

MA2001 Problem Set 3 Multiple Integrals

- Change the order of the integration in $\int_0^1 \left[\int_0^{2y} f(x, y) dx \right] dy + \int_1^3 \left[\int_0^{3-y} f(x, y) dx \right] dy$.
- Evaluate the following double integrals:
 - $\iint_S xy \, dx dy$, where S is the region bounded by the lines $x = 0$, $x = 1$, $y = x^2$ and $y = 4$.
 - $\iint_S x^2 \, dx dy$, where S is the region bounded by $y = 2x$ and $x^2 + y = 8$.
- Evaluate $\iint_S xy \, dx dy$, where S is the region enclosed by the 4 parabolas $y^2 = x$, $y^2 = 2x$, $x^2 = y$, $x^2 = 2y$ using the change of variable $u = \frac{x^2}{y}$, $v = \frac{y^2}{x}$.
- Evaluate $E_z = \frac{\sigma_0 z}{4\pi\epsilon_0} \iint_S \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \, dx dy$, where S is the disc $x^2 + y^2 \leq a^2$, which represents the z -component of the electric field at the point $(0, 0, z)$ due to a uniformly charged circular disc lying in $x^2 + y^2 \leq a^2$, $z = 0$.
- Let R be the region bounded by $x + y = 1$, $x = 0$, $y = 0$. Show that $\iint_R \cos\left(\frac{x-y}{x+y}\right) dx dy = \frac{\sin 1}{2}$, using the substitution $x - y = u$, $x + y = v$.
- Evaluate $\iint_S e^{xy} \, dx dy$, where S is the region enclosed by $xy = 1$, $xy = 2$, $y = x$, $y = 4x$ using the change of variable $xy = u$, $\frac{y}{x} = v$.
- Use the change of variables $x + y = u$, $x - y = v$ to evaluate $\iint_{|x|+|y|\leq 1} e^{(x-y)} dx dy$.
- An iterated integral like $\int_0^1 \left[\int_0^{\frac{1-x}{2}} \left(\int_0^{1-x-2y} f(x, y, z) dz \right) dy \right] dx$ is called an **iterated integral** with order $dz dy dx$. Change the order of the iterated integral $\int_0^1 \left[\int_0^{\frac{1-x}{2}} \left(\int_0^{1-x-2y} f(x, y, z) dz \right) dy \right] dx$ to an equivalent iterated integral with order $dx dz dy$.
- Let V be the region in the first octant, where $x, y, z \geq 0$ bounded by $x^2 + y^2 = 1$, $x = 0$, $y = 0$, $z = 0$, $z = 1$. Using cylindrical polar coordinate, compute $\iiint_V xy dx dy dz$.

10. (Optional)

In a sample model of the charge distribution around the positively charged (Q) nucleus of the hydrogen atom the charge density at the point (x, y, z) in the electron cloud is $f(x, y, z) = \frac{-Q}{\pi a^3} e^{-\frac{2\sqrt{x^2+y^2+z^2}}{a}}$, where a is the Bohr radius. Determine the total charge in the electron cloud.

11. (Optional)

Let V be the region enclosed by both the surfaces: $\begin{cases} x^2 + y^2 + (z-1)^2 = 1 \\ z \geq 1 \end{cases}$ and $x^2 + y^2 = z^2$.

Using spherical coordinate, compute $\iiint_V z dx dy dz$.

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