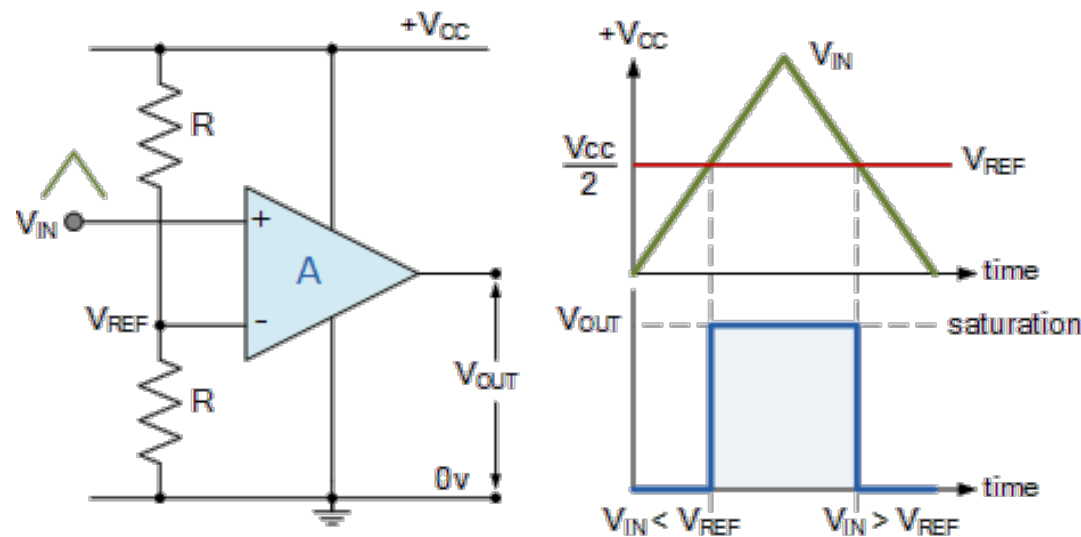


Block B Unit 2: Outline of Contents

- > Open loop:
 - Non-inverting and inverting comparators
- > Closed loop:
 - Differential and Instrumentation amplifiers
 - Integrating and differentiating amplifiers
- > Practical considerations
 - Effect of source resistance
 - Common mode rejection ratio

Non-inverting Comparator

A comparator is an example of an op amp operated in open loop
Open loop means that no feedback is applied



$$V_{out} = A(V_{+} - V_{-})$$

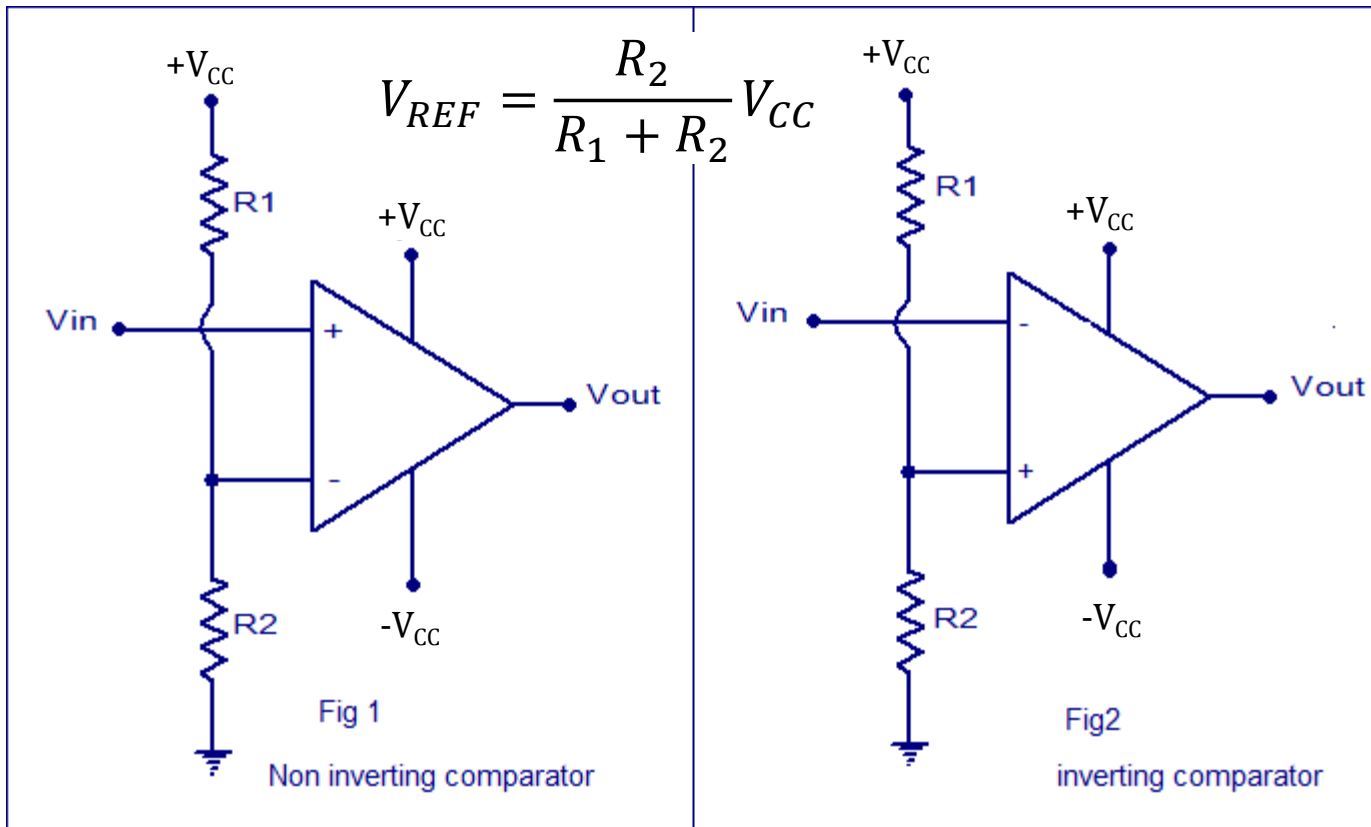
For an ideal op amp ($A \rightarrow \infty$),
given there is no feedback
between inputs and output:

When $V_{IN} > V_{REF}$: $V_{out} = +V_{CC}$;

When $V_{IN} < V_{REF}$: $V_{out} = 0$

In the above example, $V_{REF} = V_{CC}/2$ (no input current into op amp) Note
that V_{REF} can be set to any desired value by the choice of resistors

Inverting Comparator



When $V_{IN} > V_{REF}$: $V_{out} = +V_{CC}$;

When $V_{IN} < V_{REF}$: $V_{out} = -V_{CC}$

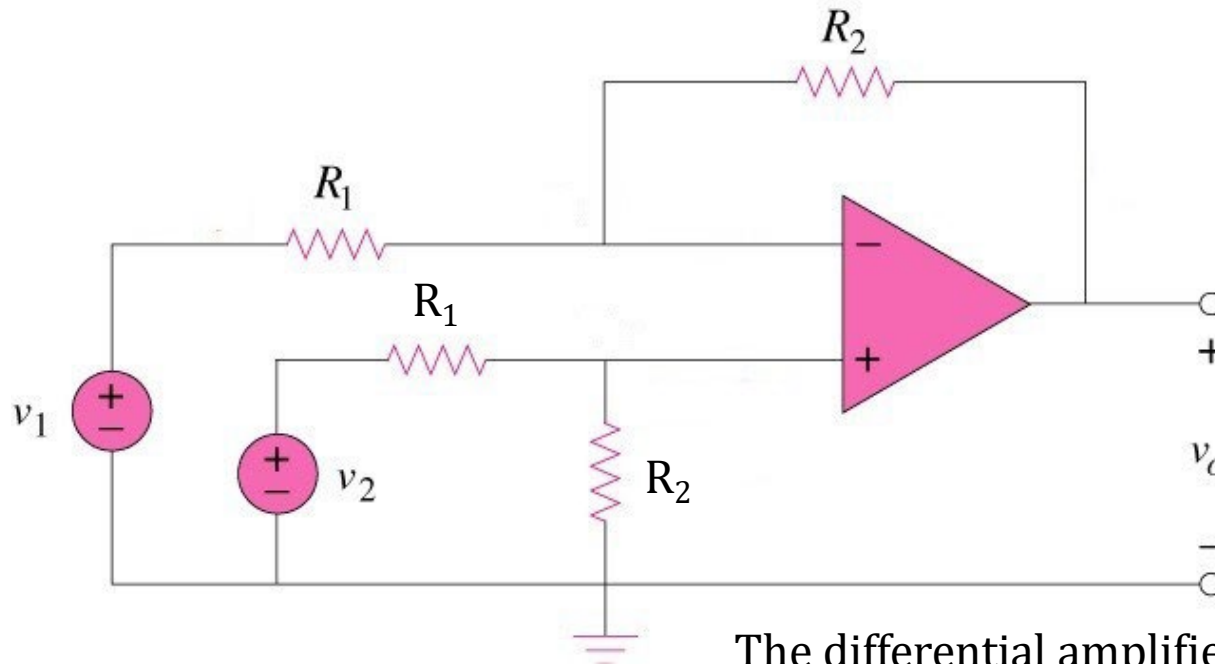
When $V_{IN} > V_{REF}$: $V_{out} = -V_{CC}$;

When $V_{IN} < V_{REF}$: $V_{out} = +V_{CC}$

Ideal differential op amp

We look at a number of op amp circuits that use negative feedback, starting with the differential amplifier from BlockBUnit1.

Negative feedback: Taking part or all of the output and returning it back to the input out of phase. The aim is to stabilize the output.



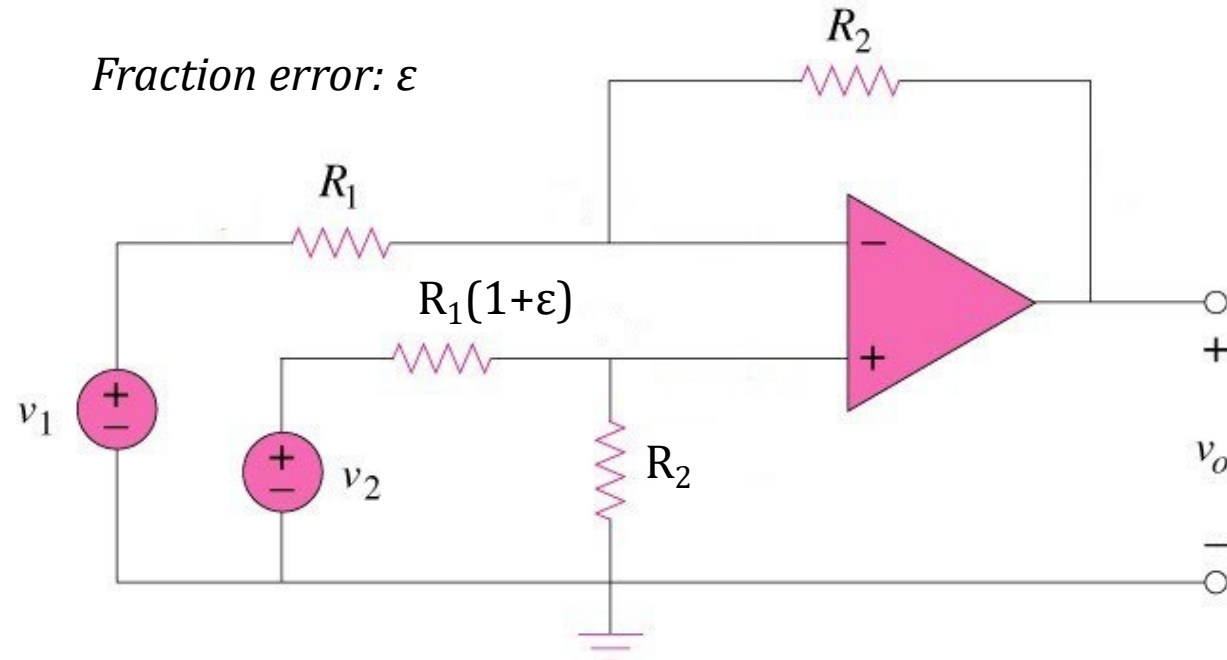
$$V_o = \frac{R_2}{R_1} (V_2 - V_1)$$

The differential amplifier is supposed to amplify only differences between the inputs.

Differential op amp in practice

In practice, it is not possible for two resistors to be exactly equal. So let us consider what happens when there is some error between just one of the resistors.

Fraction error: ε



$$V_o = \frac{R_2}{R_1} (V_2 - V_1) - \frac{\varepsilon R_2}{R_1(1 + \varepsilon) + R_2} V_2$$

We can see that V_o no longer depends on only the difference between the inputs; the value of the inputs also matter now.

Common mode

The ideal differential amplifier gives zero output when the inputs are equal and only amplifies the difference between the inputs.

This is not possible in practice. There will be some output if the inputs are equal.

We use the circuit on the previous slide as an example:

$$\text{When } V_1 = V_2: \quad V_o = -\frac{\varepsilon R_2}{R_1(1 + \varepsilon) + R_2} V_2$$

We see that in fact that V_o is proportional to the identical inputs.

In the special case when $V_1 = V_2 = V_{in}$, the resulting ratio of V_o/V_{in} is known as the common mode gain (A_{cm}). In this example:

$$A_{cm} = -\frac{\varepsilon R_2}{R_1(1 + \varepsilon) + R_2} \quad \text{As expected, } A_{cm} \text{ depends on the amount of error in the resistance value of } R_2$$

Common and differential mode

It is helpful to look at two voltage inputs ($V_1 + V_2$) in terms of common mode voltage and differential mode voltage.

$$V_2 = \frac{1}{2}(V_1 + V_2) + \frac{1}{2}(V_2 - V_1) = V_{cm} + \frac{1}{2}V_{dm}$$

Example: 3V and 7V

$$V_{cm} = 5V, V_{dm} = 4V$$

$$V_1 = \frac{1}{2}(V_1 + V_2) - \frac{1}{2}(V_2 - V_1) = V_{cm} - \frac{1}{2}V_{dm}$$

Common mode voltage: $V_{cm} = \frac{1}{2}(V_1 + V_2)$

Differential mode voltage: $V_{dm} = (V_2 - V_1)$

Common mode rejection ratio

Ideally, a differential amplifier should reject all common mode inputs and only amplify differential inputs. But in practice, this does not happen.

So for a practical (i.e. non-ideal differential amplifier), we express the output as a combination of common mode inputs and gain, and differential mode inputs and gain:

$$V_o = A_{cm}V_{cm} + A_{dm}V_{dm} = \frac{1}{2}A_{cm}(V_2 + V_1) + A_{dm}(V_2 - V_1)$$

Common mode gain (A_{cm}): Gain when both inputs are exactly the same ($V_1 = V_2 = V_{in}$); differential term goes to zero.

Differential mode gain (A_{dm}): Gain when both inputs are equal but out-of-phase ($V_2 = -V_1 = V_{in}/2$); common mode term goes to zero.

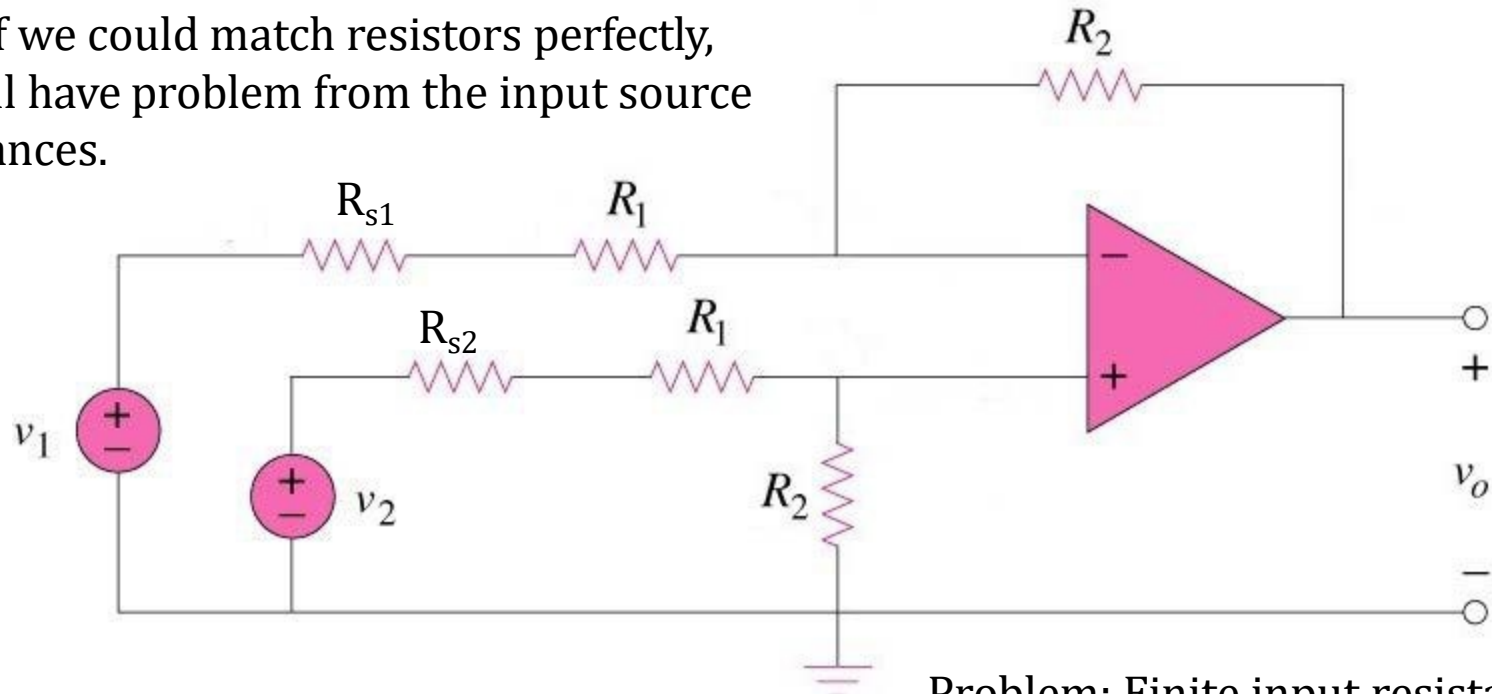
Common mode rejection ratio (CMRR) is the ratio of the differential mode gain over the common mode gain:

$$CMRR = A_{dm}/A_{cm}$$

A large CMRR is desired for a differential amplifier

Effect of source resistance

Even if we could match resistors perfectly, we will have problem from the input source resistances.



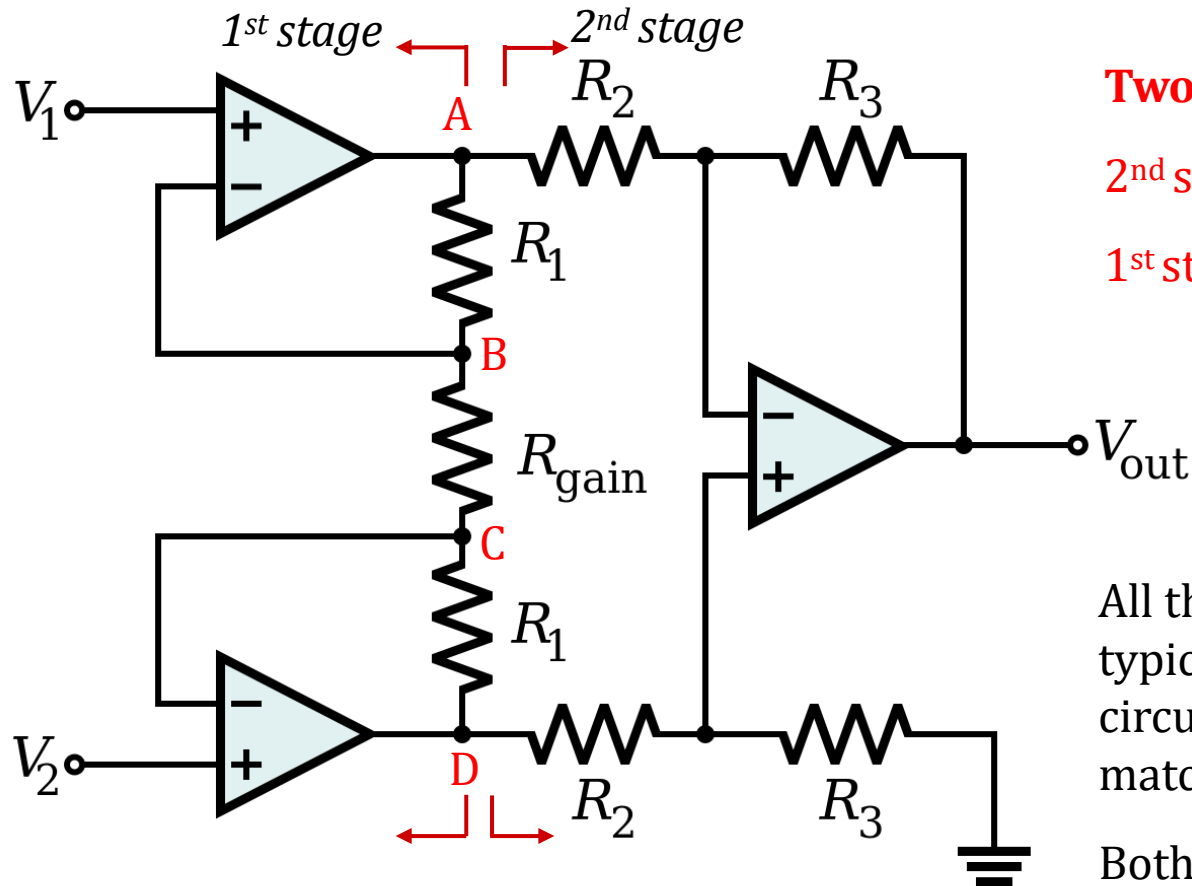
Problem: Finite input resistance

$$V_o = \left(\frac{R_2}{R_1 + R_{s1}} \right) \left[\left(\frac{R_2 + R_1 + R_{s1}}{R_1 + R_2 + R_{s2}} \right) V_2 - V_1 \right]$$

Clearly V_o does not depend just on the difference between the inputs.

If $V_2 = V_1$, V_o would not be zero.

Instrumentation amplifier



Two stage cascaded amplifier

2nd stage: Differential amplifier

1st stage: Non-inverting amplifier

All the resistors (except R_{gain}) are typically built into the integrated circuit and thus can be precisely matched.

Both inputs see the large input resistance of the op amp

⇒ Source resistance has little effect

$$V_{out} = \left(1 + \frac{2R_1}{R_{gain}}\right) \left(\frac{R_3}{R_2}\right) (V_2 - V_1)$$

Instrumentation amplifier: Analysis

Current from A $\rightarrow$ B = Current from B $\rightarrow$ C = Current from C $\rightarrow$ D

Node A: V_{o1} ; Node B: V_1 ; Node C: V_2 ; Node D: V_{o2}

1st stage

$$\frac{V_{o1} - V_1}{R_1} = \frac{V_1 - V_2}{R_{gain}} \Rightarrow V_{o1} = \left(1 + \frac{R_1}{R_{gain}}\right) V_1 - \left(\frac{R_1}{R_{gain}}\right) V_2$$
$$\frac{V_2 - V_{o2}}{R_1} = \frac{V_1 - V_2}{R_{gain}} \Rightarrow V_{o2} = \left(1 + \frac{R_1}{R_{gain}}\right) V_2 - \left(\frac{R_1}{R_{gain}}\right) V_1$$

None of the above intermediate output voltages are affected by the source resistance of the inputs

2nd stage

$$V_{out} = \frac{R_3}{R_2} (V_{o2} - V_{o1}) \quad V_{o2} - V_{o1} = \left(1 + \frac{R_1}{R_{gain}}\right) (V_2 - V_1) - \left(\frac{R_1}{R_{gain}}\right) (V_1 - V_2)$$

$$\Rightarrow V_{out} = \frac{R_3}{R_2} \left(1 + \frac{2R_1}{R_{gain}}\right) (V_2 - V_1)$$

Logarithmic amplifier

KCL at inverting input: $\frac{v_{in}}{R} = I_R = I_D$ (1)

Diode equation: $I_D = I_S \left(e^{\frac{V_D}{V_T}} - 1 \right)$

I_S : Reverse bias saturation current

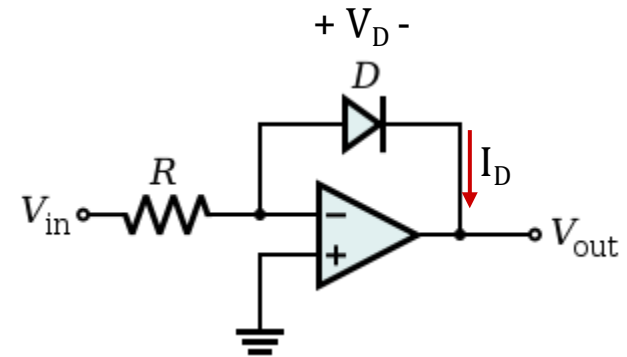
V_T : the thermal voltage (25.85 mV at 300 K)

$$e^{\frac{V_D}{V_T}} \gg 1 \quad \Rightarrow \quad I_D \approx I_S e^{\frac{V_D}{V_T}} \text{ (2)}$$

Sub (1) into (2): $v_D \approx V_T \ln \left(\frac{v_{in}}{RI_S} + 1 \right)$

But $V_{out} = -V_D$ (input voltage = 0): $v_{out} \approx -V_T \ln \left(\frac{v_{in}}{RI_S} + 1 \right)$

So we see that V_{out} is the natural log of V_{in} .



Without approximation:

$$v_{out} \approx -V_T \ln \left(\frac{v_{in}}{RI_S} + 1 \right)$$

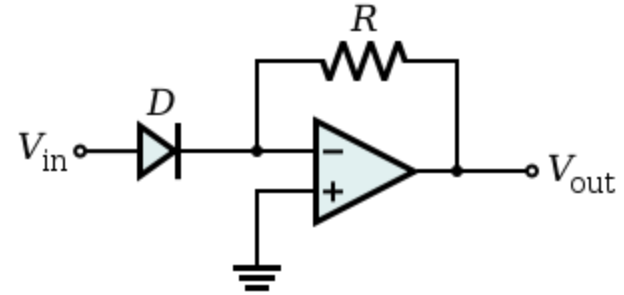
Exponential amplifier

KCL at inverting input: $v_{out} = -RI_D$ (1)

Diode equation: $I_D = I_S(e^{V_D/V_T} - 1)$

$$e^{\frac{V_D}{V_T}} \gg 1 \Rightarrow I_D \approx I_S e^{\frac{V_D}{V_T}} \text{ (2)}$$

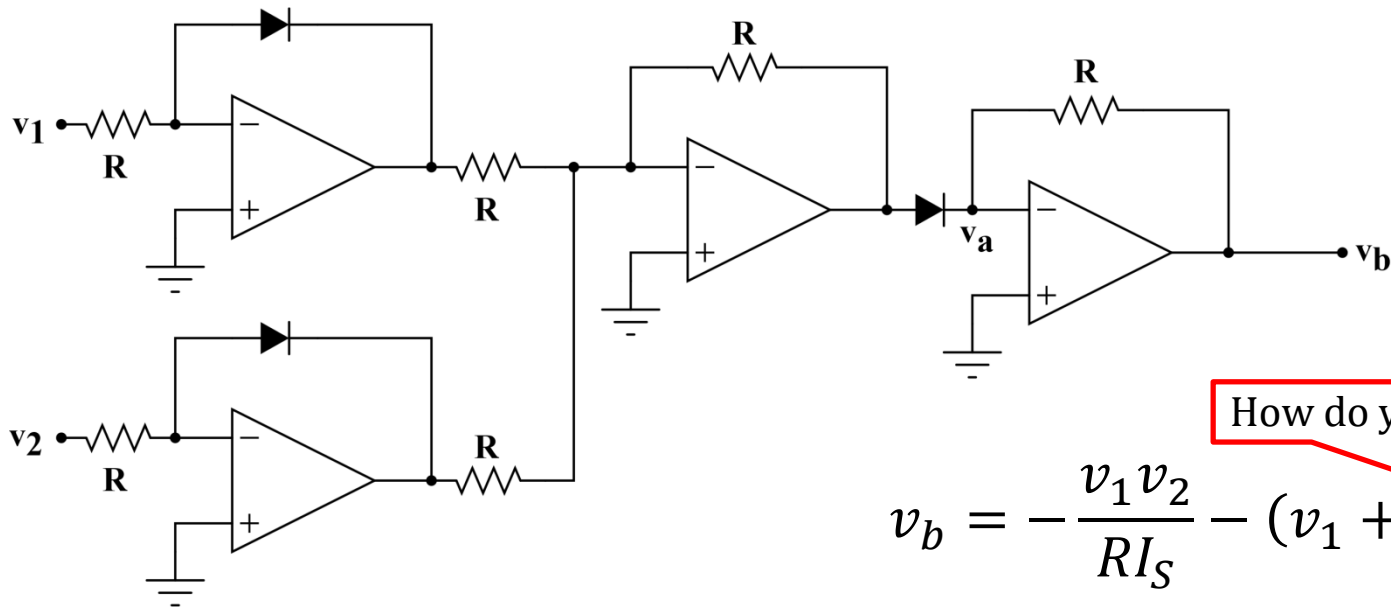
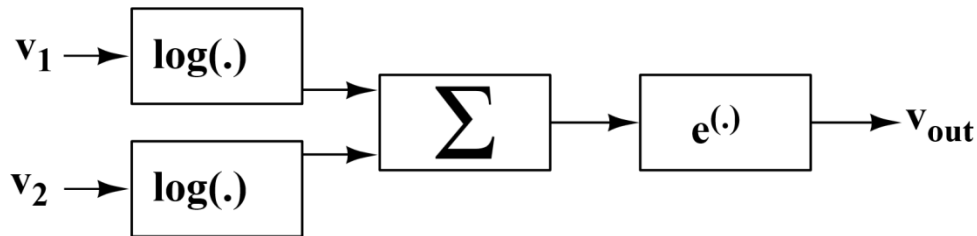
Sub (1) into (2): $V_{out} = -RI_S e^{\frac{V_{in}}{V_T}}$



So we see that V_{out} is the exponential of V_{in} .

Analogue multiplier

By combining logarithmic, exponential and summing amplifiers, we can realize an analogue multiplier.



How do you remove this term?

$$v_b = -\frac{v_1 v_2}{R I_S} - (v_1 + v_2)$$

http://en.wikibooks.org/wiki/Electronics/Analog_multipliers

Analogue multiplier: analysis

1st Stage: Logarithmic
Without approximations

$$v_{o1} \approx -V_T \left[\ln \left(\frac{v_1}{RI_S} + 1 \right) \right] \quad v_{o2} \approx -V_T \left[\ln \left(\frac{v_2}{RI_S} + 1 \right) \right]$$

2nd Stage: Summation

$$\begin{aligned} v_{os} &= -(v_{o1} + v_{o2}) \\ v_{os} &= V_T \left[\ln \left(\frac{v_1}{RI_S} + 1 \right) + \ln \left(\frac{v_2}{RI_S} + 1 \right) \right] \\ v_{os} &= V_T \left[\ln \left(\frac{v_1}{RI_S} + 1 \right) \left(\frac{v_2}{RI_S} + 1 \right) \right] \end{aligned}$$

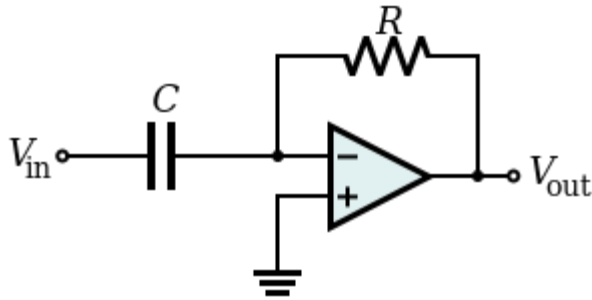
3rd Stage: Exponential
Without approximations

$$\begin{aligned} v_b &= -RI_S (e^{v_{os}/V_T} - 1) \\ v_b &= -RI_S \left[\left(\frac{v_1}{RI_S} + 1 \right) \left(\frac{v_2}{RI_S} + 1 \right) - 1 \right] \\ v_b &= -\frac{v_1 v_2}{RI_S} - (v_1 + v_2) \end{aligned}$$

Analogue multiplier: analysis

What configuration of op-amp could remove $(v_1 + v_2)$?

Differentiating amplifier



KCL at inverting input:

$$I_S = I_F = -V_{out}/R \quad \dots (1)$$

Voltage across C = V_{in} :

$$I_S = C \left(\frac{dV_{in}}{dt} \right) \quad \dots (2)$$

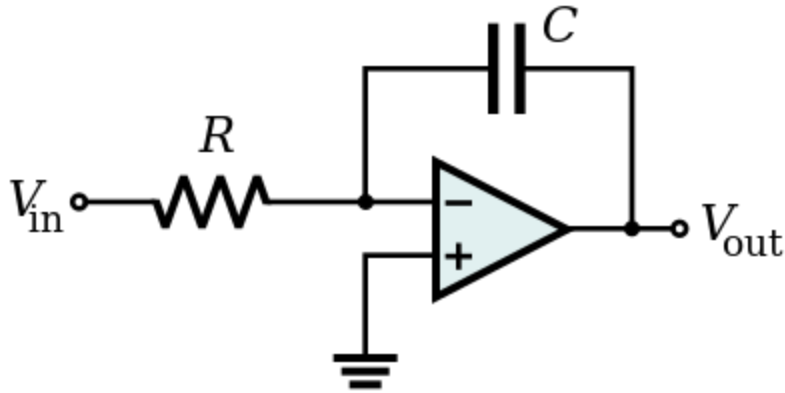
Sub (1) into (2):

$$V_{out} = -RC \left(\frac{dV_{in}}{dt} \right)$$

Output is the time-differential of the input

Ref: http://www.allaboutcircuits.com/vol_3/chpt_8/11.html

Integrating amplifier



KCL at inverting input:

$$I_S = I_F = V_{in}/R \quad \dots (1)$$

Voltage across C = $-V_{out}$:

$$I_F = -C \left(\frac{dV_{out}}{dt} \right) \quad \dots (2)$$

Sub (1) into (2):

$$\frac{V_{in}}{R} = -C \left(\frac{dV_{out}}{dt} \right)$$

$$V_{out} = -\frac{1}{RC} \int V_{in} dt$$

Output is the time-integral of the input

Ref: http://www.allaboutcircuits.com/vol_3/chpt_8/11.html