

Q1. $L: 3x - 2y = 1 \Rightarrow y = \frac{3}{2}x - \frac{1}{2}$ Its slope is $\frac{3}{2}$

Then, the line we want is

$$y - 5 = -\frac{2}{3}(x - 3) \Rightarrow y = -\frac{2}{3}x + 7$$

Q2 $P(\frac{3}{2}, 0)$ $Q(0, 2)$

The mid-point is $(\frac{3}{4}, 1)$

The slope of PQ is $\frac{2-0}{0-\frac{3}{2}} = -\frac{4}{3}$

So, the line we want is

$$y - 1 = \frac{3}{4}(x - \frac{3}{4}) \Rightarrow y = \frac{3}{4}x + \frac{7}{16}$$

Q3 Let the equation of the circle be

$$x^2 + y^2 + Dx + Ey + F = 0, \text{ where } D, E, F \text{ are constants}$$

$$\begin{cases} 4 + 4 + 2D + 2E + F = 0 \\ 1 + 1 - D - E + F = 0 \\ 12 + 8\sqrt{2} + (2 + 2\sqrt{2})D + F = 0 \end{cases} \Rightarrow \begin{cases} D = -4 \\ E = 2 \\ F = -4 \end{cases}$$

So, the equation is $x^2 + y^2 - 4x + 2y - 4 = 0$

or $(x - 2)^2 + (y + 1)^2 = 9$

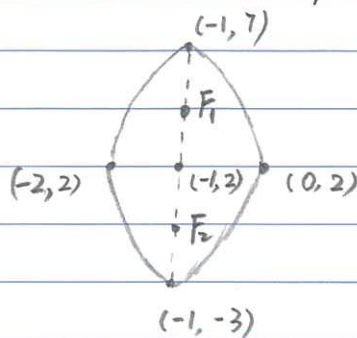
Q4. (a) $A = 25$, $C = 1$, $AC > 0$, it is an ellipse.

The standard form is

$$\frac{(y-2)^2}{5^2} + (x+1)^2 = 1$$

Foci $F_1(-1, 2 + 2\sqrt{6})$

$F_2(-1, 2 - 2\sqrt{6})$

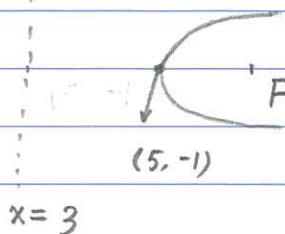


(b) $A = 0$, $C = 1$, $AC = 0$, it is a parabola

The standard form is

$$(y+1)^2 = 8(x-5)$$

Focus $(7, -1)$

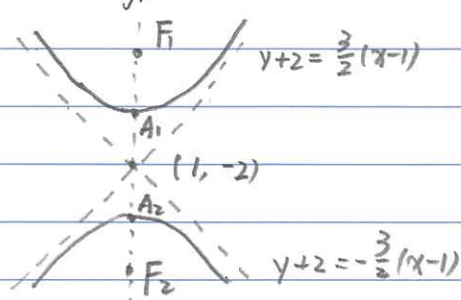


(c) $A = -9$, $C = 4$, $AC < 0$, it is a hyperbola

The standard form is

$$\frac{(y+2)^2}{3^2} - \frac{(x-1)^2}{2^2} = 1$$

vertices: $A_1(1, 1)$ $A_2(1, -5)$



Foci: $F_1(1, -2 + \sqrt{13})$, $F_2(1, -2 - \sqrt{13})$

Q5 (i) $(x-1)(2x+3) \geq 0 \Rightarrow x \geq 1 \text{ or } x \leq -\frac{3}{2}$

Domain: $(-\infty, -\frac{3}{2}] \cup [1, +\infty)$

Range: $[0, +\infty)$

(ii) $-x^2 - 2x \geq 0 \Rightarrow 1 \geq (x+1)^2 \Rightarrow -2 \leq x \leq 0$

Domain: $[-2, 0]$

$0 \leq -x^2 - 2x \leq 1 \Rightarrow 0 \leq \sqrt{-x^2 - 2x} \leq 1$ Range: $[0, 1]$

Q6 Domain of F : $\mathbb{R} \setminus \{1\}$

Range of F : $\mathbb{R} \setminus \{0\}$

Domain of G : $\mathbb{R} \setminus \{0\}$

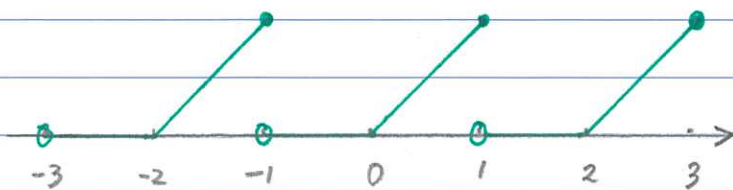
Range of G : $\mathbb{R} \setminus \{1\}$

$$(F \circ G)(x) = F(G(x)) = F\left(1 - \frac{1}{x}\right) = \frac{1}{1 - (1 - \frac{1}{x})} = \frac{1}{\frac{1}{x}} = x$$

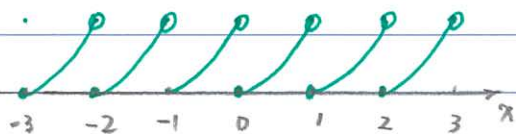
Domain of $F \circ G$: $\mathbb{R} \setminus \{0\}$

Q7

$$f(x) = \begin{cases} 0 & -1 < x \leq 0 \\ 2x & 0 < x \leq 1 \end{cases}$$



Q8



Range of f : $[0, 1)$

As $f(x+1) = (x+1 - [x+1])^2 = (x - [x])^2 = f(x)$.
 $f(x)$ is a periodic function of x with period 1.

Q9

$$y = F(x) = (x+2)^2 - 3 \Rightarrow y+3 = (x+2)^2$$

as $x \leq -2 \Rightarrow x+2 \leq 0$, then we have

$$x+2 = -\sqrt{y+3} \Rightarrow x = -2 - \sqrt{y+3}$$

$$\text{So } F^{-1}(x) = -2 - \sqrt{x+3}$$

Domain of F^{-1} : $[-3, \infty)$

Range of F^{-1} : $(-\infty, -2]$