

## EE3124 Tutorial 3 (Solution)

### AC Machine Fundamentals

Name:

Student No.:

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**Q1** - Why does switching the current flows in any two phases reverse the direction of rotation of a three-phase AC motor?

#### Solution

Switched two phases of current then gives the net magnetic flux density in the stator:

$$B_{\text{net}} = B_M \sin \omega t \angle 0^\circ + B_M \sin (\omega t - 240^\circ) \angle 120^\circ + B_M \sin (\omega t - 120^\circ) \angle 240^\circ$$

Each of the three component magnetic fields can now be broken down into its x and y components:

$$\begin{aligned} B_{\text{net}}(t) &= B_M \sin \omega t \hat{x} - [0.5 B_M \sin(\omega t - 240^\circ)] \hat{x} + \left[ \frac{\sqrt{3}}{2} B_M \sin(\omega t - 240^\circ) \right] \hat{y} \\ &\quad - [0.5 B_M \sin(\omega t - 120^\circ)] \hat{x} - \left[ \frac{\sqrt{3}}{2} B_M \sin(\omega t - 120^\circ) \right] \hat{y} \\ B_{\text{net}}(t) &= [B_M \sin \omega t - 0.5 B_M \sin(\omega t - 240^\circ) - 0.5 B_M \sin(\omega t - 120^\circ)] \hat{x} \\ &\quad + \left[ \frac{\sqrt{3}}{2} B_M \sin(\omega t - 240^\circ) - \frac{\sqrt{3}}{2} B_M \sin(\omega t - 120^\circ) \right] \hat{y} \end{aligned}$$

Using the angle-addition trigonometric identities,

$$\begin{aligned} B_{\text{net}}(t) &= \left[ B_M \sin \omega t + \frac{1}{4} B_M \sin \omega t \right. \\ &\quad \left. - \frac{\sqrt{3}}{4} B_M \cos \omega t + \frac{1}{4} B_M \sin \omega t + \frac{\sqrt{3}}{4} B_M \cos \omega t \right] \hat{x} \\ &\quad + \left[ -\frac{\sqrt{3}}{4} B_M \sin \omega t + \frac{3}{4} B_M \cos \omega t + \frac{\sqrt{3}}{4} B_M \sin \omega t + \frac{3}{4} B_M \cos \omega t \right] \hat{y} \end{aligned}$$

$$B_{\text{net}}(t) = [1.5 B_M \sin \omega t] \hat{x} + [1.5 B_M \cos \omega t] \hat{y}$$

The magnetic field rotates in a clock-wise direction.

Note:

$$B_{\text{net}} = B_M \sin \omega t \angle 0^\circ + B_M \sin (\omega t - 120^\circ) \angle 120^\circ + B_M \sin (\omega t - 240^\circ) \angle 240^\circ$$

$$B_{\text{net}}(t) = [1.5 B_M \sin \omega t] \hat{x} - [1.5 B_M \cos \omega t] \hat{y}$$

**Q2** - A three-phase  $\Delta$ - connected synchronous generator is rotating at 1800 rpm. The rotor peak magnetic field is 0.3 T. The stator diameter of the generator is 0.3 m and the coil length is 0.3 m. Each coil has 30 turns. What is the rms terminal voltage of this generator?

**Solution**

Flux in the generator:  $\phi = \text{area} \cdot B = 0.3 \times 0.3 \times 0.3 = 0.027 \text{ Wb}$

Velocity of the rotor:  $\omega = 2\pi f = 2\pi \frac{1800}{60} = 188.4 \text{ rad/s}$

The peak value of the induced voltage:  $\hat{E} = N\phi\omega = 30 \times 0.027 \times 188.4 = 152.6 \text{ V}$

The generator is  $\Delta$ - connected. phase voltage = line voltage =  $\frac{152.6}{\sqrt{2}} = 107.9 \text{ V}$

**Q3** - Use the correct word from the word bank to complete the sentence.

|           |            |           |           |                |       |                |
|-----------|------------|-----------|-----------|----------------|-------|----------------|
| increases | four       | amplifies | rotor     | output voltage | two   | output current |
| torques   | attenuates | equal to  | two-pole  | inductance     | four  | times          |
| stator    | vibration  | resonance | four-pole | reduces        | twice |                |

i) Stator windings fractional-pitch \_\_\_\_\_ the magnitude of the output voltage slightly, but \_\_\_\_\_ the harmonic components of voltage drastically, resulting in a much smoother \_\_\_\_\_ from the machine.

ii) In a \_\_\_\_\_ pole machine, the mechanical speed of rotation of the magnetic field in revolutions per second is \_\_\_\_\_ the electric frequency in hertz.

iii) A generator with \_\_\_\_\_ winding, the electrical frequency of the current is \_\_\_\_\_ the mechanical frequency of rotation.

iv) AC machines differ from dc machines in that their armature windings are almost always located on the \_\_\_\_\_, while their field windings are located on the \_\_\_\_\_.

v) The distributed windings induce harmonics in the generated voltage of ac machine. The interaction of stator and rotor slot harmonics produces parasitic \_\_\_\_\_. Thus, they introduce \_\_\_\_\_ and noise in the machine.

**Solution**

i) Stator windings fractional-pitch reduces the magnitude of the output voltage slightly, but attenuates the harmonic components of voltage drastically, resulting in a much smoother output voltage from the machine.

ii) In a two pole machine, the mechanical speed of rotation of the magnetic field in revolutions per second is equal to the electric frequency in hertz.

- iii) A generator with four-pole winding, the electrical frequency of the current is twice the mechanical frequency of rotation.
- iv) AC machines differ from dc machines in that their armature windings are almost always located on the stator, while their field windings are located on the rotor.
- v) The distributed windings induce harmonics in the generated voltage of ac machine. The interaction of stator and rotor slot harmonics produces parasitic torques. Thus, they introduce vibration and noise in the machine.

**Q4** - The first ac power system in the USA ran at a frequency of 133 Hz. If the ac power for this system were produced by a 4-pole generator, how fast would the shaft of the generator have to rotate?

**Solution**

The equation for the speed of the shaft is

$$n_{sm} = \frac{120 f_{se}}{P} = \frac{120(133 \text{ Hz})}{4} = 3990 \text{ r/min}$$

**Q5** - Develop a table showing the speed of magnetic field rotation in ac machines of 2, 4, 6, 8, 10, 12, and 14 poles operating at frequencies of 50, 60, and 400 Hz.

**Solution**

The equation relating the speed of magnetic field rotation to the number of poles and electrical frequency is

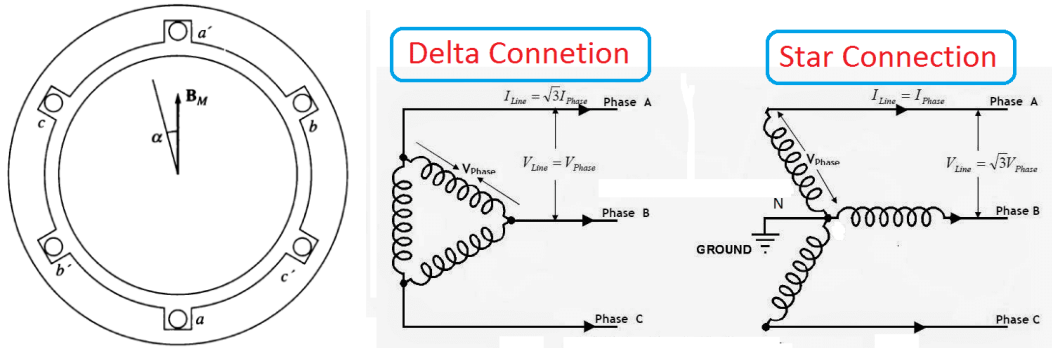
$$n_{sm} = \frac{120 f_{se}}{P}$$

The resulting table is

| Number of Poles | $f_e = 50 \text{ Hz}$ | $f_e = 60 \text{ Hz}$ | $f_e = 400 \text{ Hz}$ |
|-----------------|-----------------------|-----------------------|------------------------|
| 2               | 3000 r/min            | 3600 r/min            | 24000 r/min            |
| 4               | 1500 r/min            | 1800 r/min            | 12000 r/min            |
| 6               | 1000 r/min            | 1200 r/min            | 8000 r/min             |
| 8               | 750 r/min             | 900 r/min             | 6000 r/min             |
| 10              | 600 r/min             | 720 r/min             | 4800 r/min             |
| 12              | 500 r/min             | 600 r/min             | 4000 r/min             |
| 14              | 428.6 r/min           | 514.3 r/min           | 3429 r/min             |

**Q6** - The following information is known about the simple two-pole generator in following figure. The peak flux density of the rotor magnetic field is 0.2 T, and the mechanical rate of rotation of the shaft is 3600 r/min. The stator diameter of the machine is 0.5 m, its coil length is 0.3 m, and there are 15 turns per coil. The machine is Y-connected.

- What are the three phase voltages of the generator as a function of time?
- What is the rms phase voltage of this generator?
- What is the rms terminal voltage of this generator?



### Solution

The flux in this machine is given by

$$\phi = 2rlB = dIB$$

where  $d$  is the diameter and  $l$  is the length of the coil. Therefore, the flux in the machine is given by

$$\phi = (0.5 \text{ m})(0.3 \text{ m})(0.2 \text{ T}) = 0.03 \text{ Wb}$$

The speed of the rotor is given by

$$\omega = (3600 \text{ r/min})(2\pi \text{ rad})(1 \text{ min}/60 \text{ s}) = 377 \text{ rad/s}$$

- The magnitudes of the peak phase voltages are thus

$$\begin{aligned} E_{\max} &= N_C \phi \omega \\ &= (15 \text{ turns})(0.03 \text{ Wb})(377 \text{ rad/s}) = 169.7 \text{ V} \end{aligned}$$

and the three phase voltages are

$$\begin{aligned} e_{aa'}(t) &= 169.7 \sin 377t \quad \text{V} \\ e_{bb'}(t) &= 169.7 \sin (377t - 120^\circ) \quad \text{V} \\ e_{cc'}(t) &= 169.7 \sin (377t - 240^\circ) \quad \text{V} \end{aligned}$$

- The rms phase voltage of this generator is

$$E_A = \frac{E_{\max}}{\sqrt{2}} = \frac{169.7 \text{ V}}{\sqrt{2}} = 120 \text{ V}$$

- Since the generator is Y-connected,

$$V_T = \sqrt{3}E_A = \sqrt{3}(120 \text{ V}) = 208 \text{ V}$$

**Q7** - A three-phase Y-connected four-pole winding is installed in 24 slots on a stator. There are 40 turns of wire in each slot of the windings. All coils in each phase are connected in series. The flux per pole in the machine is 0.060 Wb, and the speed of rotation of the magnetic field is 1800 r/min.

(a) What is the frequency of the voltage produced in this winding?

(b) What are the resulting phase and terminal voltages of this stator?

**Solution**

(a) The frequency of the voltage produced in this winding is

$$f_{se} = \frac{n_{sm}P}{120} = \frac{(1800 \text{ r/min})(4 \text{ poles})}{120} = 60 \text{ Hz}$$

(b) There are 24 slots on this stator, with 40 turns of wire per slot. Since this is a four-pole machine, there are two sets of coils (in 8 slots) associated with each phase. The voltage in the coils in *one pair* of slots is

$$E_A = \sqrt{2}\pi N_c \phi f = \sqrt{2}\pi(40 \text{ t})(0.060 \text{ Wb})(60 \text{ Hz}) = 640 \text{ V}$$

There are eight slots associated with each phase, and the all of the coils in a slot are connected in series, so the total phase voltage is

$$V_\phi = 8(640 \text{ V}) = 5120 \text{ V}$$

Since the machine is Y-connected,  $V_L = \sqrt{3}V_\phi = 8868 \text{ V}$ .