EE 3109 Applied Electromagnetics
2022/23 Semester B
Week 4 Tutorial Solutions

1. (Ulaby 7.9) An RHC-polarized wave with a modulus of 2 (V/m) is traveling in free space in the negative z-direction. Write the expression for the wave's electric field vector, given that the wavelength is 3 cm.

Solution

First we calculate the values of the relevant wave parameters:

$$\lambda = 3\text{cm} = 0.03\text{m}\,;\;\; f = \frac{c}{\lambda} = \frac{3\times 10^8}{0.03} = 10^{10}\text{ Hz} = 10\text{ GHz}\,;$$
$$k = \frac{2\pi}{\lambda} = 209.4\text{ rad/m} \;\text{ and }\; \omega = 2\pi f = 2\pi\times 10^{10}\text{ rad/s}\,.$$

The modulus of a circularly polarized wave travelling in $\pm z$ direction is given as:

$$\mathbf{E}(z,t) = \sqrt{E_x^2(z,t) + E_y^2(z,t)} = \sqrt{a^2\cos^2(\omega t \mp kz) + a^2\sin^2(\omega t \mp kz)} = a = 2\text{ V/m}$$

For an RHC wave travelling in the positive z direction;

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}a\cos(\omega t - kz) + \hat{\mathbf{y}}a\cos\left(\omega t - kz - {}^{\pi}/_{2}\right)$$

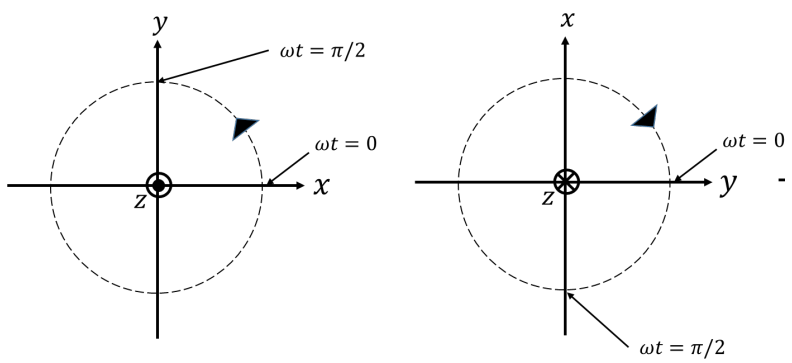
However, now we have the wave propagating in the negative z direction. Refer to Fig. 1 and apply the right-hand rule to see that, when the direction of wave propagation changes from +z to -z, the curl direction of your fingers will also change from counter-clockwise (Fig. 1a) to clockwise (Fig. 1b). One way to affect this change is to transform into a new co-ordinate system $(x', y', z') = (y, x, -z)$. Here the primed co-ordinates represent the new system. Note that (x', y', z') still form a right-hand triplet as required.

The corresponding RHC wave, travelling in the $+z'$ direction (which is the $-z$ direction), is:

$$\begin{aligned}\mathbf{E}(z,t) &= \hat{\mathbf{x}}'a\cos(\omega t - kz') + \hat{\mathbf{y}}'a\cos\left(\omega t - kz' - {}^{\pi}/_{2}\right)\\ &= \hat{\mathbf{y}}a\cos(\omega t + kz) + \hat{\mathbf{x}}a\cos\left(\omega t + kz - {}^{\pi}/_{2}\right)\\ &= \hat{\mathbf{x}}\,a\sin(\omega t + kz) + \hat{\mathbf{y}}\,a\cos(\omega t + kz)\end{aligned}$$

Substitute in the values of the parameters to get:

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\,2\sin(2\pi\times 10^{10}t + 209.4z) + \hat{\mathbf{y}}\,2\cos(2\pi\times 10^{10}t + 209.4z)\text{ V/m}$$



(a) positive z-direction travel (b) negative z-direction travel

Fig. 1 Locus of **E** versus time for a RHC wave

Q2. Sol'n.

7.10 For a wave characterized by the electric field

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\, a_x \cos(\omega t - kz) + \hat{\mathbf{y}}\, a_y \cos(\omega t - kz + \delta)$$

identify the polarization state, determine the polarization angles (γ, χ), and sketch the locus of $\mathbf{E}(0,t)$ for each of the following cases:

(a) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = 0$

(b) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = 180°$

(c) $a_x = 3$ V/m, $a_y = 3$ V/m, and $\delta = 45°$

(d) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = -135°$

Solution:

$$\psi_0 = \tan^{-1}(a_y/a_x), \quad \text{[Eq. (7.60)]},$$
$$\tan 2\gamma = (\tan 2\psi_0)\cos\delta \quad \text{[Eq. (7.59a)]},$$
$$\sin 2\chi = (\sin 2\psi_0)\sin\delta \quad \text{[Eq. (7.59b)]}.$$

Case	a_x	a_y	δ	ψ_0	γ	χ	Polarization State
(a)	3	4	0	53.13°	53.13°	0	Linear
(b)	3	4	180°	53.13°	−53.13°	0	Linear
(c)	3	3	45°	45°	45°	22.5°	Left elliptical
(d)	3	4	−135°	53.13°	−56.2°	−21.37°	Right elliptical

(a) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}4\cos(\omega t - kz)$.
(b) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) - \hat{\mathbf{y}}4\cos(\omega t - kz)$.
(c) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}3\cos(\omega t - kz + 45°)$.
(d) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}4\cos(\omega t - kz - 135°)$.

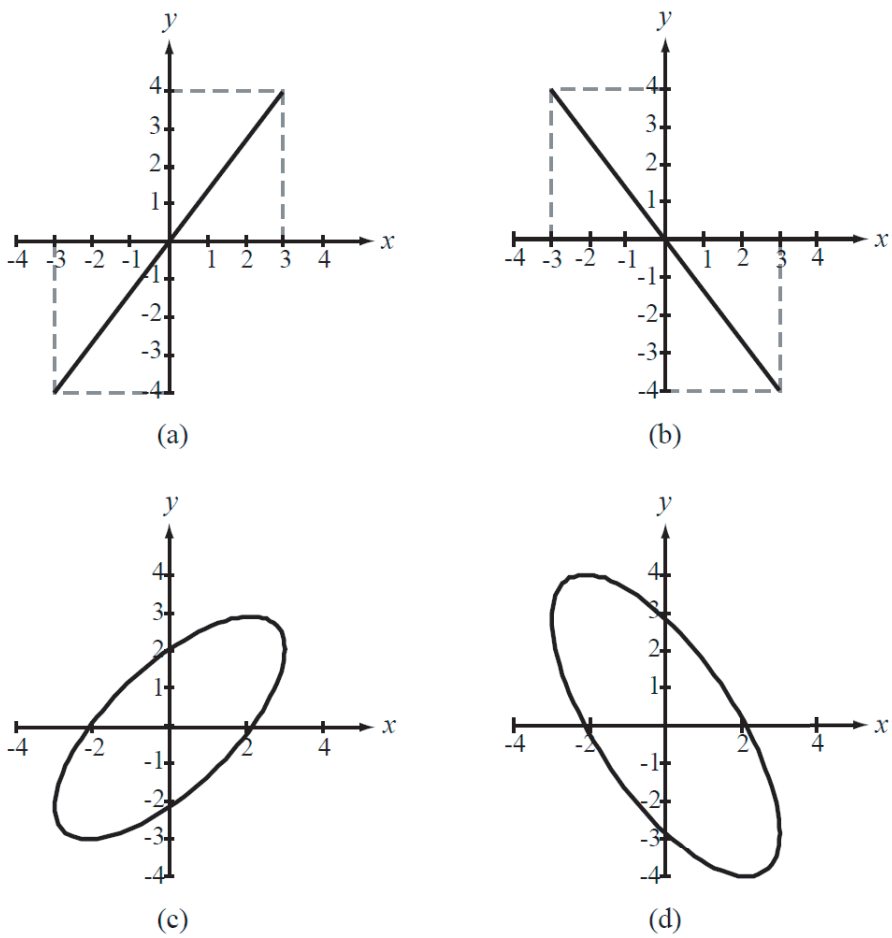


Figure P7.10 Plots of the locus of $\mathbf{E}(0,t)$.

Q3. Sol'n.

7.15 A linearly polarized plane wave of the form $\widetilde{\mathbf{E}} = \hat{\mathbf{x}} a_x e^{-jkz}$ can be expressed as the sum of an RHC polarized wave with magnitude a_{R}, and an LHC polarized wave with magnitude a_{L}. Prove this statement by finding expressions for a_{R} and a_{L} in terms of a_x.

Solution:

$$\begin{aligned}
&\widetilde{\mathbf{E}} = \hat{\mathbf{x}} a_x e^{-jkz}, \\
\text{RHC wave:}\quad &\widetilde{\mathbf{E}}_{\mathrm{R}} = a_{\mathrm{R}}(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{-j\pi/2}) e^{-jkz} = a_{\mathrm{R}}(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{-jkz}, \\
\text{LHC wave:}\quad &\widetilde{\mathbf{E}}_{\mathrm{L}} = a_{\mathrm{L}}(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{j\pi/2}) e^{-jkz} = a_{\mathrm{L}}(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) e^{-jkz}, \\
&\widetilde{\mathbf{E}} = \widetilde{\mathbf{E}}_{\mathrm{R}} + \widetilde{\mathbf{E}}_{\mathrm{L}}, \\
&\hat{\mathbf{x}} a_x = a_{\mathrm{R}}(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) + a_{\mathrm{L}}(\hat{\mathbf{x}} + j\hat{\mathbf{y}}).
\end{aligned}$$

By equating real and imaginary parts, $a_x = a_{\mathrm{R}} + a_{\mathrm{L}}$, $0 = -a_{\mathrm{R}} + a_{\mathrm{L}}$, or $a_{\mathrm{L}} = a_x/2$, $a_{\mathrm{R}} = a_x/2$.