

EE3101 Tutorial 7 Solutions

(1) For an ideal BPSK, $P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$, and the transmission power is proportional to $E_b \times (\text{bit rate})$.

(a) If the transmission power is reduced by half, i.e., E_b will be reduced by half,

$$P_B \cong Q\left(\frac{5.612}{\sqrt{2}}\right) = Q(3.968) = 3.83 \times 10^{-5}$$

i.e., The P_B will be increased by $\sim 3,800$ time for 3dB power loss.

(b) Similarly, if the bit rate is doubled, then $P_B = 3.83 \times 10^{-5}$.

(2) (a) BPSK: $BW = 1M/\log_2 2 = 1\text{MHz}$

(b) The average signal levels at the integrator output are (*the average noise component equals zero*):

$$Z(T) = \int_0^T S(t)\Psi(t)dt = AB \int_0^T \sin(\omega t) \cos(\omega t)dt \pm B^2 \int_0^T \cos^2(\omega t)dt = \pm \frac{B^2 T}{2}$$

where the plus and negative signs correspond to binary “1” and “0” respectively and all the high frequency components are filtered.

The distance of the mean signal level of “1” to the threshold is $\frac{B^2 T}{2}$,

The distance of the mean signal level of “0” to the threshold is $\frac{B^2 T}{2}$

The overall noise power at the integrator output is: $\sigma^2 = \frac{N_0}{2} \int_0^T B^2 \cos^2(\omega t)dt = \frac{N_0 B^2 T}{4}$

The bit error probability is:

$$P_B = \frac{1}{2} Q\left[\frac{B^2 T/2}{\sigma}\right] + \frac{1}{2} Q\left[\frac{B^2 T/2}{\sigma}\right] = Q\left[\sqrt{\frac{B^2 T}{N_0}}\right]$$

(c) The average power of the transmitted signal P_t is:

$$P_t = \frac{1}{2} A^2 + \frac{1}{2} B^2$$

Since 15% of the transmitted signal power is the additional carrier, this implies:

$$\frac{A^2}{A^2 + B^2} = 15\%, \quad \text{or} \quad \frac{B^2}{A^2 + B^2} = 85\%$$

For probability of bit error = $10^{-5} \rightarrow (B^2 T/N_0)^{1/2} \cong 4.26$

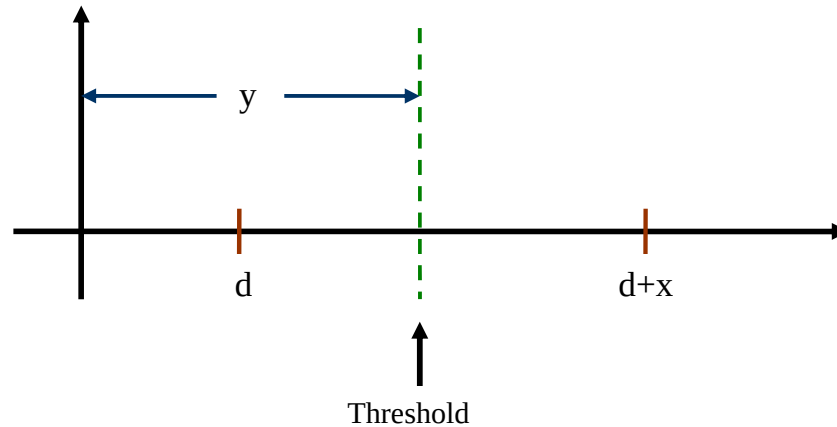
The probability of bit error for a traditional BPSK is:

$$P'_B = Q\left[\sqrt{\frac{B'^2 T}{N_0}}\right]$$

The total transmission power remains the same, $\rightarrow B'^2 = A^2 + B^2 = B^2 / 0.85$

$$P'_B = Q\left[\sqrt{\frac{B'^2 T}{N_0}}\right] = Q\left[\frac{4.26}{\sqrt{0.85}}\right] \approx 2 \times 10^{-6}$$

(3) (a) Assume the decision threshold is set at y :



For sending a binary “0”, the probability of error is:

$$Q\left(\frac{y-d}{\sqrt{\sigma'^2}}\right) \quad \text{where } \sigma'^2 \text{ is the variance of the noise}$$

For sending a binary “1”, the probability of error is:

$$Q\left(\frac{d+x-y}{\sqrt{\sigma'^2}}\right) \quad \text{where } \sigma'^2 \text{ is the variance of the noise}$$

Total average probability of bit error is:

$$\begin{aligned} P_B &= (\text{prob of sending binary “0”}) * (\text{prob of error when sending binary “0”}) \\ &\quad + (\text{prob of sending binary “1”}) * (\text{prob of error when sending binary “1”}) \\ &= \frac{1}{2} Q\left(\frac{y-d}{\sqrt{\sigma'^2}}\right) + \frac{1}{2} Q\left(\frac{d+x-y}{\sqrt{\sigma'^2}}\right) \end{aligned}$$

For minimum P_B , i.e. $\frac{dP_B}{dy} = 0$

$$\frac{d}{dy} Q\left(\frac{y-d}{\sqrt{\sigma'^2}}\right) = -\frac{1}{\sqrt{2\pi\sigma'^2}} \exp\left[-\frac{(y-d)^2}{2\sigma'^2}\right] \}$$

$$\frac{d}{dy} Q\left(\frac{d+x-y}{\sqrt{\sigma'^2}}\right) = -(-1) \frac{1}{\sqrt{2\pi\sigma'^2}} \exp\left[-\frac{(d+x-y)^2}{2\sigma'^2}\right]$$

$$\text{i.e. } \frac{1}{2\sqrt{2\pi\sigma'^2}} \left\{ -\exp\left[-\frac{(y-d)^2}{2\sigma'^2}\right] - (-1) \exp\left[-\frac{(d+x-y)^2}{2\sigma'^2}\right] \right\} = 0$$

$$\rightarrow \exp\left[-\frac{(y-d)^2}{2\sigma'^2}\right] = \exp\left[-\frac{(d+x-y)^2}{2\sigma'^2}\right]$$

$$\rightarrow y = d + x/2$$

Alternated Method, using equation (10) of Chapter 1:

$$\frac{\exp[-(z-d)^2/2\sigma'^2]}{\exp[-(z-d-x)^2/2\sigma'^2]} \underset{H_1}{\overset{H_0}{>}} \frac{P_1}{P_0} = 1$$

z : signal level at the decision device input (i.e. after the correlator)

H_0 : decide that "0" is transmitted

H_1 : decide that "1" is transmitted

Take natural log on both side, i.e.,
$$-\frac{(z-d)^2}{\sigma'^2} \underset{H_1}{\overset{H_0}{>}} -\frac{(z-d-x)^2}{\sigma'^2}$$

$$\underset{H_1}{\overset{H_0}{>}} 2xz - 2xd - x^2 \quad \Rightarrow \quad \underset{H_1}{\overset{H_0}{>}} d + \frac{x}{2} > z$$

i.e. the optimum threshold is at $d + x/2$

(4) (i) “1”: $E\sqrt{\frac{2}{T}}\cos(\omega t)$; “0”: $-E\sqrt{\frac{2}{T}}\cos(\omega t)$

and with the reference signal of $\sqrt{\frac{2}{T}}\cos(\omega t)$, the signal component at the integrator output for “1” is:

$$\int_0^T E \frac{2}{T} \cos^2(\omega t) dt = \frac{E}{T} \int_0^T [1 + \cos(2\omega t)] dt = E$$

Similarly, the signal component at the integrator output for “0” is: $-E$

The noise variance at the integrator output is $\sigma^2 = \frac{N_0}{2} \int_0^T \frac{2}{T} \cos^2(\omega t) dt = \frac{N_0}{2}$

Average bit energy is: $E_b = [(-E\sqrt{\frac{2}{T}})^2 \frac{T}{2} P_0 + (E\sqrt{\frac{2}{T}})^2 \frac{T}{2} P_1] = E^2$

The distance of the mean signal level of “1” to the threshold is $E - y$,

The distance of the mean signal level of “0” to the threshold is $E + y$

$$P_B = P_0 Q\left(\frac{E + y}{\sigma}\right) + P_1 Q\left(\frac{E - y}{\sigma}\right)$$

$$P_B = P_0 Q\left[\sqrt{\frac{2E_b}{N_0}}\left(1 + \frac{y}{\sqrt{E_b}}\right)\right] + P_1 Q\left[\sqrt{\frac{2E_b}{N_0}}\left(1 - \frac{y}{\sqrt{E_b}}\right)\right]$$

(ii) For minimum P_B , i.e. $\frac{dP_B}{dy} = 0$

$$\text{i.e. } -\frac{P_0}{\sqrt{\pi N_0}} \exp\left[-\frac{(\sqrt{E_b} + y)^2}{N_0}\right] - (-1) \frac{P_1}{\sqrt{\pi N_0}} \exp\left[-\frac{(\sqrt{E_b} - y)^2}{N_0}\right] = 0$$

$$\exp\left\{-\frac{(\sqrt{E_b} - y)^2 + (\sqrt{E_b} + y)^2}{N_0}\right\} = \frac{P_0}{P_1}$$

$$\exp\left\{-\frac{4y\sqrt{E_b}}{N_0}\right\} = \frac{P_0}{P_1}$$

$$\text{i.e. } y = \frac{N_0}{4\sqrt{E_b}} \ln\left(\frac{P_0}{P_1}\right)$$