

Name: _____

Student Number: _____

EE3101 Communication Engineering
Quiz 1a

Attempt ALL Questions [Total: 16 marks]

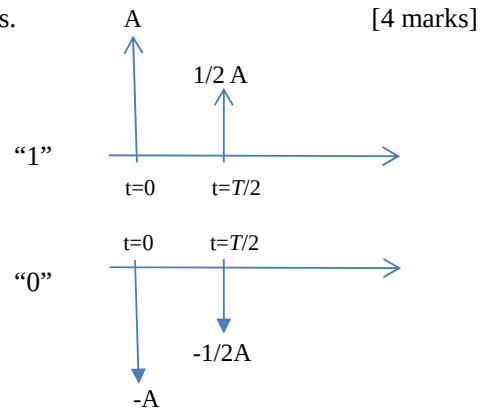
- (1) In a binary transmission system, binary “1” is represented by two positive discrete impulses with amplitudes $+A$ and $+1/2 A$, and separation $T/2$ (where T is the bit separation). Binary “0” is represented by two negative discrete impulses with amplitudes $-A$ and $-1/2 A$, and separation $T/2$, as shown in the following figure. Assume random data is used.

- (a) Calculate R_0 and R_1 of the auto-correlation function of the discrete impulse sequence. You need to show detailed calculation in every step. [5 marks]
- (b) It is given that $R_r = 0$ for $|r| > 1$. Show that the power spectral density of the discrete impulse sequence can be expressed as:

$$P(\omega) = \frac{A^2}{T} [a + b \cos(c\omega T)]$$

where a , b , and c are constants. Determine their values.

{ Note: $\cos\theta = \frac{\exp(j\theta) + \exp(-j\theta)}{2}$ }



$$\begin{aligned} (a) \quad R_0 &= \frac{A^2}{4} + \frac{(0.5A)^2}{4} + \frac{(-A)^2}{4} + \frac{(-0.5A)^2}{4} = \frac{2.5A^2}{4} \\ R_1 &= \frac{A A}{4 \cdot 2} + \frac{1 A}{4 \cdot 2} \left[\frac{A}{2} + \frac{-A}{2} \right] + \frac{-A - A}{4 \cdot 2} + \frac{1 - A}{4 \cdot 2} \left[\frac{A}{2} + \frac{-A}{2} \right] \\ &\quad \begin{matrix} a & + & b & + & c & + & d \end{matrix} \\ R_1 &= \frac{A^2}{4} \end{aligned}$$

2 marks

0.5 marks for each item

1 mark

$$(b) \quad P(\omega) = \frac{1}{T'} \sum_{r=-\infty}^{\infty} R_r \exp(-r\omega T')$$

or use $T/2$ to replace T'

1 mark

0.5 marks if T' is replaced with T

$$P(\omega) = \frac{2}{T} \left[\frac{A^2}{4} \exp\left(\frac{j\omega T}{2}\right) + \frac{2.5A^2}{4} + \frac{A^2}{4} \exp\left(\frac{-j\omega T}{2}\right) \right]$$

$$P(\omega) = \frac{A^2}{T} \left[1.25 + \cos\left(\frac{\omega T}{2}\right) \right]$$

i.e. $a = 1.25, b = 1, c = 0.5$ (or -0.5)

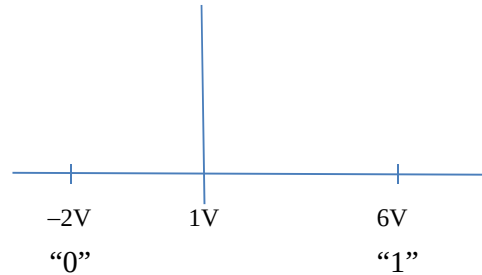
1 mark each

- (2) Consider a receiver of a binary system. The mean signal levels at the threshold comparator input are -2V and $+6\text{V}$ for “0” and “1” signals respectively. The threshold is set at 1V and the rms of the AWGN at the threshold comparator input is 1.5V .
- (a) Assume that the probabilities of “0” and “1” are 0.4 and 0.6 respectively, show that the probability of error is:

$$P_e = aQ(c) + bQ(d)$$

where $Q(x)$ is the Q function. Determine the values of a, b, c and d . [6 marks]

- (b) Will the optimum threshold depends on the probabilities of “0” and “1”? [1 mark]



(a) $P_e = Q\left(\frac{d_1}{\sigma}\right) \text{Prob}(0) + Q\left(\frac{d_2}{\sigma}\right) \text{Prob}(1)$

$$d_1 = 1 - (-2) = 3$$

$$d_2 = 6 - 1 = 5$$

i.e. $P_e = Q(2)0.4 + Q\left(\frac{10}{3}\right)0.6$

6 marks total

$$a = 0.4, b = 0.6 \quad \text{OR} \quad a = 0.6, b = 0.4$$

1 mark each, 2 marks max

With each correct matching of c and d

2 marks each, 4 marks max

Correct ans for c and d but wrong matching for a and b

4 marks

(i.e. wrong a and b values or the values of a and b are swapped in $Q(c)$ & $Q(d)$)

- (b) yes

1 mark

EE3101 Communication Engineering
Quiz 1b

Attempt ALL Questions [Total: 16 marks]

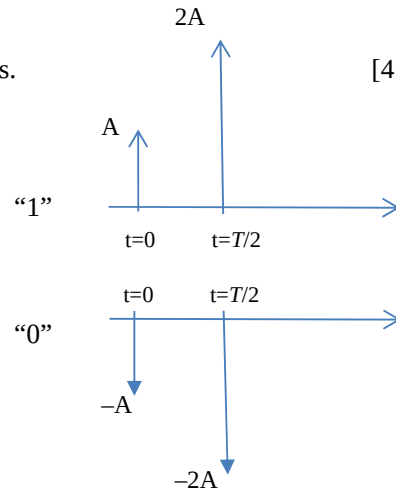
- (1) In a binary transmission system, binary “1” is represented by two positive discrete impulses with amplitudes $+A$ and $+2A$, and separation $T/2$ (where T is the bit separation). Binary “0” is represented by two negative discrete impulses with amplitudes $-A$ and $-2A$, and separation $T/2$, as shown in the following figure. Assume random data is used.

- (a) Calculate R_0 and R_1 of the auto-correlation function of the discrete impulse sequence. You need to show detailed calculation in every step. [5 marks]
(b) It is given that $R_r = 0$ for $|r| > 1$. Show that the power spectral density of the discrete impulse sequence can be expressed as:

$$P(\omega) = \frac{A^2}{T} [a + b \cos(c\omega T)]$$

where a , b , and c are constants. Determine their values. [4 marks]

{ Note: $\cos\theta = \frac{\exp(j\theta) + \exp(-j\theta)}{2}$ }



(a) $R_0 = \frac{A^2}{4} + \frac{(2A)^2}{4} + \frac{(-A)^2}{4} + \frac{(-2A)^2}{4} = \frac{10A^2}{4}$

$$R_1 = \frac{A}{4} 2A + \frac{1}{4} 2A \left[\frac{A}{2} + \frac{-A}{2} \right] + \frac{-A}{4} (-2A) + \frac{1}{4} (-2A) \left[\frac{A}{2} + \frac{-A}{2} \right]$$

$a \quad + \quad b \quad + \quad c \quad + \quad d$

$$R_1 = A^2$$

2 marks

0.5 marks for each item

1 mark

(b) $P(\omega) = \frac{1}{T} \sum_{r=-\infty}^{\infty} R_r \exp(-r\omega T')$

or use $T/2$ to replace T'

1 mark

0.5 marks if T' is replaced with T

$$P(\omega) = \frac{2}{T} \left[A^2 \exp\left(\frac{j\omega T}{2}\right) + \frac{10A^2}{4} + A^2 \exp\left(\frac{-j\omega T}{2}\right) \right]$$

$$P(\omega) = \frac{A^2}{T} \left[5 + 4 \cos\left(\frac{\omega T}{2}\right) \right]$$

i.e. $a = 5, b = 4, c = 0.5$ or (-0.5)

1 mark each

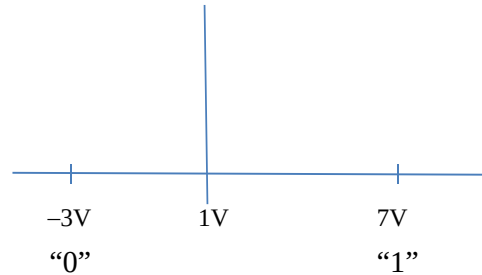
- (2) Consider a receiver of a binary system. The mean signal levels at the threshold comparator input are -3V and $+7\text{V}$ for “0” and “1” signals respectively. The probabilities of “0” and “1” are 0.3 and 0.7 respectively. The threshold is set at 1V .

(a) Assume that the rms of the AWGN at the threshold comparator input is 2V , show that the probability of error is:

$$P_e = aQ(c) + bQ(d)$$

where $Q(x)$ is the Q function. Determine the values of a, b, c and d . [6 marks]

(b) Will the optimum threshold depends on the rms of the AWGN ? [1 mark]



(a) $P_e = Q\left(\frac{d_1}{\sigma}\right) \text{Prob}(0) + Q\left(\frac{d_2}{\sigma}\right) \text{Prob}(1)$

$$d_1 = 1 - (-3) = 4$$

$$d_2 = 7 - 1 = 6$$

i.e. $P_e = Q(2)0.3 + Q(3)0.7$

6 marks total

$a = 0.3, b = 0.7$ OR $a = 0.7, b = 0.3$

1 mark each, 2 marks max

With each correct matching of c and d

2 marks each, 4 marks max

Correct ans for c and d but wrong matching for a and b 4 marks

(i.e. wrong a and b values or the values of a and b are swapped in $Q(c)$ & $Q(d)$)

(b) yes

1 mark