

MA1200 Hand-in Assignment #3 due Nov. 27

Instructions to students:

1. Please submit it via Canvas in a PDF file (you can handwrite the answers and take photos by your phone, then make it into a PDF file, see, for example, <https://www.wikihow.com/Convert-JPG-to-PDF> for how to combine JPG files to a PDF; you can also do it by note-taking apps on an iPad or a Surface)
2. The assignment is due on **23:59 of November 27 (Sunday)**. Late submissions will **NOT** be marked.
3. Please write down your name and student ID.

Questions:

1. Compute the following limits:

$$(a) \lim_{x \rightarrow 0} \frac{\cos x - \cos(3x)}{x^2}, \quad (b) \lim_{x \rightarrow \infty} \left(\sqrt{x + \sqrt{x}} - \sqrt{x} \right), \quad (c) \lim_{x \rightarrow 0} \frac{x^2 \sin(\frac{1}{x})}{x}.$$

2. Let

$$f(x) = \begin{cases} 3x^2 - 1, & \text{if } x < 0, \\ cx + d, & \text{if } 0 \leq x \leq 1, \\ \sqrt{x + 8}, & \text{if } x > 1. \end{cases}$$

Determine the values of c and d , such that $f(x)$ is continuous everywhere.

3. Which of the following functions is differentiable at $x = 0$?

$$f(x) = x|x|, \quad g(x) = x\sqrt{|x|}, \quad h(x) = x + |x|, \quad j(x) = \begin{cases} x \sin\left(\frac{\pi}{x}\right), & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

4. Find derivatives of the following functions:

$$(a) \tan \sqrt{x}, \quad (b) \sqrt{\frac{1-x^2}{1+x^2}}, \quad (c) \ln(\csc x + \cot x),$$

$$(d) (\cosh x)^x, \quad (e) \tan^{-1} \left(\frac{2}{1-x^2} \right), \quad (f) \sin x \sin \left(\frac{1}{x} \right).$$

5. Find the tangent line of the curve $xy + \ln y = 1$ at the point $(1, 1)$.

6. Find the tangent line of the curve $\begin{cases} x = \frac{2t+t^2}{1+t^3}, \\ y = \frac{2t-t^2}{1+t^3}, \end{cases}$ at the point when $t = 1$.

7. A stone thrown into a still pond produces concentric circular ripples. If the radius of the largest ripple increases at the rate of 0.60 meters per second, how fast is the area within this ripple increasing when the radius is 50 meters?

8. A farmer wishes to fence off a rectangular pasture along a straight river, the side along the river requiring no fence. He has wire to fence a distance of 800 meters. What is the area of the largest pasture of the above description that can be fenced?

9. Let $f(x) = (1 + x^2)^{-\frac{1}{2}}$.

(a) Show that

$$(1 + x^2)f'(x) + xf(x) = 0.$$

(b) Let n be a non-negative integer, show that

$$(1 + x^2)f^{(n+2)}(x) + (2n + 3)xf^{(n+1)}(x) + (n + 1)^2f^{(n)}(x) = 0.$$

(c) Hence, or otherwise, find the Taylor series of $(1 + x^2)^{-\frac{1}{2}}$ at $x = 0$ up to the term x^4 .

End