

Due date: Monday, 4 December 2023 before 11:59 p.m.

- Answer **all** questions. Show **all** steps.
- Please submit the softcopy of your work (as a **single pdf file**) to Canvas on or before the due date.

Questions

1. Evaluate the double integral $\iint_R \left(\frac{y^2}{x^4} + \frac{x^2}{y^4} \right) dx dy$, where R is the region bounded by four parabolas $y = x^2$, $y = 2x^2$, $x = y^2$ and $x = 4y^2$ using the change of variables $u = \frac{y}{x^2}$ and $v = \frac{x}{y^2}$.
2. Compute the following triple integrals.
 - (a) $\iiint_D x^2 dx dy dz$, where D is the region bounded by the parabolic cylinders $z = x^2$, $z = 2 - x^2$ and the planes $y = 0$ and $y + z = 4$.
 - (b) $\iiint_V \frac{1}{\sqrt{4-x^2-y^2}} dx dy dz$, where V is the region which is bounded above by the upper half of the sphere $x^2 + y^2 + z^2 = 4$ and is bounded below by a plane $z = 1$.
3. Let $\vec{F}(x, y, z) = (ax \cos z - x^2)\vec{i} + (bz - 2y)\vec{j} + (4y + z - x^2 \sin z)\vec{k}$ be a vector field on $\mathbb{R}^3$, where a and b are real constants.
 - (a) Find the values of a and b such that $\vec{F}$ is irrotational.
 - (b) With the values of a and b obtained in (a), determine a scalar function $\varphi(x, y, z)$ on $\mathbb{R}^3$ for which $\nabla\varphi = \vec{F}$.
4.
 - (a) Find the mass of a wire in the shape of the semi-circle $y = \sqrt{4 - x^2}$ from $(2, 0)$ to $(-2, 0)$, and the density ρ at a point (x, y) on the wire is $\rho(x, y) = y^2$.
 - (b) Find the work done by the force field $\vec{F} = z\vec{i} - x\vec{j} + y\vec{k}$ in moving a particle from $(1, 1, 1)$ $(2, 8, 4)$ along the curve $C: \begin{cases} y = x^3 \\ z = x^2 \end{cases}$.
5.
 - (a) Evaluate the surface integral $\iint_S z dS$, where S is the portion of the cone $z = \sqrt{x^2 + y^2}$ lying below the plane $z = 3$.
 - (b) Let S be an open surface which is the portion of the paraboloid $z = 9 - x^2 - y^2$ lying above the plane $z = 5$. Evaluate the surface integral $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$, and the normal to S is pointing away from the xy -plane.