

CB2203 Data-Driven Business Modeling

Tutorial 12

NLP

Question 1

Thermolock Corporation produces massive rubber washers and gaskets like the type used to seal joints on the NASA space shuttles. To do so, it combines two ingredients, rubber and oil. The upper bound of each of the two ingredients is 50 pounds. The cost of the industrial-quality rubber used is \$5 per pound, and the cost of the high-viscosity oil is \$7 per pound. Two of the three constraints Thermolock faces are nonlinear. If R and O denote the number of pounds used of rubber and oil, respectively, the firm's objective function and constraints can be written as follows:

$$\text{Minimize cost} = \$5R + \$7O$$

subject to the constraints

$$3R + 0.25R^2 + 4O + 0.3O^2 \geq 125 \text{ (hardness constraint)}$$

$$13R + R^3 \geq 80 \text{ (tensile strength constraint)}$$

$$0.7R + O \geq 17 \text{ (elasticity constraint)}$$

$$R, O \geq 0$$

Set up and solve Thermolock's NLP model using Solver.

Solution:

Let's use the [Multistart](#) option available in the [GRG Nonlinear](#) method in [Solver](#) to solve this NLP model. Recall that for NLP models, we should select [GRG Nonlinear](#) as the solving method to use.

We then click the [Options](#) button in [Solver](#) and check the boxes labeled [Use Multi-start](#) and [Require Bounds on Variables](#) in the [GRG Nonlinear](#) tab (refer to Figure 6.19). As noted previously, the nonnegativity constraints provide lower bounds for the variables. In Thermolock's case, we have specified a value of 50 as an upper bound for each of the two decision variables. When we now solve the model, we get the result shown in Figure 6.24, which specifies that Thermolock should use 3.325 pounds of rubber and 14.672 pounds of oil, at a total cost of \$119.33.

Question 2

A retailer orders three different products from a manufacturer. The unit ordering cost of each product is 10HKD. As these three products are under the same category, they are substitutable to each other, i.e. when price of one product is increased, not only its own demand decreases, but also demands of other products increase. Specifically, we have the following demand form for each product.

$$d_1(p_1, p_2, p_3) = 160 - 4p_1 + p_2 + 2p_3,$$

$$d_2(p_1, p_2, p_3) = 180 + p_1 - 4p_2 + p_3,$$

$$d_3(p_1, p_2, p_3) = 200 + 2p_1 + p_2 - 4p_3.$$

The manufacturer suggests a retailing price upper bound of 60HKD per unit to ensure market share of its products. The retailer has to agree on this price constraint to receive orders from the manufacturer. Suppose that you are the retailer.

- Please write down the formulation of this problem to maximize the total profit by optimizing the prices of the products.
- Please implement your optimization problem in excel spreadsheet. What should be the optimal prices and corresponding demands? What is the maximum total profit?
- If the unit ordering cost of product 2 is increased to 14HKD, how will it affect the optimal prices and maximum total profit? (Hint: changing the parameter in the model)

Solution:

- a) Let p_1, p_2, p_3 be the price of products 1, 2, 3

$$\begin{aligned} & \text{Max } (160 - 4p_1 + p_2 + 2p_3)(p_1 - 10) \\ & \quad + (180 + p_1 - 4p_2 + p_3)(p_2 - 10) \\ & \quad + (200 + 2p_1 + p_2 - 4p_3)(p_3 - 10) \\ \text{s.t. } & p_1 \leq 60 \\ & p_2 \leq 60 \\ & p_3 \leq 60 \\ & p_1, p_2, p_3 \geq 0 \end{aligned}$$

- b) Set Objective: B9

To: Max

By Changing Variable Cells: B3:D3

Subject to the Constraints: E13:E15 <= G13:G15

Click Make Unconstrained variables

Non-Negative

Select a Solving Method: GRG Nonlinear

In Options => GRG Nonlinear tab,

Derivatives: Central

Click Use Multistart

Price of product 1: \$60

Price of product 2: \$55

Price of product 3: \$60

Maximum total profit: \$15100

c) By changing the cost of product \$2 to \$14,

Price of product 1: \$60

Price of product 2: \$57

Price of product 3: \$60

Maximum total profit: \$14796