

EE3124 Tutorial 1 (Solution)

Basic Magnetism

Name:

Student No.:

Q1 - A motor's shaft is spinning at a speed of 1800 r/min. What is the shaft speed in radians per second?

Solution

$$\omega = (1800 \text{ r/min}) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ r}} \right) = 188.5 \text{ rad/s}$$

Q2 - A motor is supplying 50 Nm of torque to its load. If the motor's shaft is turning at 1500 r/min, what is the mechanical power supplied to the load in watts? In horsepower?

Solution

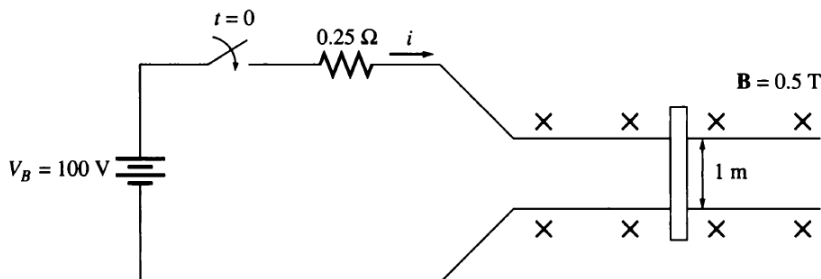
The mechanical power supplied to the load is

$$P = \tau \omega = (50 \text{ N} \cdot \text{m}) (1500 \text{ r/min}) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ r}} \right) = 7854 \text{ W}$$

$$P = (7854 \text{ W}) \left(\frac{1 \text{ hp}}{746 \text{ W}} \right) = 10.5 \text{ hp}$$

Q3 - A linear machine shown as follows has a magnetic flux density of 0.5 T directed into the page, a resistance of 0.25Ω , a bar length $l = 1.0 \text{ m}$, and a battery voltage of 100 V.

- (a) What is the initial force on the bar at starting? What is the initial current flow?
- (b) What is the no-load steady-state speed of the bar?
- (c) If the bar is loaded with a force of 25 N opposite to the direction of motion, what is the new steady-state speed? What is the efficiency of the machine under these circumstances?



Solution

(a) The current in the bar at starting is

$$i = \frac{V_B}{R} = \frac{100 \text{ V}}{0.25 \Omega} = 400 \text{ A}$$

Therefore, the force on the bar at starting is

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) = (400 \text{ A})(1 \text{ m})(0.5 \text{ T}) = 200 \text{ N, to the right}$$

(b) The no-load steady-state speed of this bar can be found from the equation

$$V_B = e_{\text{ind}} = vBl$$

$$v = \frac{V_B}{Bl} = \frac{100 \text{ V}}{(0.5 \text{ T})(1 \text{ m})} = 200 \text{ m/s}$$

(c) With a load of 25 N opposite to the direction of motion, the steady-state current flow in the bar will be given by

$$F_{\text{app}} = F_{\text{ind}} = i l B$$

$$i = \frac{F_{\text{app}}}{Bl} = \frac{25 \text{ N}}{(0.5 \text{ T})(1 \text{ m})} = 50 \text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 100 \text{ V} - (50 \text{ A})(0.25 \Omega) = 87.5 \text{ V}$$

and the velocity of the bar will be

$$v = \frac{V_B}{Bl} = \frac{87.5 \text{ V}}{(0.5 \text{ T})(1 \text{ m})} = 175 \text{ m/s}$$

The *input* power to the linear machine under these conditions is

$$P_{\text{in}} = V_B i = (100 \text{ V})(50 \text{ A}) = 5000 \text{ W}$$

The *output* power from the linear machine under these conditions is

$$P_{\text{out}} = V_B i = (87.5 \text{ V})(50 \text{ A}) = 4375 \text{ W}$$

Therefore, the efficiency of the machine under these conditions is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{4375 \text{ W}}{5000 \text{ W}} \times 100\% = 87.5\%$$

Q4 - A linear machine (same topology as the upper figure) has the following characteristics:

$$B = 0.5 \text{ T into page}$$

$$R = 0.25 \Omega$$

$$l = 0.5 \text{ m}$$

$$V_B = 120 \text{ V}$$

(a) If this bar has a load of 20 N attached to it opposite to the direction of motion, what is the steady-state speed of the bar?

(b) If the bar runs off into a region where the flux density falls to 0.45 T, what happens to the bar? What is its final steady-state speed?

(c) Suppose V_B is now decreased to 100 V with everything else remaining as in part (b). What is the new steady-state speed of the bar?

(d) From the results for parts (b) and (c), what are two methods of controlling the speed of a linear machine (or a real dc motor)?

Solution

(a) With a load of 20 N opposite to the direction of motion, the steady-state current flow in the bar will be given by

$$F_{\text{app}} = F_{\text{ind}} = i l B$$

$$i = \frac{F_{\text{app}}}{B l} = \frac{20 \text{ N}}{(0.5 \text{ T})(0.5 \text{ m})} = 80 \text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - i R = 120 \text{ V} - (80 \text{ A})(0.25 \Omega) = 100 \text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{B l} = \frac{100 \text{ V}}{(0.5 \text{ T})(0.5 \text{ m})} = 400 \text{ m/s}$$

(b) If the flux density drops to 0.45 T while the load on the bar remains the same, there will be a speed transient until $F_{\text{app}} = F_{\text{ind}} = 20 \text{ N}$ again. The new steady state current will be

$$F_{\text{app}} = F_{\text{ind}} = i l B$$

$$i = \frac{F_{\text{app}}}{B l} = \frac{20 \text{ N}}{(0.45 \text{ T})(0.5 \text{ m})} = 88.9 \text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - i R = 120 \text{ V} - (88.9 \text{ A})(0.25 \Omega) = 97.8 \text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{B l} = \frac{97.8 \text{ V}}{(0.45 \text{ T})(0.5 \text{ m})} = 433 \text{ m/s}$$

(c) If the battery voltage is decreased to 100 V while the load on the bar remains the same, there will be a speed transient until $F_{\text{app}} = F_{\text{ind}} = 20 \text{ N}$ again. The new steady state current will be

$$F_{\text{app}} = F_{\text{ind}} = i l B$$

$$i = \frac{F_{\text{app}}}{B l} = \frac{20 \text{ N}}{(0.45 \text{ T})(0.5 \text{ m})} = 88.9 \text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - i R = 100 \text{ V} - (88.9 \text{ A})(0.25 \Omega) = 77.8 \text{ V}$$

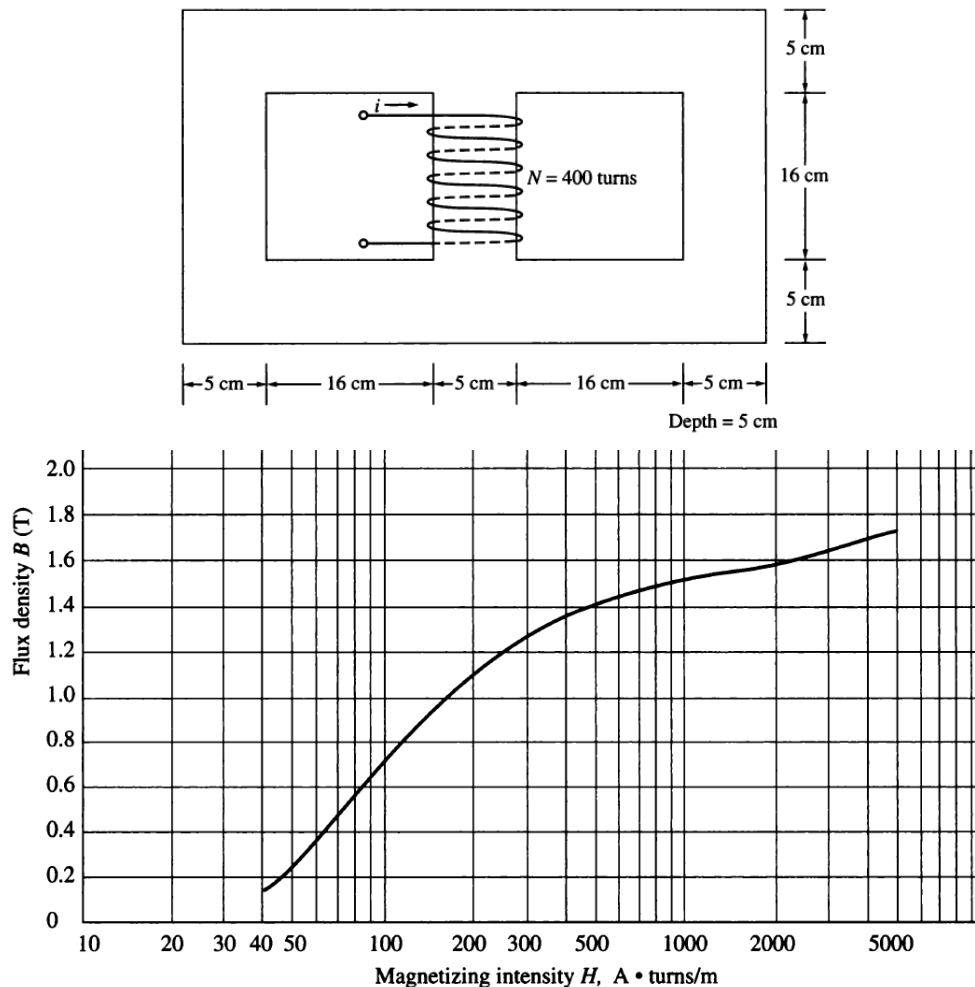
and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{B l} = \frac{77.8 \text{ V}}{(0.45 \text{ T})(0.5 \text{ m})} = 344 \text{ m/s}$$

(d) From the results of the two previous parts, we can see that there are two ways to control the speed of a linear dc machine. *Reducing* the flux density B of the machine *increases* the steady-state speed, and *reducing* the battery voltage V_B *decreases* the steady-state speed of the machine. Both of these speed control methods work for real dc machines as well as for linear machines.

Q5 - A core with three legs is shown as follows. Its depth is 5 cm, and there are 400 turns on the center leg. The remaining dimensions are shown in the figure. The core is composed of a steel having the magnetization curve as follows. Answer the following questions about this core:

- What current is required to produce a flux density of 0.5 T in the central leg of the core?
- What current is required to produce a flux density of 1.0 T in the central leg of the core?
Is it twice the current in part (a)?
- What are the reluctances of the central and right legs of the core under the conditions in part (a)?
- What are the reluctances of the central and right legs of the core under the conditions in part (b)?
- What conclusion can you make about reluctances in real magnetic cores?



Solution

(a) A flux density of 0.5 T in the central core corresponds to a total flux of

$$\phi_{\text{TOT}} = BA = (0.5 \text{ T})(0.05 \text{ m})(0.05 \text{ m}) = 0.00125 \text{ Wb}$$

By symmetry, the flux in each of the two outer legs must be $\phi_1 = \phi_2 = 0.000625 \text{ Wb}$, and the flux density in the other legs must be

$$B_1 = B_2 = \frac{0.000625 \text{ Wb}}{(0.05 \text{ m})(0.05 \text{ m})} = 0.25 \text{ T}$$

The magnetizing intensity H required to produce a flux density of 0.25 T can be found from Figure 1-10c. It is $50 \text{ A} \cdot \text{t/m}$. Similarly, the magnetizing intensity H required to produce a flux density of 0.50 T is $75 \text{ A} \cdot \text{t/m}$. The mean length of the center leg is 21 cm and the mean length of each outer leg is 63 cm, so the total MMF needed is

$$\begin{aligned}\mathcal{F}_{\text{TOT}} &= H_{\text{center}} l_{\text{center}} + H_{\text{outer}} l_{\text{outer}} \\ \mathcal{F}_{\text{TOT}} &= (75 \text{ A} \cdot \text{t/m})(0.21 \text{ m}) + (50 \text{ A} \cdot \text{t/m})(0.63 \text{ m}) = 47.3 \text{ A} \cdot \text{t}\end{aligned}$$

and the required current is

$$i = \frac{\mathcal{F}_{\text{TOT}}}{N} = \frac{47.3 \text{ A} \cdot \text{t}}{400 \text{ t}} = 0.12 \text{ A}$$

(b) A flux density of 1.0 T in the central core corresponds to a total flux of

$$\phi_{\text{TOT}} = BA = (1.0 \text{ T})(0.05 \text{ m})(0.05 \text{ m}) = 0.0025 \text{ Wb}$$

By symmetry, the flux in each of the two outer legs must be $\phi_1 = \phi_2 = 0.00125 \text{ Wb}$, and the flux density in the other legs must be

$$B_1 = B_2 = \frac{0.00125 \text{ Wb}}{(0.05 \text{ m})(0.05 \text{ m})} = 0.50 \text{ T}$$

The magnetizing intensity H required to produce a flux density of 0.50 T can be found from Figure 1-10c. It is $75 \text{ A} \cdot \text{t/m}$. Similarly, the magnetizing intensity H required to produce a flux density of 1.00 T is about $160 \text{ A} \cdot \text{t/m}$. Therefore, the total MMF needed is

$$\begin{aligned}\mathcal{F}_{\text{TOT}} &= H_{\text{center}} l_{\text{center}} + H_{\text{outer}} l_{\text{outer}} \\ \mathcal{F}_{\text{TOT}} &= (160 \text{ A} \cdot \text{t/m})(0.21 \text{ m}) + (75 \text{ A} \cdot \text{t/m})(0.63 \text{ m}) = 80.8 \text{ A} \cdot \text{t}\end{aligned}$$

and the required current is

$$i = \frac{\mathcal{F}_{\text{TOT}}}{N} = \frac{80.8 \text{ A} \cdot \text{t}}{400 \text{ t}} = 0.202 \text{ A}$$

This current is *not* twice the current in part (a).

(c) The reluctance of the central leg of the core under the conditions of part (a) is:

$$\mathcal{R}_{\text{cent}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(75 \text{ A} \cdot \text{t/m})(0.21 \text{ m})}{0.00125 \text{ Wb}} = 12.6 \text{ kA} \cdot \text{t/Wb}$$

The reluctance of the right leg of the core under the conditions of part (a) is:

$$\mathcal{R}_{\text{right}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(50 \text{ A} \cdot \text{t/m})(0.63 \text{ m})}{0.000625 \text{ Wb}} = 50.4 \text{ kA} \cdot \text{t/Wb}$$

(d) The reluctance of the central leg of the core under the conditions of part (b) is:

$$\mathcal{R}_{\text{cent}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(160 \text{ A} \cdot \text{t/m})(0.21 \text{ m})}{0.0025 \text{ Wb}} = 13.4 \text{ kA} \cdot \text{t/Wb}$$

The reluctance of the right leg of the core under the conditions of part (b) is:

$$\mathcal{R}_{\text{right}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(75 \text{ A} \cdot \text{t/m})(0.63 \text{ m})}{0.00125 \text{ Wb}} = 37.8 \text{ kA} \cdot \text{t/Wb}$$

(e) The reluctances in real magnetic cores are not constant.

Q6 - A magnetic circuit with a single air gap is shown as follows. The core dimensions are

Cross-sectional Area $A_c = 3.5 \text{ cm}^2$

Mean core length $l_c = 25 \text{ cm}$

Gap length $g = 2.4 \text{ mm}$

$N = 95$ turns

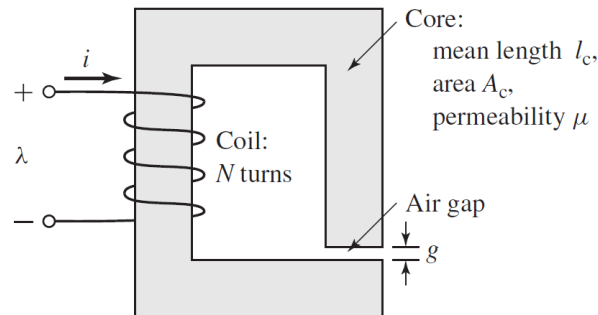
Assume that the core is of infinite permeability ($\mu \rightarrow \infty$) and neglect the effects of fringing fields at the air gap and leakage flux.

(a) Calculate the reluctance of the core $\mathcal{R}_c$ and that of the gap $\mathcal{R}_g$. For a current of $i = 1.4 \text{ A}$, calculate

(b) the total flux ϕ ,

(c) the flux linkages λ of the coil, and

(d) the coil inductance L .



Solution

Part (a):

$$\mathcal{R}_c = \frac{l_c}{\mu A_c} = \frac{l_c}{\mu_r \mu_0 A_c} = 0 \text{ A/Wb} \quad \text{Part (b):}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_c} = 5.457 \times 10^6 \text{ A/Wb}$$

$$\Phi = \frac{NI}{\mathcal{R}_c + \mathcal{R}_g} = 2.437 \times 10^{-5} \text{ Wb}$$

Part (c):

$$\lambda = N\Phi = 2.315 \times 10^{-3} \text{ Wb}$$

Part (d):

$$L = \frac{\lambda}{I} = 1.654 \text{ mH}$$