- Please make sure to write your name and student ID on top of the answer sheet.
- You don't need to return the question sheet.

1. (20 points) Solve the following Differential Equation (DE) using the Exact DE method.

$$(2x - y^2) \cdot dx + xy \cdot dy = 0, \quad \text{where } y = y(x)$$

2. (20 points) (a) Solve the following DE and find the general solution.

$$\frac{dx}{dt} + 2x = \sin(3t), \quad \text{where } x = x(t)$$

- (b) Solve the following Initial Value Problem (IVP) and derive the complete solution.

$$\frac{d^2y}{dx^2} - 2 \cdot \frac{dy}{dx} + 2y = 0, \quad y(0) = 0, \quad y'(0) = 5, \quad \text{where } y = y(x)$$

3. (20 points) Please answer whether the following ODE is Bernoulli equation or not. Also, determine whether it is linear DE. Make sure to explain the answer to get full mark.

(a)

$$y' = 2xy, \quad \text{where } y = y(x)$$

(b)

$$y' + 3ty = 0, \quad \text{where } y = y(t)$$

(c)

$$2y' + ty = y^2, \quad \text{where } y = y(t)$$

(d)

$$y' = 3xy^3 - 2xy, \quad \text{where } y = y(x)$$

(e)

$$y^2 y' - y = x, \quad \text{where } y = y(x)$$

4. (20 points) Solve the following DE and find the general solution. For the Initial Value Problem (IVP), please find the complete solution.

(a)

$$(x+2)^2 \cdot \frac{d^2y}{dx^2} - (x+2) \cdot \frac{dy}{dx} + y = 0, \quad \text{where } y = y(x)$$

(b)

$$\frac{d^3y}{dx^3} + 9 \cdot \frac{dy}{dx} = 0, \quad y(0) = 3, \quad y'(0) = -1, \quad y''(0) = 2, \quad \text{where } y = y(x)$$

(c)

$$9 \cdot \frac{d^2y}{dx^2} - 12 \cdot \frac{dy}{dx} + 4y = 0, \quad \text{where } y = y(x)$$

(d)

$$\frac{d^2y}{dx^2} - 3 \cdot \frac{dy}{dx} - 28y = 0, \quad y(0) = 4, \quad y'(0) = 1, \quad \text{where } y = y(x)$$

5. (20 points) A person borrowed Z_0 amount of funds from a bank that charges a fixed interest rate of r . The borrower's periodical payment to the bank is $Q_0 \cdot t$, where Q_0 is constant, and t is time. Then, the time-varying loan balance $Z(t)$ has the following relationship:

$$dZ = Zr \cdot dt - Q_0 t \cdot dt,$$

where the decrement in loan due to the payments (dZ) is equal to the difference between the total amount with interest ($Zr \cdot dt$) and payment ($Q_0 t \cdot dt$).

(a) What is the initial value condition in this problem?

(b) Please solve the equation on $Z = Z(t)$ and express it explicitly.