

EE1004 Test 1 2022-23 Sem A

1. Six unbiased coins are tossed. What is the probability of getting at most four heads?

[10 marks]

1. Solution = $1 - (1/2)^6 \times [1+6] = 57/64$

2. Suppose that a die is tossed 6 times. What is the probability of getting exactly 4 fives?

[10 marks]

2. Solution:

This is a binomial experiment in which the number of trials is equal to 6, the number of successes is equal to 4, and the probability of success on a single trial is $1/6$ or about 0.167.

Therefore, the binomial probability is:

$$b(4; 6, 0.167) = \{ 6! / [4! (6 - 4)!] \} * (0.167)^4 * (0.833)^2 = 8.0956 \times 10^{-3}$$

3. In a class, there are 10 boys and 16 girls. Four students are selected at random. What is the probability that 2 girls and 2 boys are selected? Note that the number of significant digits after the decimal point should be limited to four during the calculation.

[10 marks]

3. Solution = $6X(16X15x10x9)/(26X25x24x23) = 0.3612$

4. A system is composed of four components, each of which is either working or failed. Assume the probability that a component fails is 20%. Suppose that the system will work if component 1 is working, or if components 2, 3 and 4 are all working. What is the probability that the system works?

[10 marks]

4. Solution: The probability that the system works = $1 - \text{Prob}[\text{component 1 fails}] \times \{1 - \text{Prob}[\text{components 2, 3 and 4 are all working}]\} = 1 - 0.2 \times [1 - 0.8 \times 0.8 \times 0.8] = 0.9024$

5. Suppose that the average times that a viewer watches tigers on YouTube is 8 in two days. What is the probability that a viewer will see more than three tigers one day on YouTube?

[15 marks]

5. Solution: This is a Poisson experiment in which we know the following:

- $\mu = 4$; since 4 tigers are seen per day on average.
- $x = 4, 5, 6, \dots$; since we want to find the likelihood that a viewer will see more than 3 tigers.
- $e = 2.71828$; since e is a constant equal to approximately 2.71828.

To solve this problem, we first need to find the probability that a viewer will see 0, 1, 2, or 3 tigers. Thus, we need to calculate the sum of four probabilities: $P(0; 4) + P(1; 4) + P(2; 4) + P(3; 4)$.

To compute this sum, we use the Poisson formula:

$$\begin{aligned} P(x \leq 3, 4) &= P(0; 4) + P(1; 4) + P(2; 4) + P(3; 4) \\ &= [(e^{-4})(4^0) / 0!] + [(e^{-4})(4^1) / 1!] + [(e^{-4})(4^2) / 2!] + [(e^{-4})(4^3) / 3!] \\ &= [(0.006738)(1) / 1] + [(0.006738)(4) / 1] + [(0.006738)(16) / 2] + [(0.006738)(64) / 6] \end{aligned}$$

$$= 0.4335$$

Thus, the probability of seeing more than 3 tigers is

$$P(x > 3, 4) = 1 - P(x \leq 3, 4) = 1 - 0.4335 = 0.5665.$$

6. Suppose that scores on an IQ test are normally distributed, with a population mean of 110 and a population standard deviation of 16. Suppose 10 people are randomly selected and tested. What is the probability that the average test score in the sample group will be at least 120? Please choose the closest value in the table for the answer. Note that the number of significant digits after the decimal point should be limited to four during the calculation.

[15 marks]

6. Solution:

$$\begin{aligned} z &= [\bar{x} - \mu] / [\delta / \sqrt{n}] \\ &= (120 - 110) / [16 / \sqrt{10}] \\ &= 1.9764 \end{aligned}$$

where $\bar{x}$ is the sample mean, μ is the population mean, δ is the population standard deviation, and n is the sample size.

Now, use the standard normal distribution table and choose the closest value in the standard normal distribution table. We obtain the cumulative probability: 0.9762. Hence, there is a $100\% - 97.62\% = 2.38\%$ chance that the sample average will be at least 120.

7. John was tested positive for COVID-19. We assume that the probability that a person gets COVID-19 is 3%. When a person doesn't get COVID-19, the test correctly predicts negative 80% of the time. When a person actually gets COVID-19, the test incorrectly predicts negative 10% of the time. What is the probability that John gets COVID-19? Note that the number of significant digits after the decimal point should be limited to four during the calculation.

[15 marks]

7. Solution: The sample space is defined by two mutually-exclusive events – a person gets COVID-19, or a person does not get COVID-19. Additionally, a third event occurs when the test predicts positive for COVID-19. Notation for these events appears below.

- Event A_1 . A person gets COVID-19.
- Event A_2 . A person does not get COVID-19.
- Event B . The test predicts positive for COVID-19.

In terms of probabilities, we know the following:

- $P(A_1) = 0.03$
- $P(A_2) = 0.97$
- $P(B | A_1) = 0.9$ [When a person actually gets COVID-19, the test correctly predicts positive 90% of the time.]
- $P(B | A_2) = 0.2$ [When a person doesn't get COVID-19, the test incorrectly predicts positive 20% of the time.]

We want to know $P(A_1 | B)$, the probability that Join gets COVID-19, given that he was tested positive for COVID-19. The answer can be determined from Bayes' theorem, as shown below.

$$\begin{aligned} P(A_1 | B) &= [P(A_1) P(B | A_1)] / [P(A_1) P(B | A_1) + P(A_2) P(B | A_2)] \\ &= [0.03 \times 0.9] / [0.03 \times 0.9 + 0.97 \times 0.2] \\ &= 0.1222 \end{aligned}$$

8. If a random sample of 15 students had a sample mean test score of 90 and a sample standard deviation of test scores 12, find a 90 percent confidence interval estimate for the average score of all students. Note that the number of significant digits after the decimal point should be limited to four during the calculation.

[15 marks]

8. Solution: Degree of Freedom = 14. Use the T-table to find the critical t-score = 1.761. The 90 percent confidence interval estimate is $90 \pm 1.761 (12) / \sqrt{15} = 90 \pm 5.4563$