

CB2203 Data-Driven Business Modeling

Tutorial 11

IP Part 2

Question 1

A bank needs to install ATMs to serve customers living in six regions: *A, B, C, D, E* and *F*. The bank would like to install the minimum number of ATM machines possible. However, the bank needs to ensure that there is at least one ATM within a ten-minute walk from each region. The times required to walk between two regions are as follows:

Walking time between regions (min)	A	B	C	D	E	F
A	0	5	10	15	20	15
B	5	0	8	10	15	12
C	10	8	0	15	20	10
D	15	10	15	0	10	12
E	20	15	20	10	0	12
F	15	12	10	12	12	0

For example, if there is an ATM in region A, then customers who live in region B can walk to the ATM in region A within 5 minutes.

The bank wants to minimize the number of ATMs to install. Use binary integer programming to model the problem faced by the bank.

- Write a programming model to help the bank decide where to install its ATMs.
- Solve it on a spreadsheet for this problem.

Solution

a) We introduce the binary variables A, B, C, D, E, F to indicate whether (value of 1) or not (value of 0) the bank places an ATM machine in regions A, B, C, D, E and F, respectively.

The model is given as

$$\begin{aligned} \min \quad & A + B + C + D + E + F \\ \text{s.t.} \quad & A + B + C \geq 1 \\ & A + B + C + D \geq 1 \\ & A + B + C + F \geq 1 \\ & B + D + E \geq 1 \\ & D + E \geq 1 \\ & C + F \geq 1 \\ & A, B, C, D, E, F \in \{0,1\}. \end{aligned}$$

b) Set Objective: H2

To: Min

By Changing Variable Cells: B2:H2

Subject to the Constraints:

H5:H10 >= J5:J10

B2:G2 = binary

Click Make Unconstrained variables

Non-Negative

Select a Solving Method: Simplex LP

Minimum number of ATMs to install = 2

Locating ATMs in regions C and D

Question 2

Michael Manufacturing is planning to build at least one new plant, and three cities are being considered: Baytown, Texas; Lake Charles, Louisiana; and Mobile, Alabama. Once the plant or plants have been constructed, the company wishes to have sufficient capacity to produce at least 38,000 units each year. The costs associated with the possible locations are given in the table below.

Michael Manufacturing wants to minimize the total cost. Further constraints are as follows:

- (1) Number of units produced at the Baytown plant is 0 if the plant is not built, and it is no more than 21,000 if the plant is built;
- (2) Number of units produced at the Lake Charles plant is 0 if the plant is not built, and it is no more than 20,000 if the plant is built;
- (3) Number of units produced at the Mobile plant is 0 if the plant is not built, and it is no more than 19,000 if the plant is built.

SITE	ANNUAL FIXED COST	VARIABLE COST PER UNIT	ANNUAL CAPACITY
Baytown, TX	\$340,000	\$32	21,000
Lake Charles, LA	\$270,000	\$33	20,000
Mobile, AL	\$290,000	\$30	19,000

Formulate the IP problem.

Solution

a) Then we define the decision variables as below:

$X_1 = 1$ if the factory is built in Baytown or 0 otherwise

$X_2 = 1$ if the factory is built in Lake Charles or 0 otherwise

$X_3 = 1$ if the factory is built in Mobile or 0 otherwise

X_4 = number of units produced at Baytown plant

X_5 = number of units produced at Lake Charles plant

X_6 = number of units produced at Mobile plant

The integer programming problem formulation becomes

Minimize cost = $340,000X_1 + 270,000X_2 + 290,000X_3 + 32X_4 + 33X_5 + 30X_6$

subject to $X_4 + X_5 + X_6 \geq 38,000$

$$X_4 - 21,000 X_1 \leq 0$$

$$X_5 - 20,000 X_2 \leq 0$$

$$X_6 - 19,000 X_3 \leq 0$$

$X_1, X_2, X_3 = 0 \text{ or } 1$; $X_4, X_5, X_6 \geq 0$ and integer

Question 3

Triangle Utilities provides electricity for three cities. The company has four electric generators that are used to provide electricity. The main generator operates 24 hours per day, with an occasional shutdown for routine maintenance. Three other generators (1, 2, and 3) are available to provide additional power when needed. A start-up cost is incurred each time one of these generators is started. The start-up costs are \$6,000 for 1, \$5,000 for 2, and \$4,000 for 3. These generators are used in one of the following ways: a generator may be started at 6:00 a.m. and run for either 8 hours or 16 hours, or it may be started at 2:00 p.m. and run for 8 hours (until 10:00 p.m.). All generators except the main generator are shut down at 10:00 p.m. Forecasts indicate the need for 3,200 megawatts more than provided by the main generator before 2:00 p.m., and this need goes up to 5,700 megawatts between 2:00 and 10:00 p.m. Generator 1 may provide up to 2,400 megawatts, generator 2 may provide up to 2,100 megawatts, and generator 3 may provide up to 3,300 megawatts. The cost per megawatt used per 8-hour period is \$8 for 1, \$9 for 2, and \$7 for 3.

- (a) Formulate this problem as an integer programming problem to determine the least-cost way to meet the needs of the area.
- (b) Create the spreadsheet to solve this model.

Solutions:

- (a) Let $X_{ij} = 1$ if generator i is functioning during time period j , and 0 otherwise; where $i = 1, 2, 3$ and $j = 1$ for 6-2 time period; $j = 2$ for 2-10 time period; $j = 3$ for 6-10 time period.

Let Y_{ij} = megawatts produced by generator i in time period j , where $i = 1, 2, 3$ and $j = 1$ for 6-2 time period; $j = 2$ for 2-10 time period.

$$\text{Minimize cost} = 6,000(X_{11} + X_{12} + X_{13}) + 5,000(X_{21} + X_{22} + X_{23}) + 4,000(X_{31} + X_{32} + X_{33}) + 8(Y_{11} + Y_{12}) + 9(Y_{21} + Y_{22}) + 7(Y_{31} + Y_{32})$$

Subject to:

$$Y_{11} + Y_{21} + Y_{31} \geq 3,200 \text{ (megawatts requirements from 6-2)}$$

$$Y_{12} + Y_{22} + Y_{32} \geq 5,700 \text{ (megawatts requirements from 2-10)}$$

$$Y_{11} \leq 2,400(X_{11} + X_{13}) \text{ (maximum megawatts from \#1 from 6-2)}$$

$$Y_{12} \leq 2,400(X_{12} + X_{13}) \text{ (maximum megawatts from \#1 from 2-10)}$$

$$Y_{21} \leq 2,100(X_{21} + X_{23}) \text{ (maximum megawatts from \#2 from 6-2)}$$

$$Y_{22} \leq 2,100(X_{22} + X_{23}) \text{ (maximum megawatts from \#2 from 2-10)}$$

$$Y_{31} \leq 3,300(X_{31} + X_{33}) \text{ (maximum megawatts from \#3 from 6-2)}$$

$$Y_{32} \leq 3,300(X_{32} + X_{33}) \text{ (maximum megawatts from \#3 from 2-10)}$$

$$X_{11} + X_{12} + X_{13} \leq 1 \quad \text{(generator \#1 starts up at most once)}$$

$$X_{21} + X_{22} + X_{23} \leq 1 \quad \text{(generator \#2 starts up at most once)}$$

$$X_{31} + X_{32} + X_{33} \leq 1 \quad \text{(generator \#3 starts up at most once)}$$

$$X_{ij} = 0 \text{ or } 1 \text{ for all } i, j$$

$$Y_{ij} \geq 0 \text{ for all } i, j$$

- (b) The solution is: $X_{12} = 1$, $X_{33} = 1$, $Y_{12} = 2,400$, $Y_{31} = 3,200$, $Y_{32} = 3,300$, total cost = \$74,700.

Thus, generator #1 will be utilized in the period 2-10 and will generate 2,400 megawatts of electricity. Generator #3 will be started at 6 and utilized for the entire 16 hours. It will generate 3,200 megawatts during the 6-2 time period, and 3,300 megawatts during the 2-10 time period.