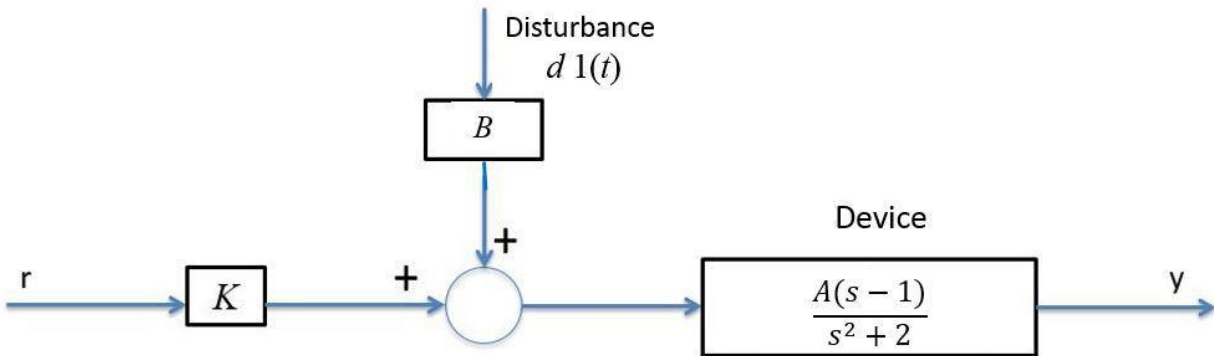


## EE3114 HW-6 Solutions

**6-1** Consider the following open-loop system, where A and B are given constants, K is the control gain to be designed, and  $1(t)$  is the unit step function.



Find:

- Output  $Y(s)$  in terms of both the unit step reference input  $r(t) = r_0 1(t)$  and the disturbance  $d 1(t)$  **(1.5points)**
- Find the steady-state output  $y(\infty) = ?$  **(1points)**
- Design an appropriate open-loop gain  $K = K_0 = ?$  so as to simplify  $y(\infty)$  if possible **(1.5points)**
- Show how much  $y(\infty)$  deviates from the reference value  $r_0$  **(1points)**

**SOLUTION:**

a)

$$R(s) = \mathcal{L}(r_0 1(t)) = \frac{r_0}{s}, \quad D(s) = \mathcal{L}(d 1(t)) = \frac{d}{s}$$

$$Y(s) = \frac{A(s-1)}{s^2+2} (KR(s) + BD(s)) = \frac{r_0 AK(s-1)}{s^3+2s} + \frac{dAB(s-1)}{s^3+2s}$$

b)

$$y(\infty) = \lim_{s \rightarrow 0} sY(s) = -\frac{1}{2}(r_0 AK + dAB)$$

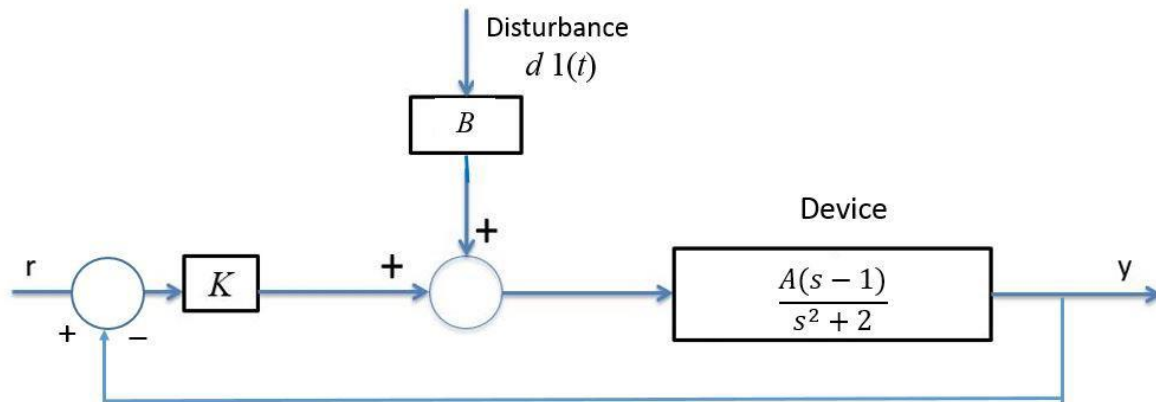
c)

$$\text{Open-loop design} \rightarrow K_0 = -2A^{-1} \rightarrow y(\infty) = r_0 - \frac{1}{2}dAB$$

d)

$y(\infty)$  deviates from the reference value  $r_0$  by  $e(\infty) = -\frac{1}{2}dAB$ , if  $d = 0$ , then  $e(\infty) = 0$ .

6-2 Consider the corresponding closed-loop system:



Find:

a) Output  $Y(s)$  in terms of both the unit step reference input  $r(t) = r_0 1(t)$  and the disturbance  $d 1(t)$  **(1.5points)**

b) Find the steady-state output  $y(\infty) = ?$  **(1points)**

c) Design an appropriate closed-loop gain  $K = K_c = ?$  so as to simplify  $y(\infty)$  if possible **(1.5points)**

d) Design a closed-loop gain  $K = K_c = ?$  so as to achieve  $e(\infty) = r_0 - y(\infty) = 0$  if possible **(1points)**

**SOLUTION:**

a)

$$R(s) = \mathcal{L}(r_0 1(t)) = \frac{r_0}{s}, \quad D(s) = \mathcal{L}(d 1(t)) = \frac{d}{s},$$

$$Y(s) = \frac{A(s-1)}{s^2+2} \left( K(R(s) - Y(s)) + BD(s) \right) \rightarrow$$

$$\begin{aligned} Y(s) &= \left( 1 + \frac{AK(s-1)}{s^2+2} \right)^{-1} \left( \frac{r_0 AK(s-1)}{s^3+2s} + \frac{dAB(s-1)}{s^3+2s} \right) \\ &= \frac{A(s-1)(r_0 K + dB)}{s(s^2+2+AK(s-1))} \end{aligned}$$

b)

$$y(\infty) = \lim_{s \rightarrow 0} sY(s) = -\frac{1}{2} \left( 1 - \frac{1}{2} AK \right)^{-1} (r_0 AK + dAB) = -\frac{r_0 AK + dAB}{2 - AK}$$

c) Open-loop design  $\rightarrow AK_c \gg 2 \rightarrow y(\infty) \approx r_0 + \frac{dB}{K_c}$

d)

$y(\infty)$  deviates from the reference value  $r_0$  by  $\frac{dB}{K_c}$ , thus  $e(\infty) \rightarrow 0$  when  $K_c \rightarrow \infty$ .

**Remarks:**

For c), some students choose  $K_c = \frac{1}{A}$  and then derive that  $y(\infty) = -r_0 - dAB$ . However, our reference value is  $r_0$  instead of  $-r_0$ . Please recall Lecture 7.

For d), some students cannot take a limitation approach to tackle this problem.