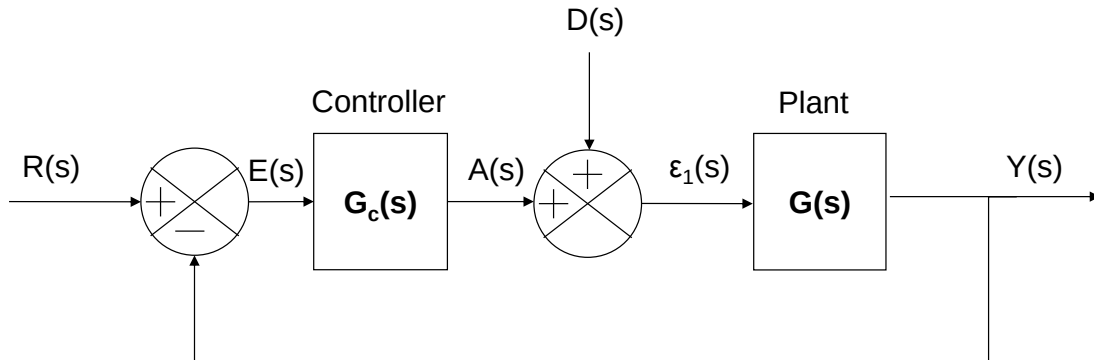


EE3114 Tutorial Week 9 (Solution)

System with Disturbance

1. Consider the following feedback control system with the distribution input $D(s)$:



Given that $G_c(s) = \frac{s-100}{s+200}$, and $G(s) = \frac{s+400}{s-100}$

- (a) Prove that $\frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}$

Answer:

When we consider $\frac{Y(s)}{R(s)}$,

we assume $D(s)=0$.

Firstly, from the block diagram, we have the relation:

$$Y(s) = E(s)G_c(s)G(s)$$

$$\Leftrightarrow E(s) = \frac{Y(s)}{G_c(s)G(s)} \quad \dots\dots\dots(1)$$

On the other hand, from the block diagram, we have another relation:

$$E(s) = R(s) - Y(s) \quad \dots\dots\dots(2)$$

Substitute (1) into (2), we get:

$$\begin{aligned}\frac{Y(s)}{G_c(s)G(s)} &= R(s) - Y(s) \\ \Leftrightarrow Y(s) &= G_c(s)G(s)R(s) - G_c(s)G(s)Y(s) \\ \Leftrightarrow Y(s)[1 + G_c(s)G(s)] &= R(s)G_c(s)G(s) \\ \Leftrightarrow \frac{Y(s)}{R(s)} &= \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}\end{aligned}$$

(b) Use the result of (a) to find $\frac{Y(s)}{R(s)}$

Answer:

$$\begin{aligned}\frac{Y(s)}{R(s)} &= \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} \\ &= \frac{\frac{s-100}{s+200} \times \frac{s+400}{s-100}}{1 + \frac{s-100}{s+200} \times \frac{s+400}{s-100}} \\ &= \frac{\frac{s+400}{s+200}}{1 + \frac{s+400}{s+200}} \\ &= \frac{s+400}{2s+600}\end{aligned}$$

(c) Prove that $\frac{Y(s)}{D(s)} = \frac{G(s)}{1 + G_c(s)G(s)}$

Answer:

When we consider $\frac{Y(s)}{D(s)}$,

we assume $R(s)=0$.

Firstly, from the block diagram, we have the relation:

$$\begin{aligned}Y(s) &= \varepsilon_1(s)G(s) \\ \Leftrightarrow \varepsilon_1(s) &= \frac{Y(s)}{G(s)} \dots\dots\dots(1)\end{aligned}$$

On the other hand, from the block diagram, we have another relation:

$$\varepsilon_1(s) = D(s) - G_c(s)Y(s) \quad \dots\dots\dots(2)$$

Substitute (1) into (2), we get:

$$\begin{aligned} \frac{Y(s)}{G(s)} &= D(s) - G_c(s)Y(s) \\ \Leftrightarrow Y(s) &= G(s)D(s) - G_c(s)G(s)Y(s) \\ \Leftrightarrow Y(s)[1 + G_c(s)G(s)] &= G(s)D(s) \\ \Leftrightarrow \frac{Y(s)}{D(s)} &= \frac{G(s)}{1 + G_c(s)G(s)} \end{aligned}$$

(d) Use the result of (c) to find $\frac{Y(s)}{D(s)}$

Answer:

$$\begin{aligned} \frac{Y(s)}{D(s)} &= \frac{G(s)}{1 + G_c(s)G(s)} \\ &= \frac{s + 400}{1 + \frac{s - 100}{s + 200} \times \frac{s + 400}{s - 100}} \\ &= \frac{s + 400}{1 + \frac{s + 400}{s + 200}} \\ &= \frac{(s + 400)(s + 200)}{2(s - 100)(s + 300)} \\ &= \frac{s^2 + 600s + 80000}{2s^2 + 400s - 60000} \end{aligned}$$

$$G_c(s) = \frac{s - 100}{s + 200}, \text{ and } G(s) = \frac{s + 400}{s - 100}$$

(e) Comment on the stability of the system.

Answer:

For the transfer function of $\frac{Y(s)}{R(s)}$, the characteristic equation is $2s + 600 = 0$:

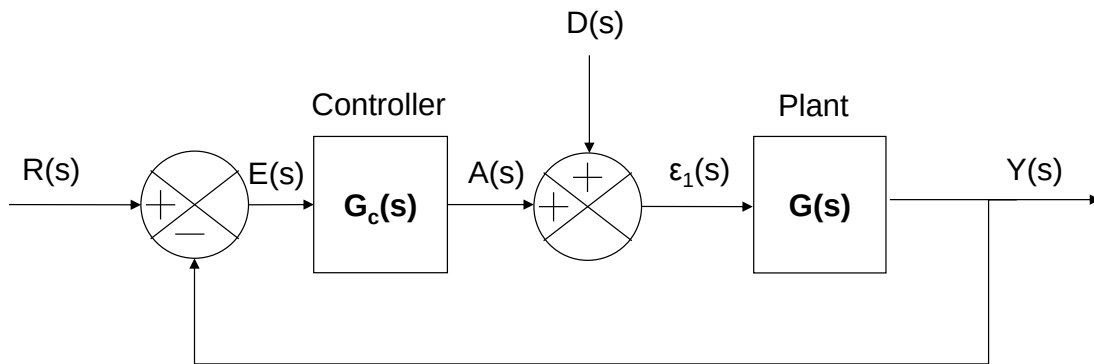
Hence the pole is $s = -300$.

For the transfer function of $G_{DY}(s)$, the characteristic equation is $2(s - 100)(s + 300) = 0$

Hence the poles are $s = -300, s = 100$.

Since one the pole is $s = 100$, which is placed on the right-handed s-plane, the system is unstable.

2. Consider the following feedback control system with the distribution input $D(s)$:



Given that $G_c(s) = \frac{s+3}{s+5}$, and $G(s) = \frac{s+10}{s+3}$

(a) Derive the transfer function for $G_{RY}(s) = \frac{Y(s)}{R(s)}$

Answer:

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{s+10}{2s+15}$$

(b) Derive the transfer function for $G_{DY}(s) = \frac{Y(s)}{D(s)}$

Answer:

$$\frac{Y(s)}{D(s)} = \frac{G(s)}{1 + C(s)G(s)}$$

$$\frac{Y(s)}{D(s)} = \frac{\frac{s+10}{s+3}}{1 + \frac{(s+3)(s+10)}{(s+5)(s+3)}} = \frac{(s+5)(s+10)}{(s+3)(2s+15)}$$

(c) Comment on the stability of the system.

Answer:

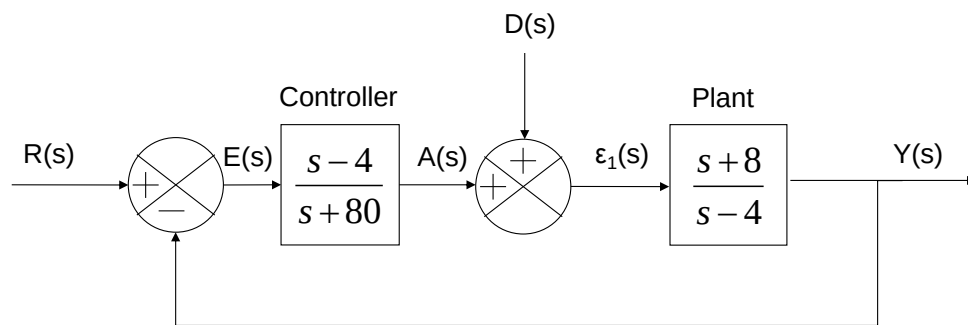
For the transfer function of $G_{RY}(s)$, the characteristic equation is $2s + 15 = 0$:

Hence the pole is $s = -7.5$.

For the transfer function of $G_{DY}(s)$, the poles are $s = -7.5$ and -3 .

Since all poles are on LHP, the system is stable.

3. Consider the following feedback control system with the distribution input $D(s)$:



(a) Find $\frac{Y(s)}{R(s)}$

Answer:

$$\begin{aligned}
& \frac{Y(s)}{R(s)} \\
&= \frac{\frac{s-4}{s+80} \times \frac{s+8}{s-4}}{1 + \frac{s-4}{s+80} \times \frac{s+8}{s-4}} \\
&= \frac{\frac{s+8}{s+80}}{1 + \frac{s+8}{s+80}} \\
&= \frac{s+8}{2s+88}
\end{aligned}$$

(b) Find $\frac{Y(s)}{D(s)}$

Answer:

$$\begin{aligned}
& \frac{Y(s)}{D(s)} \\
&= \frac{G(s)}{1 + G_c(s)G(s)} \\
&= \frac{\frac{s+8}{s-4}}{1 + \frac{s-4}{s+80} \times \frac{s+8}{s-4}} \\
&= \frac{\frac{s+8}{s-4}}{1 + \frac{s+8}{s+80}} \\
&= \frac{(s+8)(s+80)}{2(s-4)(s+44)} \\
&= \frac{s^2 + 88s + 640}{2s^2 + 80s - 352}
\end{aligned}$$

(c) Determine the stability of the system.

Answer:

For the transfer function of $\frac{Y(s)}{R(s)}$, the characteristic equation is $2s + 88 = 0$:

Hence the pole is $s = -44$.

For the transfer function of $\frac{Y(s)}{D(s)}$, the characteristic equation is $2(s - 4)(s + 44) = 0$

Hence the poles are $s = -44$, $s = 4$.

Since one the pole is $s = 4$, which is placed on the right-handed s-plane, the system is unstable.