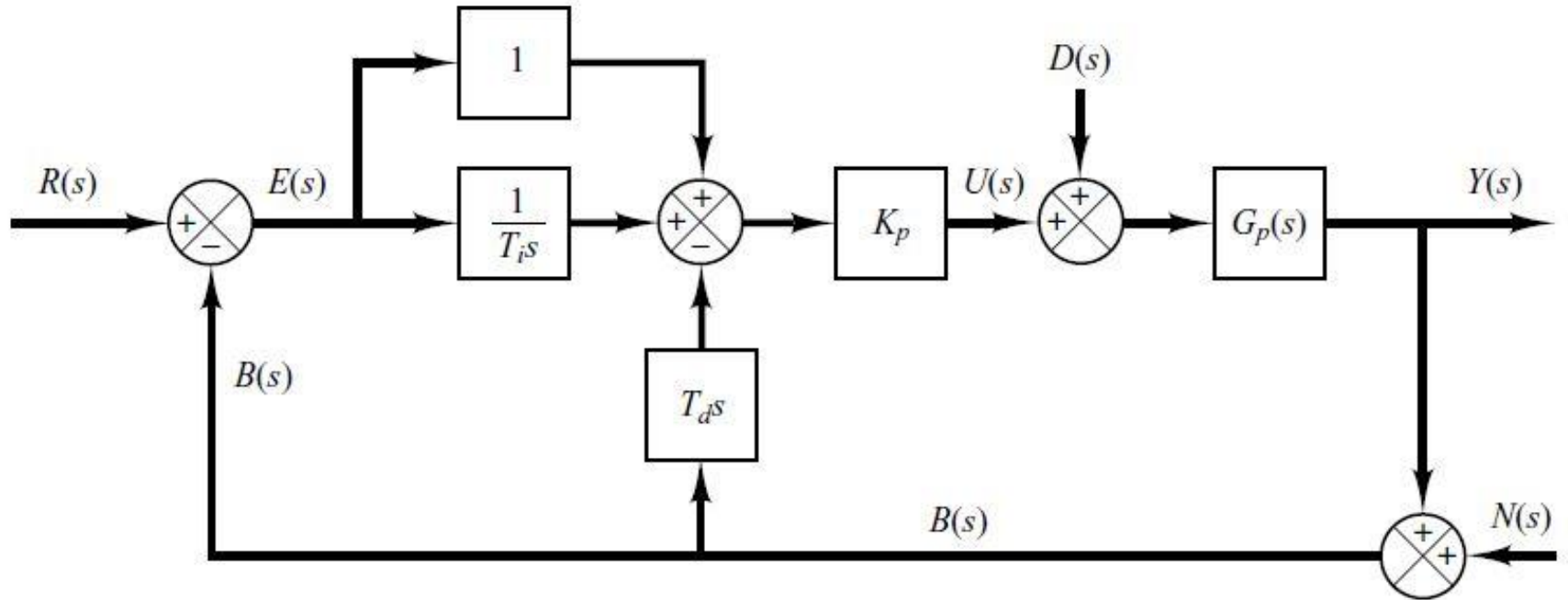


# EE3114 Systems and Control

## **Tutorial - 5**

Semester B, 2017-18

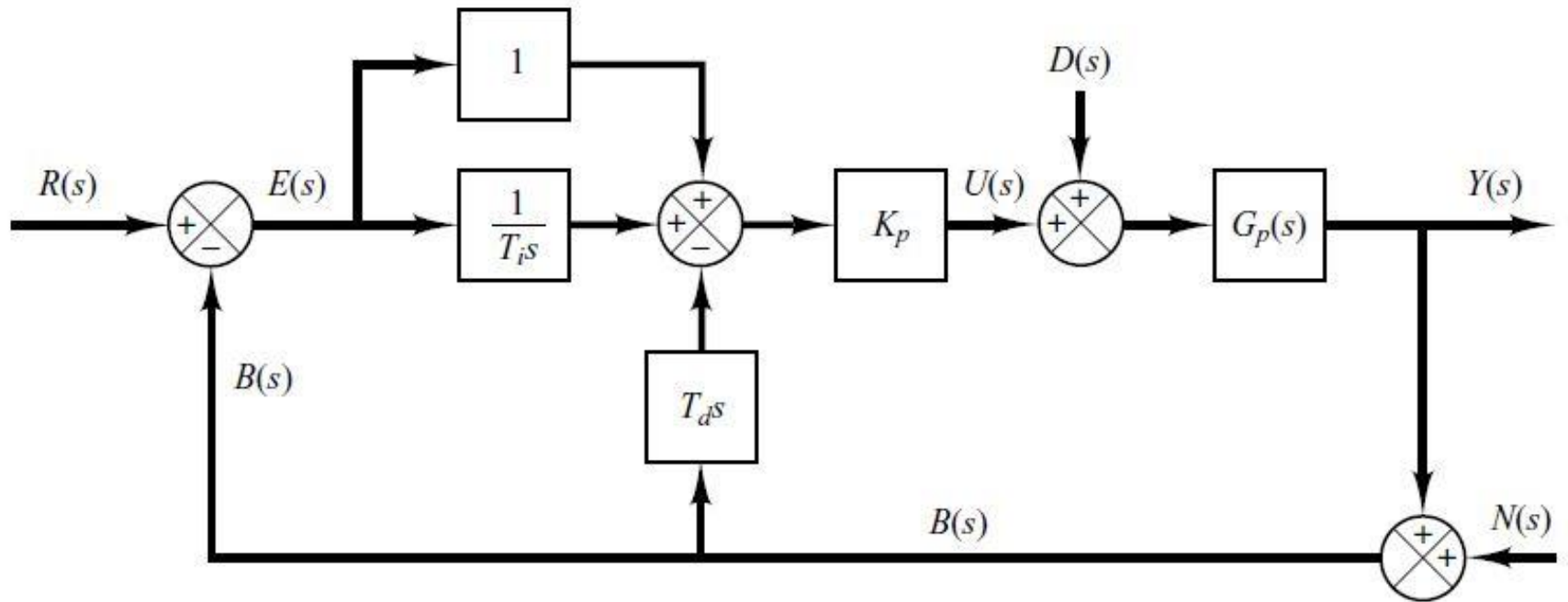
# PI + D Control System



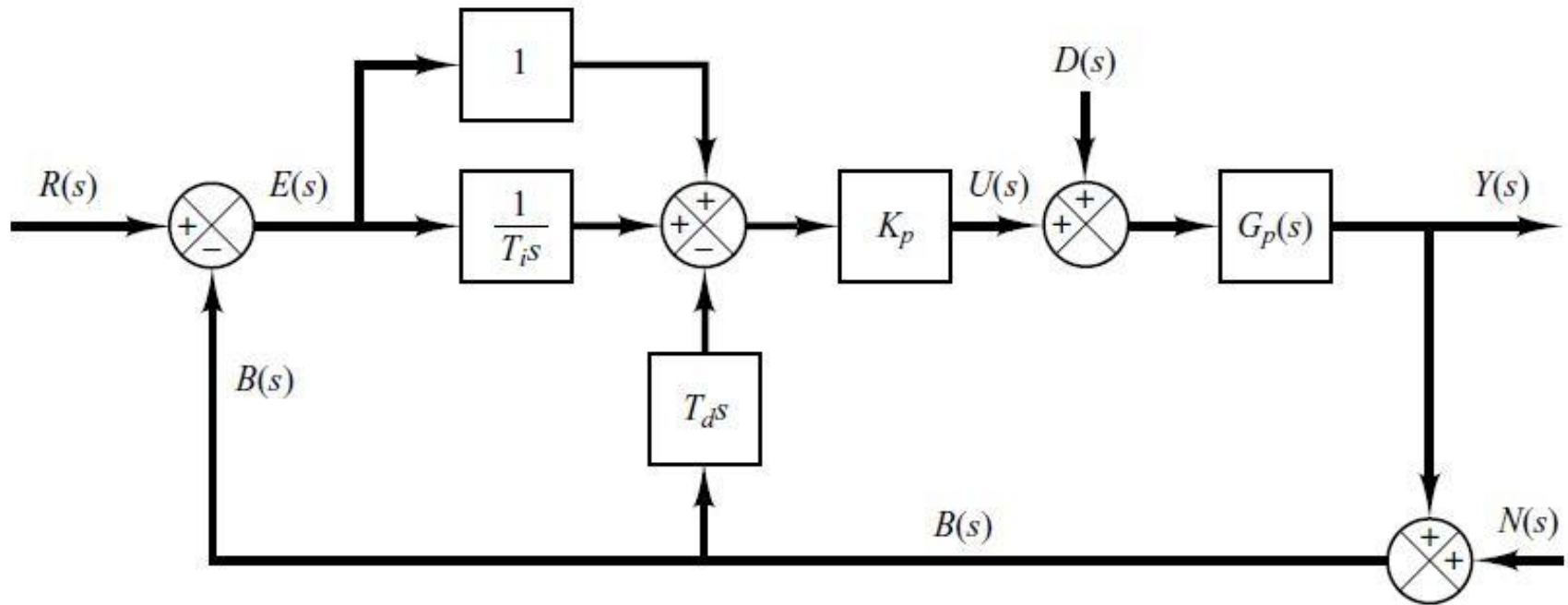
$D(s)$  --- Disturbance to the plant

$N(s)$  --- Noise to the measurement feedback

Let  $N(s) = D(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{R(s)} = ?$



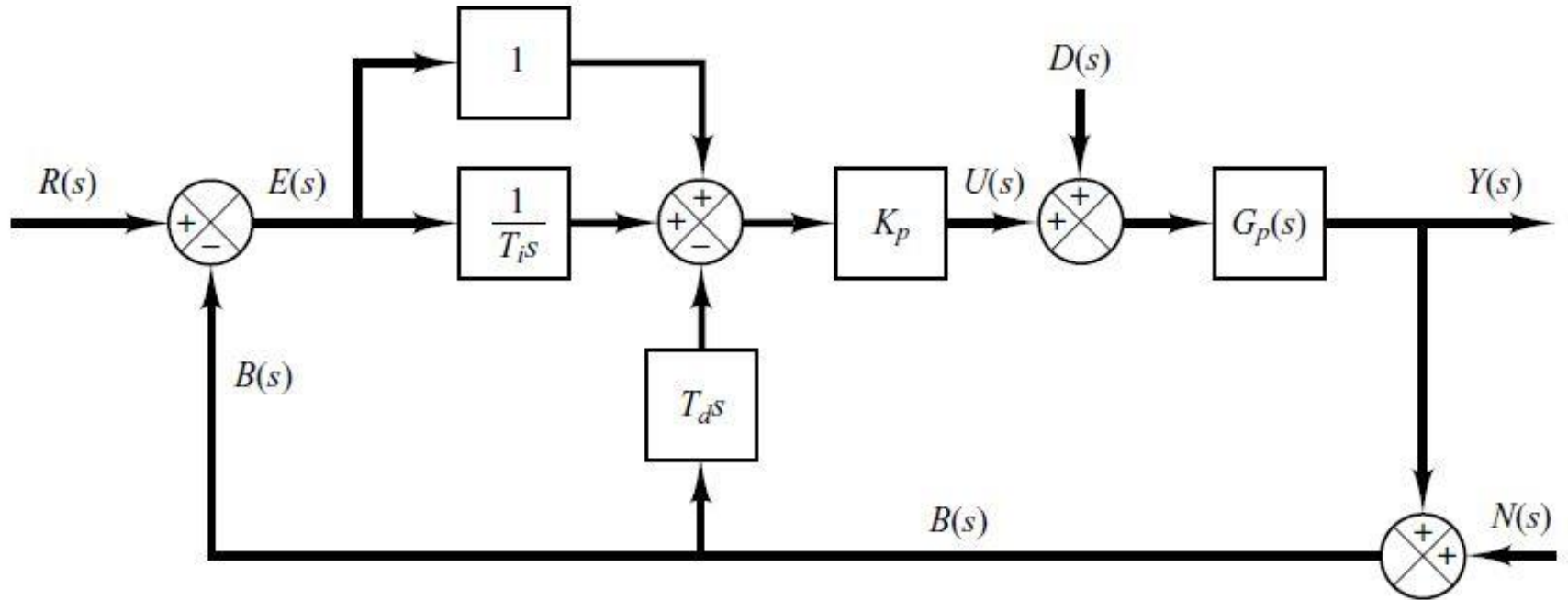
Let  $N(s) = D(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{R(s)} = ?$



**Solution:**

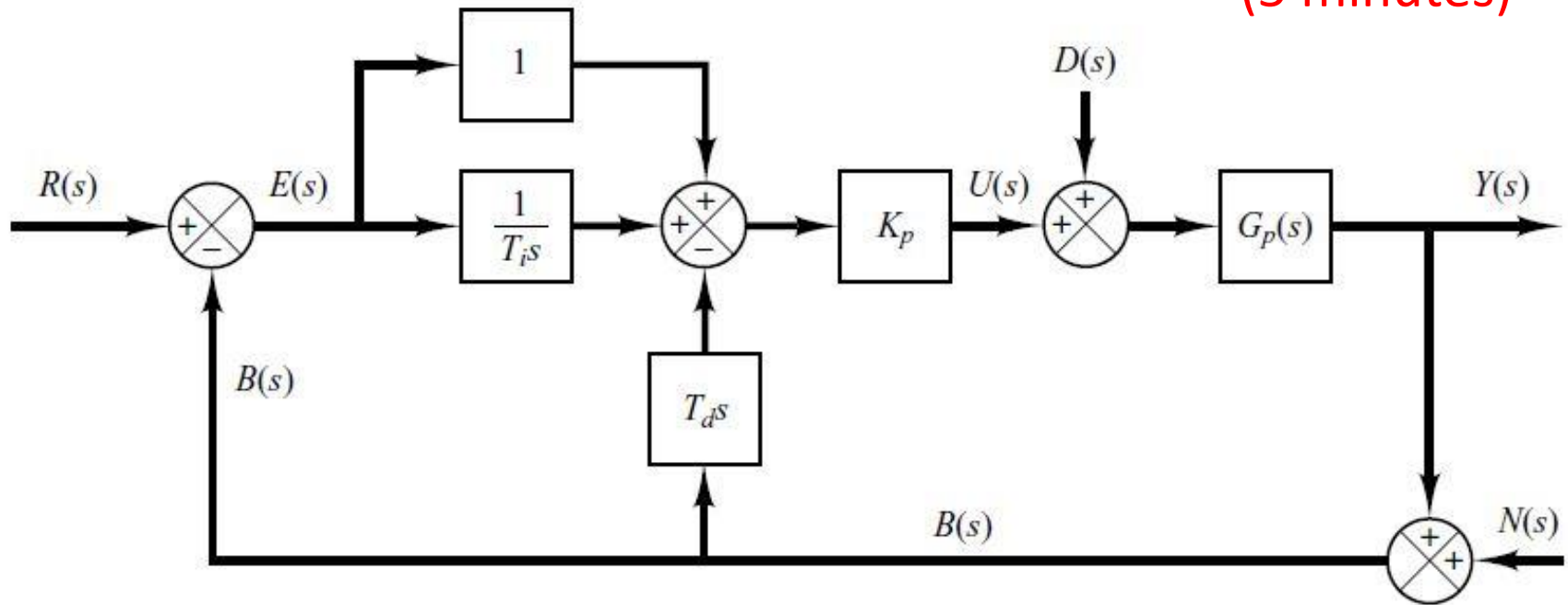
$$\frac{Y(s)}{R(s)} = \frac{K_p G_p(s) \left(1 + \frac{1}{T_i s}\right)}{1 + K_p G_p(s) \left(1 + \frac{1}{T_i s} + T_d s\right)}$$

# Disturbance Rejection

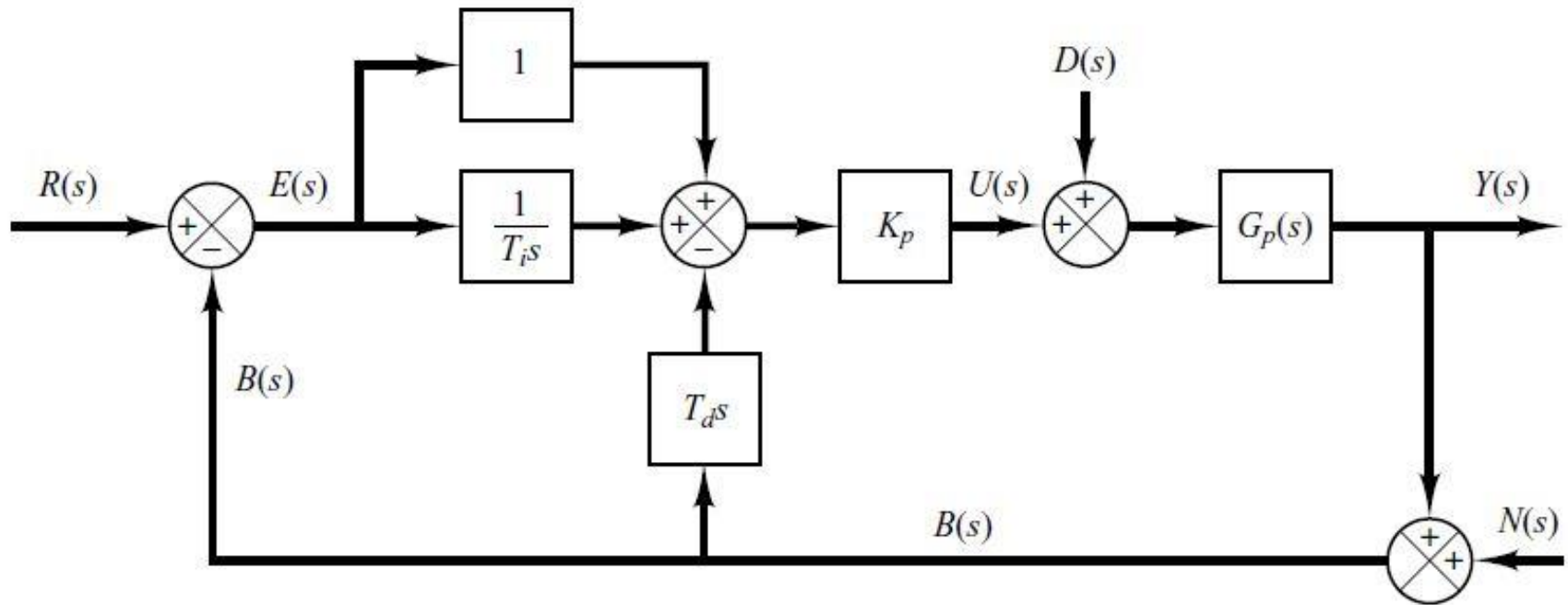


(i) Let  $N(s) = R(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{D(s)} = ?$

(5 minutes)



(i) Let  $N(s) = R(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{D(s)} = ?$



Solution:

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

(ii) Find the steady-state response to unit ramp input  $D(s)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ? \quad (5 \text{ minutes})$$

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

(ii) Find the steady-state response to unit ramp input  $D(s)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ?$$

Solution:

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) = \frac{T_i}{K_p}$$

(iii) Comment on the ability in disturbance rejection of the PI+D controller (5 minutes)

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

(ii) Find the steady-state response to unit ramp input  $D(s)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ?$$

Solution:

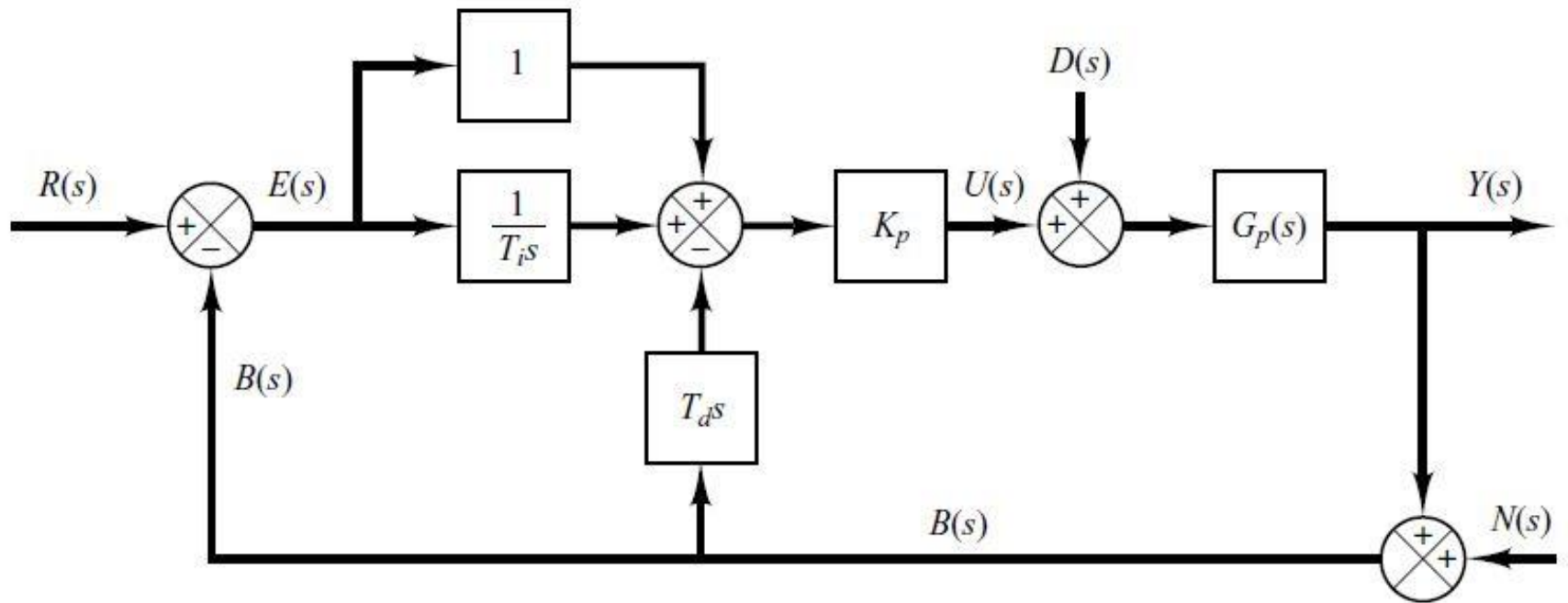
$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) = \frac{T_i}{K_p}$$

(iii) Comment on the ability in disturbance rejection of the PI+D controller

Comments:

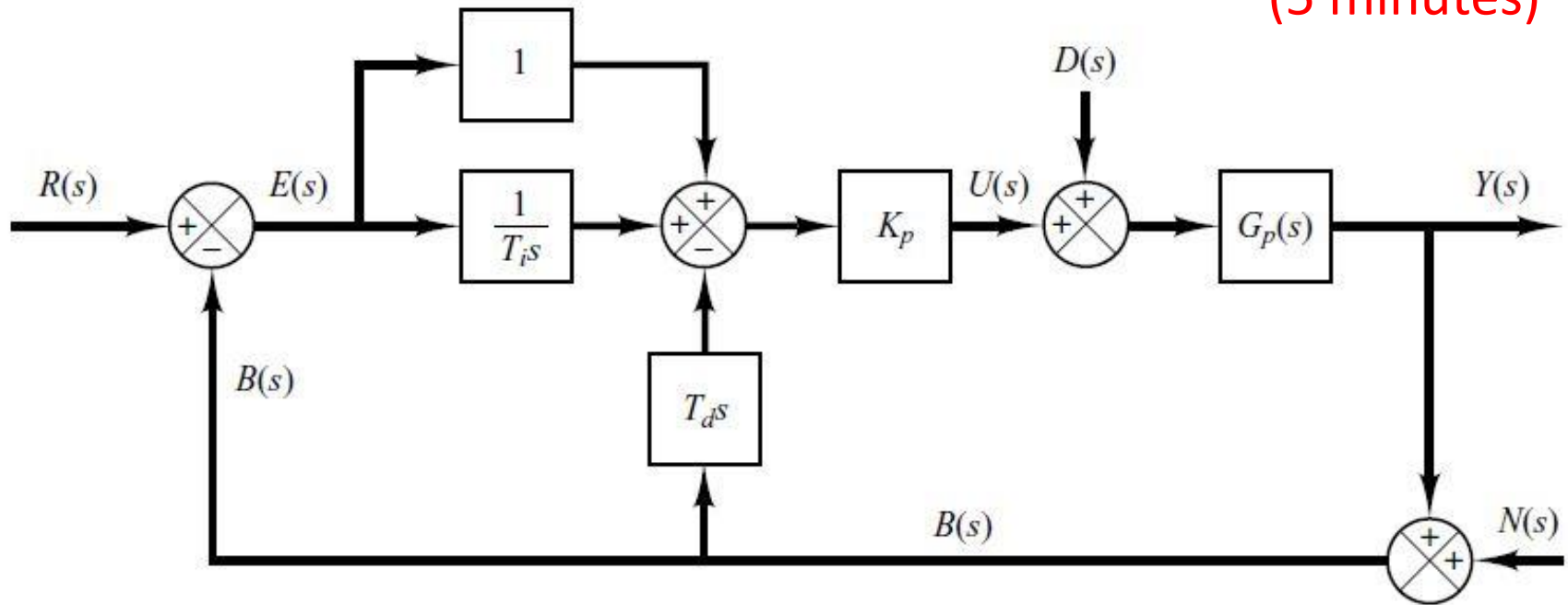
- It cannot completely attenuate disturbance
- It can reduce the disturbance effect by large  $P$ -gain  $K_p$

# Noise Rejection

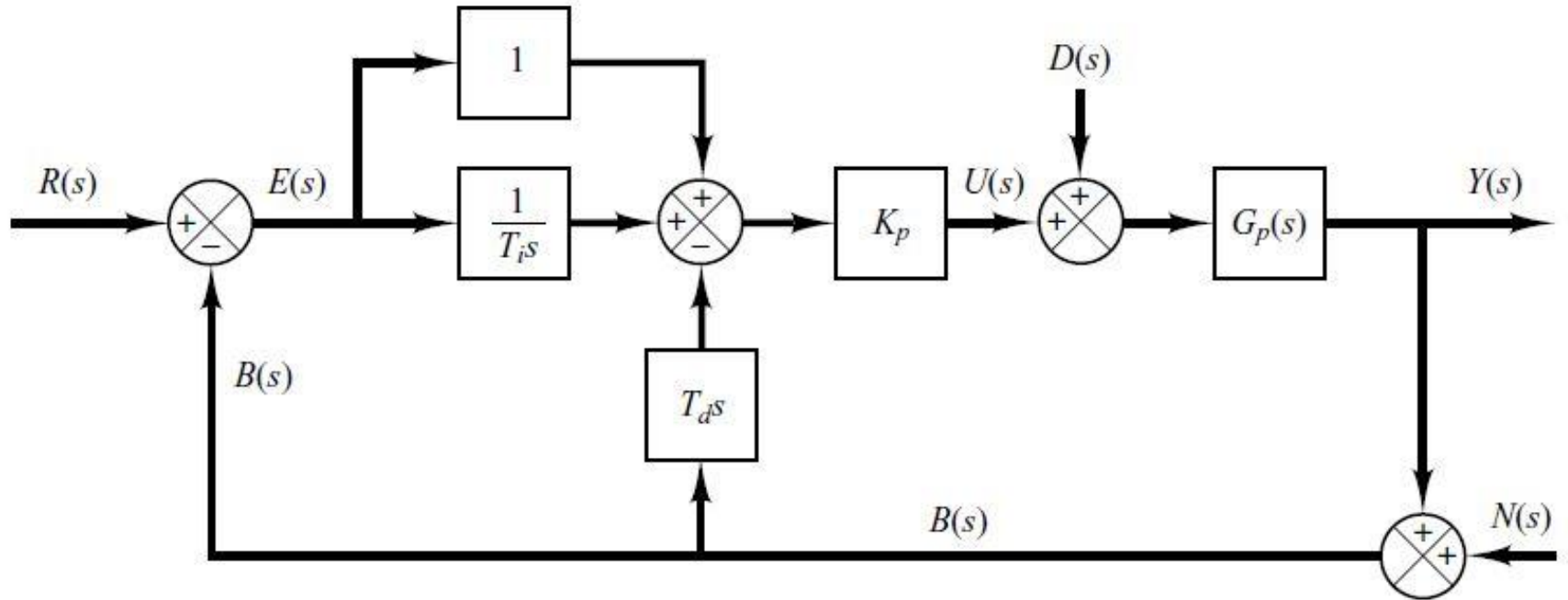


(i) Let  $D(s) = R(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{N(s)} = ?$

(5 minutes)



(i) Let  $D(s) = R(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{N(s)} = ?$



Solution:

$$\frac{Y(s)}{N(s)} = \frac{-G_p K_p \left( \frac{1}{T_i s} + 1 + T_d s \right)}{1 + G_p K_p \left( \frac{1}{T_i s} + 1 + T_d s \right)}$$

$$\frac{Y(s)}{N(s)} = \frac{-G_p K_p \left( \frac{1}{T_i s} + 1 + T_d s \right)}{1 + G_p K_p \left( \frac{1}{T_i s} + 1 + T_d s \right)}$$

(ii) Find the steady-state response to unit step input  $N(t)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ?$$

(5 minutes)

$$\frac{Y(s)}{N(s)} = \frac{-G_p K_p \left( \frac{1}{T_i s} + 1 + T_d s \right)}{1 + G_p K_p \left( \frac{1}{T_i s} + 1 + T_d s \right)}$$

(ii) Find the steady-state response to unit step input  $N(t)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ?$$

**Solution:** 
$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = \frac{-G_p K_p \frac{1}{T_i}}{G_p K_p \frac{1}{T_i}} = -1$$

(iii) Comment on the ability in noise rejection (filtering) of the PI+D controller (5 minutes)

$$\frac{Y(s)}{N(s)} = \frac{-G_p K_p \left( \frac{1}{T_i s} + 1 + T_d s \right)}{1 + G_p K_p \left( \frac{1}{T_i s} + 1 + T_d s \right)}$$

(ii) Find the steady-state response to unit step input  $N(t)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ?$$

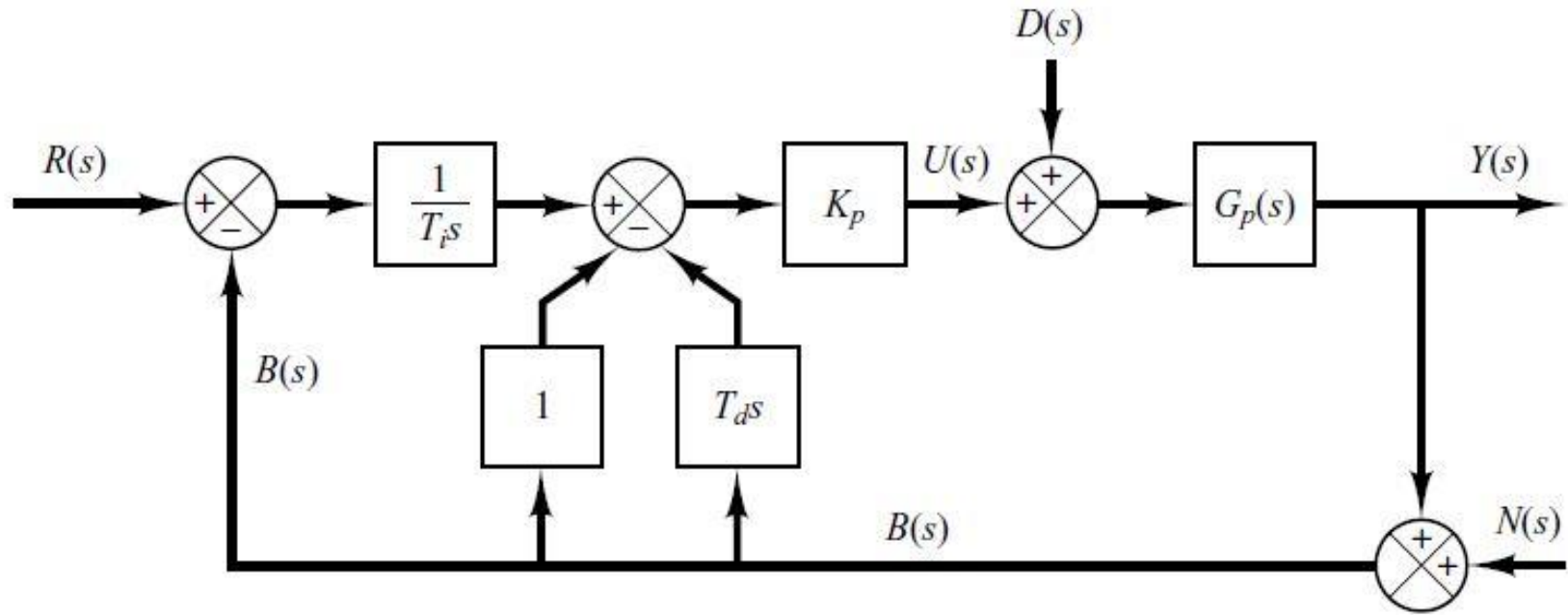
**Solution:** 
$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = \frac{-G_p K_p \frac{1}{T_i}}{G_p K_p \frac{1}{T_i}} = -1$$

(iii) Comment on the ability in noise rejection (filtering) of the PI+D controller

**Comments:**

PI+D controller has no effect on step noise input

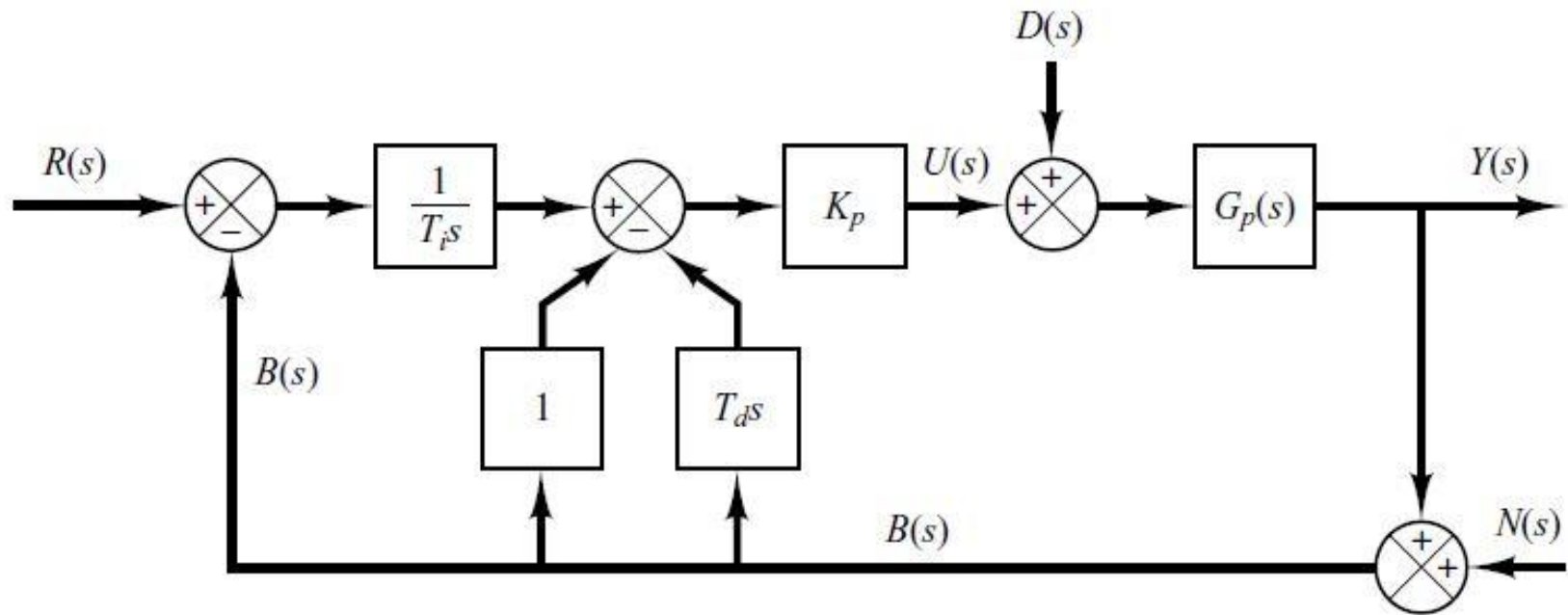
# I + PD Control System



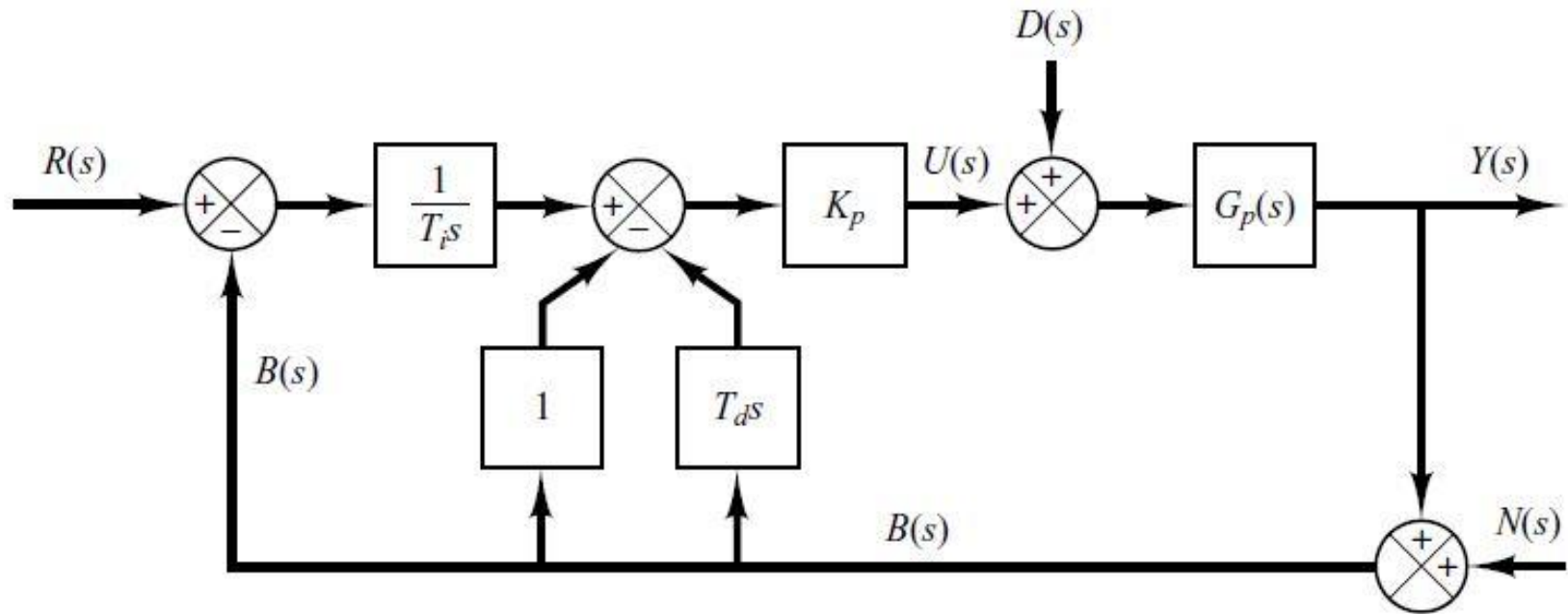
$D(s)$  --- Disturbance to the plant

$N(s)$  --- Noise to the measurement feedback

Let  $N(s) = D(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{R(s)} = ?$



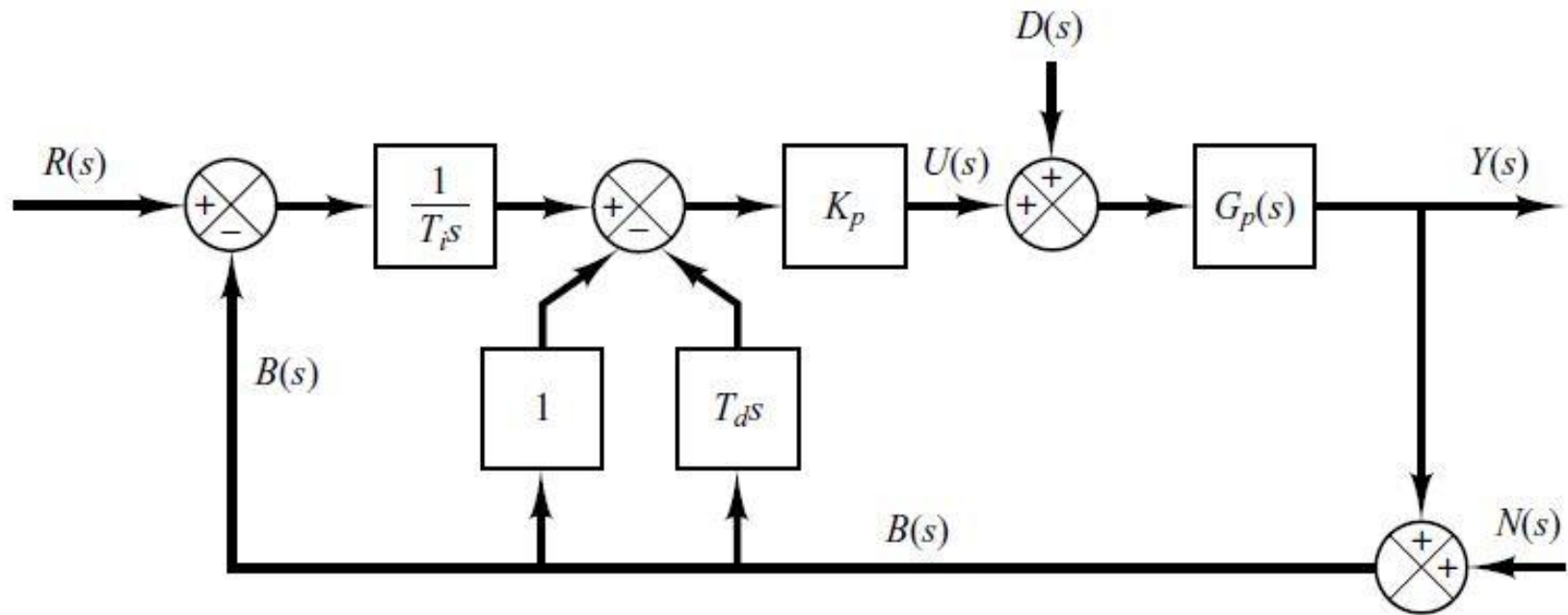
Let  $N(s) = D(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{R(s)} = ?$



**Solution:**

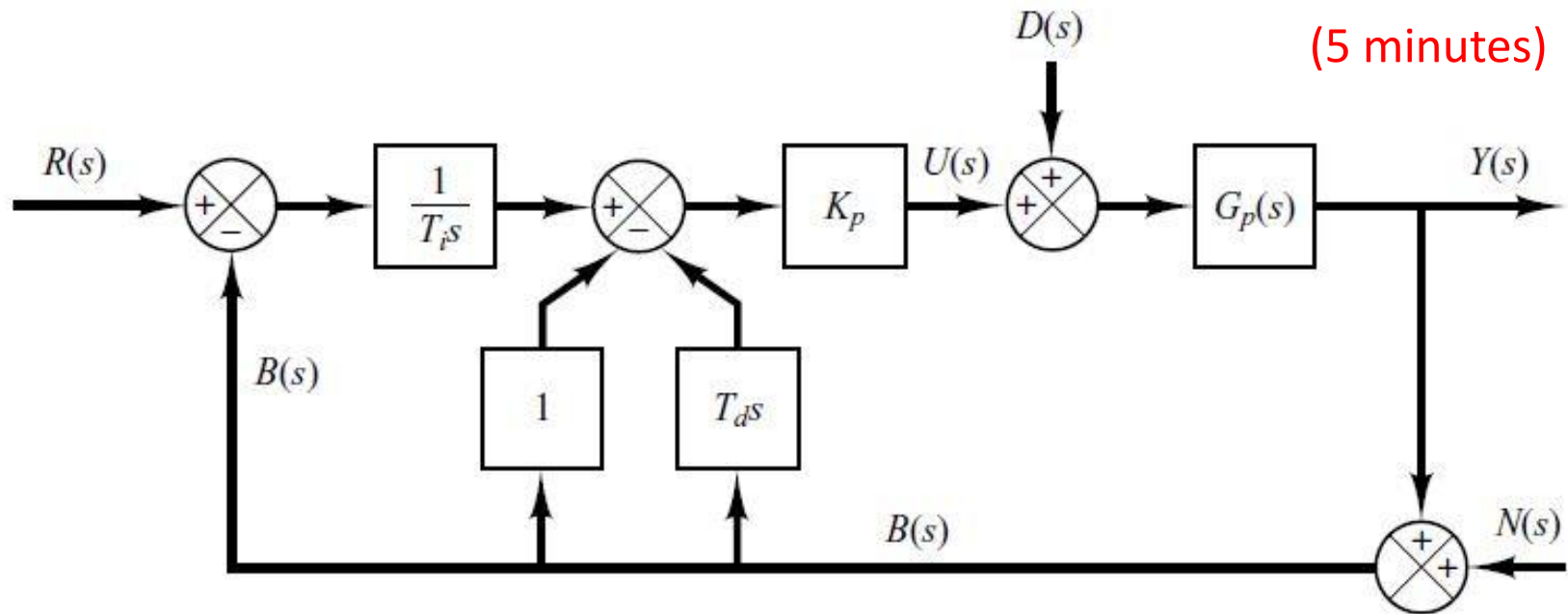
$$\frac{Y(s)}{R(s)} = \frac{K_p G_p(s) \frac{1}{T_i s}}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

# Disturbance Rejection

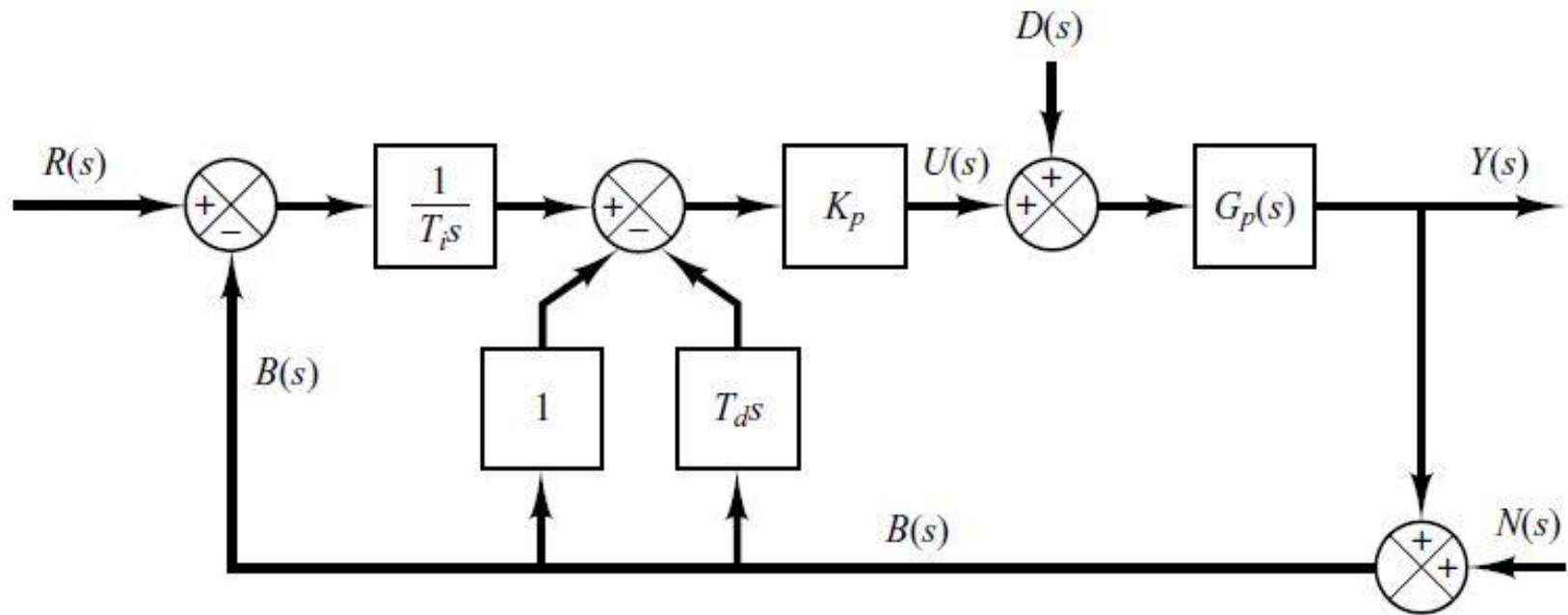


(i) Let  $N(s) = R(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{D(s)} = ?$

(5 minutes)



(i) Let  $N(s) = R(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{D(s)} = ?$



**Solution:**

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

(ii) Find the steady-state response to unit ramp input  $N(t)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ? \quad (5 \text{ minutes})$$

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

(ii) Find the steady-state response to unit ramp input  $N(t)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ?$$

**Solution:** 
$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = \frac{T_i}{K_p}$$

(iii) Comment on the ability in disturbance rejection of the I+PD controller (5 minutes)

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

(ii) Find the steady-state response to unit ramp input  $N(t)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ?$$

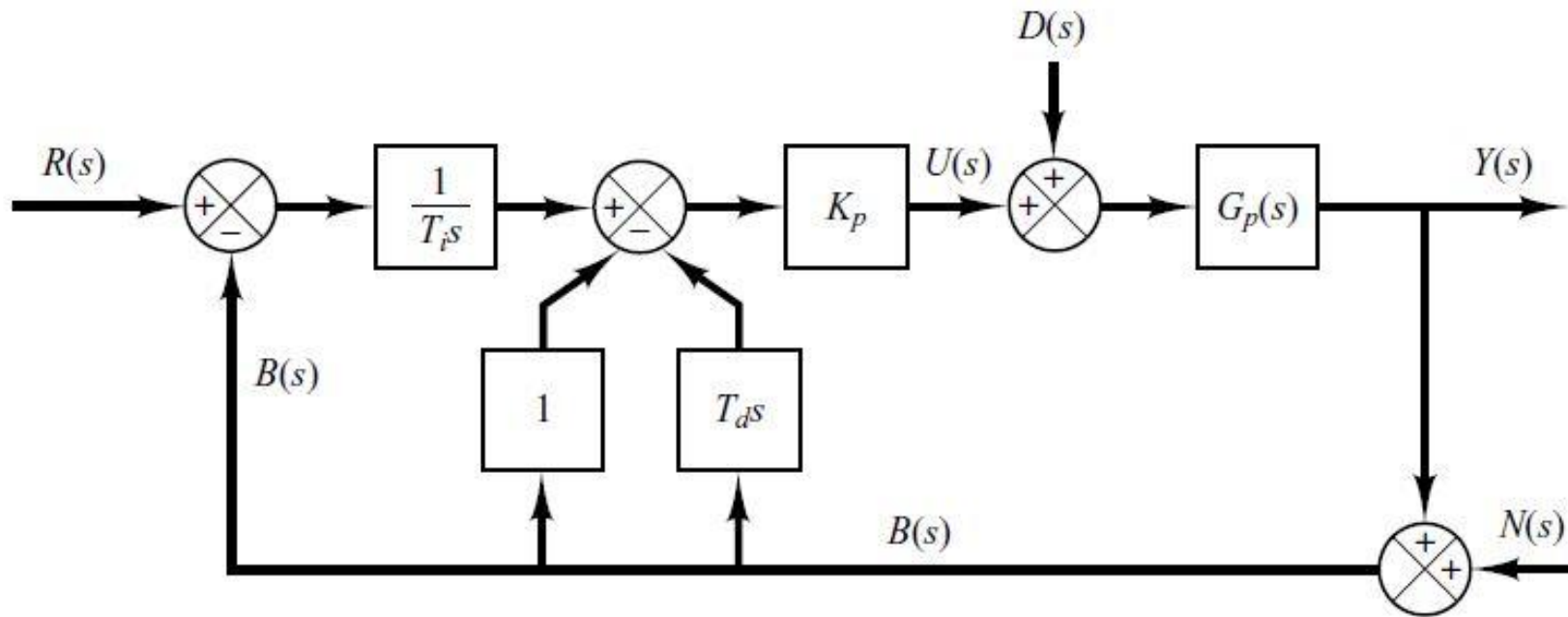
**Solution:** 
$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = \frac{T_i}{K_p}$$

(iii) Comment on the ability in disturbance rejection of the I+PD controller

**Comments:**

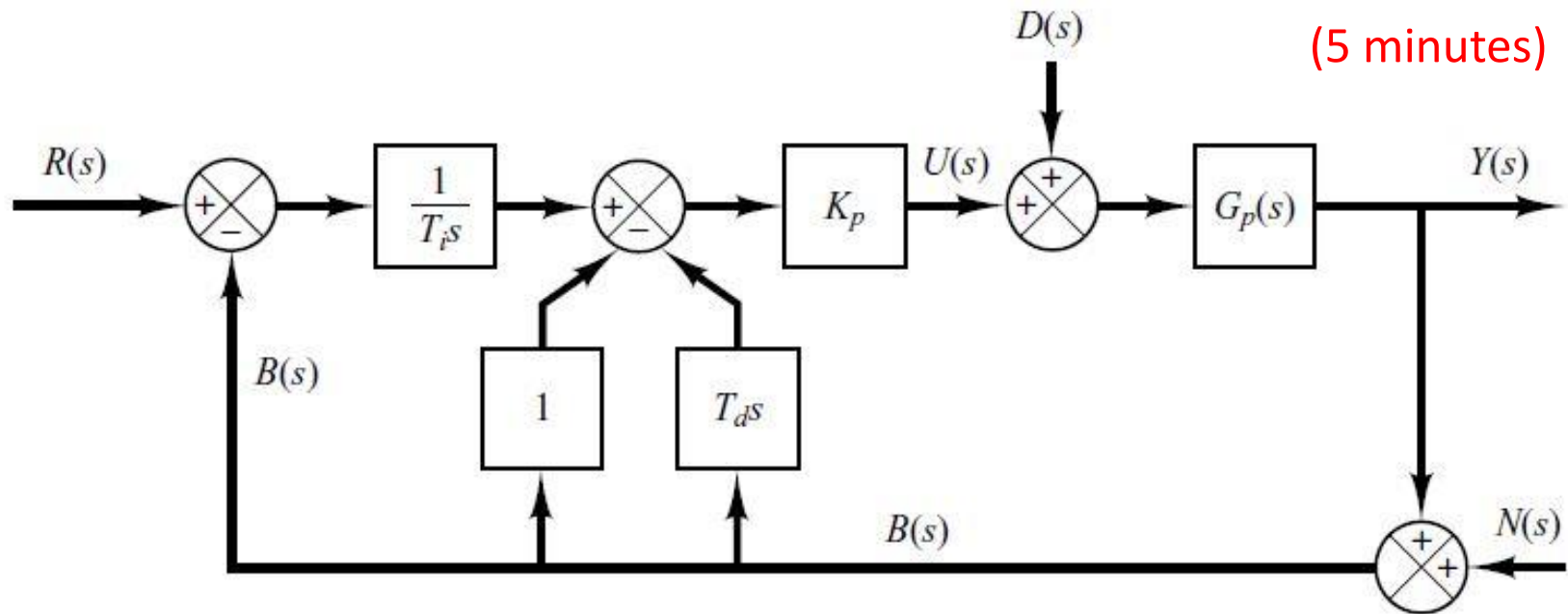
- It cannot completely attenuate disturbance
- It can reduce the disturbance effect by large  $P$ -gain  $K_p$
- It is better than PI+D controller (input is unit ramp)

# Noise Rejection

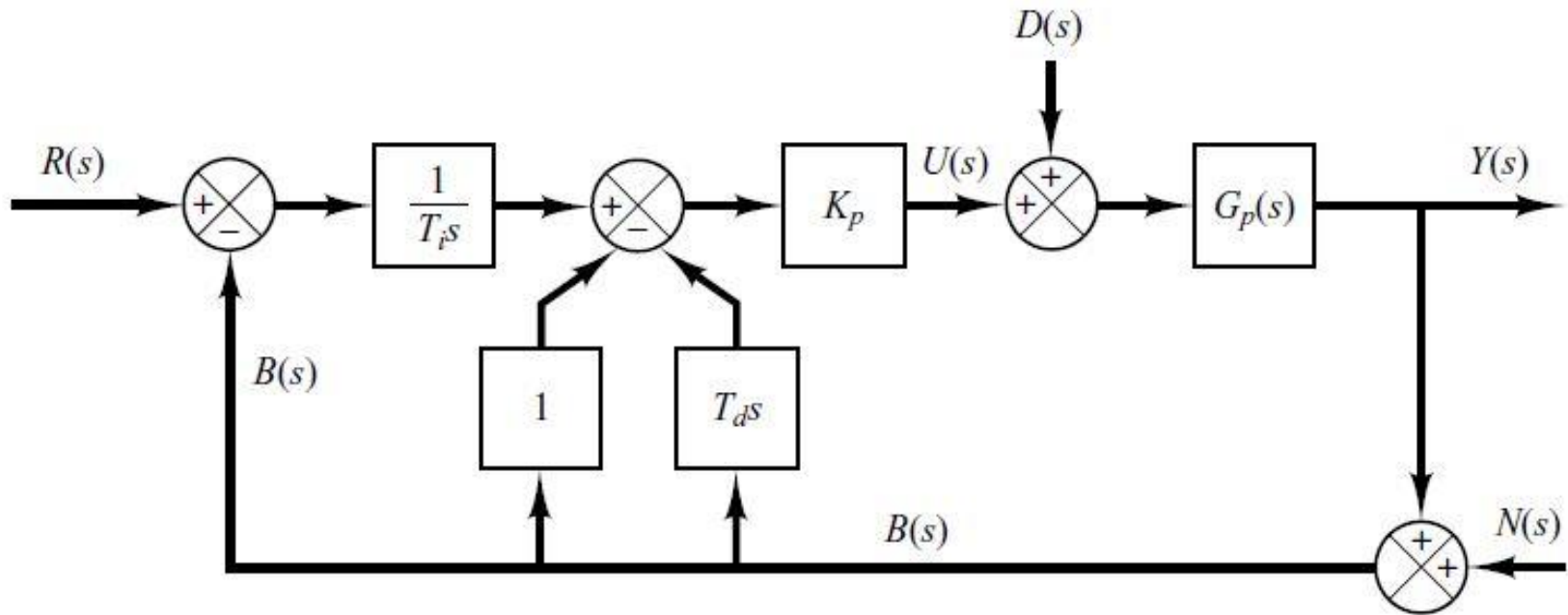


(i) Let  $D(s) = R(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{N(s)} = ?$

(5 minutes)



(i) Let  $D(s) = R(s) = 0$ . Find the Transfer Function  $\frac{Y(s)}{N(s)} = ?$



**Solution:**

$$\frac{Y(s)}{N(s)} = \frac{K_p G_p(s) \left( \frac{1}{T_i s} - T_d s - 1 \right)}{1 - K_p G_p(s) \left( \frac{1}{T_i s} - T_d s - 1 \right)}$$

$$\frac{Y(s)}{N(s)} = \frac{K_p G_p(s) \left( \frac{1}{T_i s} - T_d s - 1 \right)}{1 - K_p G_p(s) \left( \frac{1}{T_i s} - T_d s - 1 \right)}$$

(ii) Find the steady-state response to unit step input  $N(t)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ? \quad (5 \text{ minutes})$$

$$\frac{Y(s)}{N(s)} = \frac{K_p G_p(s) \left( \frac{1}{T_i s} - T_d s - 1 \right)}{1 - K_p G_p(s) \left( \frac{1}{T_i s} - T_d s - 1 \right)}$$

(ii) Find the steady-state response to unit step input  $N(t)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ?$$

**Solution:** 
$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = \frac{K_p G_p(s) \frac{1}{T_i}}{-K_p G_p(s) \frac{1}{T_i}} = -1$$

(iii) Comment on the ability in noise rejection of the I+PD controller (5 minutes)

$$\frac{Y(s)}{N(s)} = \frac{K_p G_p(s) \left( \frac{1}{T_i s} - T_d s - 1 \right)}{1 - K_p G_p(s) \left( \frac{1}{T_i s} - T_d s - 1 \right)}$$

(ii) Find the steady-state response to unit step input  $N(t)$ :

$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = ?$$

**Solution:** 
$$y_{ss} = \lim_{t \rightarrow \infty} y(t) = \frac{K_p G_p(s) \frac{1}{T_i}}{-K_p G_p(s) \frac{1}{T_i}} = -1$$

(iii) Comment on the ability in noise rejection of the I+PD controller

**Comments:**

- It has no effect on step noise input
- It is similar to the PI+D controller

**END**