

Problem Set 1: First Order Ordinary Differential Equations

1. Solve the following ordinary differential equations:

(a) $xy \frac{dy}{dx} = 1 + x^2 + y^2 + x^2 y^2$

(b) $\frac{dy}{dx} = \frac{y-x}{y+x}$

(c) $\cos x \frac{dy}{dx} + y = \sin^2 x$

(d) $2 \frac{dy}{dx} - y \cot x = y^3 \cos x$

(e) $(x+y)^{-1} dx + [(x+y)^{-1} + \tan y] dy = 0$

(f) $\frac{dy}{dx} = \frac{2x-y-4}{y-2x-2}$

(g) $\frac{dy}{dx} = \frac{x-2y}{2x-y-3}$

2. Solve the following boundary value problems:

(a) $(x^2 + 1) \frac{dy}{dx} + 4xy = x$ with $y(2) = 1$

(b) $(x-2) \frac{dy}{dx} - y = (x-2)^3$ with $y(4) = 10$

(c) $xy \frac{dy}{dx} = 2(x^2 + y^2)$ with $y(2) = -1$

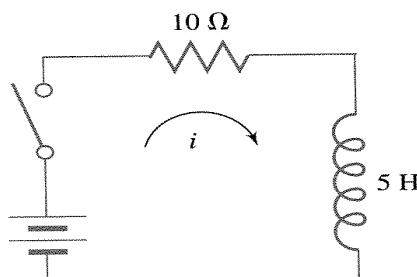
3. Determine values of C and K so that $(Cx^2ye^y + 2\cos y)dx + (x^3e^y y + x^3e^y + Kx\sin y)dy = 0$ is exact and then solve the corresponding differential equation.

4. A resistor of $R = 10$ ohms, and an inductor of $L = 5$ henries and a battery of E volts are connected in series with a switch K as shown in the following figure. If the switch K is closed at time $t = 0$ and the current $I = 0$, set up a differential equation for the current. Find the current I for $t > 0$ if (a) $E = 40$, (b) $E = 20e^{-3t}$, (c) $E = 50\sin 5t$.

(Hint: By Kirchhoff's laws:

Potential drop across L + Potential drop across R + Potential drop across $E = 0$,

that is, $5 \frac{dI}{dt} + 10I + (-E) = 0$.)



5. A radioactive substance S_1 is changing into another radioactive substance S_2 , while S_2 is changing into a stable substance S_3 . If A_1, A_2, A_3 are the amount of the substances present at time t , the following differential equations are satisfied:

$$\begin{cases} \frac{dA_1}{dt} = -\lambda_1 A_1 \\ \frac{dA_2}{dt} = \lambda_1 A_1 - \lambda_2 A_2 \\ \frac{dA_3}{dt} = \lambda_2 A_2 \end{cases}$$

where $\lambda_1, \lambda_2, \lambda_3$ are constants peculiar to the substances S_1, S_2, S_3 respectively.

If $A_1 = C, A_2 = 0, A_3 = 0$ at $t = 0$, find the amount of the different substances present at time t .

6. Consider the Riccati equation $\frac{dy}{dx} = \frac{1}{x}y^2 + \frac{1}{x}y - \frac{2}{x}$. Define z by $y = 1 + \frac{1}{z}$.
- (a) Convert the Riccati equation $\frac{dy}{dx} = \frac{1}{x}y^2 + \frac{1}{x}y - \frac{2}{x}$ into a linear differential equation involving x and z .
- (b) Find the general solution of the deduced linear differential equation and then find the general solution of the Riccati equation.

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