

EE3109 Applied Electromagnetics

Tutorial II

- Q1. The instantaneous expression for the magnetic field intensity of a uniform plane wave propagating in the +y direction in air is given by

$$\vec{H} = 4 \times 10^{-6} \cos\left(10^7 \pi t - k_o y + \frac{\pi}{4}\right) \hat{a}_z \text{ A/m}$$

- (a) Determine k_o and the location where H_z vanishes at $t=3\text{ms}$.
- (b) Write the instantaneous expression for E

$$(a) k_o = \omega \sqrt{\mu_o \epsilon_o} = \frac{10^7 \pi}{3 \times 10^8} = \frac{\pi}{30} = 0.105$$

At $t = 3 \times 10^{-3}$ (s), we require

$$10^7 \pi (3 \times 10^{-3}) - \frac{\pi}{30} y + \frac{\pi}{4} = \pm n\pi + \frac{\pi}{2} \quad n=0, 1, 2, 3 \dots \dots$$

$$y = \pm 30n - 7.5$$

But $\lambda = \frac{2\pi}{k_o} = 60 \text{ (m)}$, so

$$y = 22.5 \pm n \frac{\lambda}{2}$$

$$(b) \quad \vec{E} = -\eta_o \hat{a}_y \times \vec{H}$$

$$= -\hat{a}_x 1.508 \times 10^{-3} \cos(10^7 \pi t - \frac{\pi}{30} y + \frac{\pi}{4}) \text{ (V/m)}$$

- Q2. The magnetic field intensity of a linearly polarized uniform plane wave propagating in the +y direction in seawater ($\epsilon_r=80$, $\mu_r=1$ and $\sigma=4$ S/m) is

$$\vec{H} = 0.1 \sin\left(10^{10} \pi t - \frac{\pi}{3}\right) \hat{a}_x \text{ A/m}$$

at $y=0$.

- Determine the attenuation constant, the phase constant, the intrinsic impedance, the phase velocity, the wavelength and the skin depth.
- Find the location at which the amplitude of H is 0.01 A/m.
- Write the expressions for $E(y,t)$ and $H(y,t)$ at $y=0.5$ m as functions of t .

$$\frac{\sigma}{\omega \epsilon} = \frac{4}{(10^{10} \pi) \cdot (80 \cdot \frac{1}{36\pi} \times 10^{-9})} = 0.18 \ll 1$$

$$\alpha = \omega \sqrt{\mu \varepsilon'} \frac{\varepsilon''}{2\varepsilon'}$$

$$= 84 \text{ (Np/m)}$$

$$\beta = \omega \sqrt{\mu \varepsilon} \left[1 + \frac{1}{8} \left(\frac{\varepsilon''}{\varepsilon'} \right)^2 \right]$$

$$= 300\pi \text{ (rad/m)}$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu_0}{\varepsilon}} \left(1 + j \frac{\varepsilon''}{\varepsilon'} \right)$$

$$= 42.3 e^{j0.0286\pi}$$

$$u_p = \frac{\omega}{\beta} = 33.3 \times 10^6 (m / s)$$

$$\lambda = \frac{2\pi}{\beta} = 6.67 \times 10^{-3} (m)$$

$$\delta = \frac{1}{\alpha} = 11.9 \times 10^{-3} (m)$$

(b)

$$e^{-\alpha y} = 0.1$$

$$\begin{aligned} y &= \frac{1}{\alpha} \ln(10) \\ &= 27.4 \times 10^{-3} (m) \end{aligned}$$

$$(c) \quad \vec{H}(y, t) = \hat{a}_x 0.1 e^{-\alpha y} \sin(10^{10} \pi t - \beta y - \frac{\pi}{3})$$

At $y=0.5$ (m),

From (2)

$$\vec{H}(y, t) = \hat{a}_x 0.1 e^{-84 \times 0.5} \sin(10^{10} \pi t - 300 \pi \times 0.5 - \frac{\pi}{3})$$

$$= \hat{a}_x 5.75 \times 10^{-20} \sin(10^{10} \pi t - \frac{\pi}{3}) (A / m)$$

$$\vec{E}(y, t) = -\eta_c \hat{a}_y \times \vec{H}(y, t)$$

$$= \hat{a}_z 2.43 \times 10^{-18} \sin(10^{10} \pi t - \frac{\pi}{3} + 0.0286 \pi) (V / m)$$

- Q3 There is continuing discussion on the radiation hazards to human health. The following calculation will provide a rough comparison.
 - (a) The U.S. standard for personal safety in a microwave environment is that the power density be less than 10mW/cm^2 . Calculate the corresponding standard in terms of electric field intensity and in terms of magnetic field intensity.
 - (b) It is estimated that the earth receives radiant energy from the sun at a rate of about 1.3 kW/cm^2 on a sunny day. Assuming a monochromatic plane wave (which is not), calculate the equivalent amplitudes of the electric and magnetic field intensity vectors.

$$P_{av} = \frac{|E|^2}{2\eta_0} = 0.01(\text{W/cm}^2)$$

$$|E| = \sqrt{0.02\eta_0} = 2.75(\text{V/cm}) = 275(\text{V/m})$$

$$|H| = \frac{1}{\eta_0}|E| = 7.28 \times 10^{-3}(\text{A/cm}) = 0.728(\text{A/m})$$

$$P_{av} = \frac{|E|^2}{2\eta_0} = 1.3 \times 10^3 (\text{W/cm}^2)$$

$$|E| = 990 (\text{V/cm}) = 99 \times 10^3 (\text{V/m})$$

$$|H| = 2.63 (\text{A/cm}) = 263 (\text{A/m})$$

- Q4 From the point of view of electromagnetics, the power transmitted by a lossless coaxial cable can be considered in terms of the Poynting vector inside the dielectric medium between the inner conductor and the outer sheath. Assume that a dc voltage V_0 causes a current I to flow to a load resistance, verify that the integration of the Poynting vector over the cross-sectional area of the dielectric medium equals the power $V_0 I$ that is transmitted to the load.

From Gauss's law, $\vec{E} = \hat{a}_r \frac{\rho}{2\pi\epsilon r}$, where ρ is the line charge density on the inner conductor.

$$V_0 = -\int_b^a \vec{E} \cdot d\vec{r} = \frac{\rho}{2\pi\epsilon} \ln\left(\frac{b}{a}\right)$$

$$\vec{E} = \hat{a}_r \frac{V_0}{r \ln\left(\frac{b}{a}\right)}$$

From Ampere's circuital law, $\vec{H} = \hat{a}_\phi \frac{I}{2\pi r}$

Poynting vector $\vec{P} = \vec{E} \times \vec{H} = \hat{a}_z \frac{V_0 I}{2\pi r^2 \ln(\frac{b}{a})}$

Power transmitted of cross-sectional area:

$$\begin{aligned} P &= \int_s \vec{P} \cdot d\vec{s} = \frac{V_0 I}{2\pi \ln(\frac{b}{a})} \int_0^{2\pi} \int_a^b \frac{1}{r^2} r dr d\phi \\ &= V_0 I \end{aligned}$$

- Q5 Airplanes use radar altimeter to determine their low-level altitude accurately. If an airplane is flying over the ocean ($\epsilon_r \cong 81$, $\mu_r \cong 1$ and $\sigma \cong 4$ S/m) and the radar frequency is 7GHz, determine the percent reduction in received power due to the ocean.

$$\hat{\eta}_{ocean} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\frac{\mu}{\epsilon}}{1 - j\frac{\sigma}{\omega\epsilon}}} = 41.72 \angle 3.618^\circ$$

$$\hat{\Gamma} = \frac{\hat{\eta}_{ocean} - \hat{\eta}_{air}}{\hat{\eta}_{ocean} + \hat{\eta}_{air}} = 0.8011 \angle 179.2^\circ$$

$$P_{average\ reflected} \approx |\hat{\Gamma}|^2 \Rightarrow 64.2\% \text{ of incident power is in the reflected wave.}$$

So, the percent of reduction is 35.8%

- Q6. A Doppler radar is used to determine the speed of a moving vehicle by measuring the frequency shift of the wave reflected from the vehicle.
 - (a) Assuming that the reflecting surface of the vehicle can be represented by a perfectly conducting plane and the transmitted signal is a time-harmonic uniform plane wave of a frequency f incident normally on the reflecting surface, find the relation between the frequency shift Δf and the speed u of the vehicle.
 - (b) Determine u both in (km/hr) and in (miles/hr) if $\Delta f = 2.33\text{kHz}$ with $f = 10.5\text{ GHz}$

For a time harmonic source moving towards a stationary observer, the apparent frequency of the received wave is:

$$\begin{aligned}
 f' &\approx f \left(1 + \frac{u}{c} \cos \theta\right) \\
 &= f \left(1 + \frac{u}{c}\right) \quad \text{for } \theta = 0^\circ
 \end{aligned}$$

In Doppler radar involving a moving target, the signal undergoes the Doppler shift when impinging upon the target. This target becomes the “source” of the reflected waves. Therefore the Doppler effect is encountered a second time, and the overall effect is double. Hence the frequency shift is:

$$\Delta f = \frac{2uf}{c}$$

(b) For $\Delta f = 2.33\text{kHz}$ and $f = 10.5\text{GHz}$, we have

$$\begin{aligned} u &= \frac{c\Delta f}{2f} = 33.3 \text{ (m/s)} \\ &= 120 \text{ (km/hr)} \\ &= 74.5 \text{ (miles/hr)} \end{aligned}$$

Q7. For each uniform plane wave traveling in vacuum characterized below, describe the polarization and direction of propagation.

$$(i) \vec{E} = \hat{x}e^{+jkz}$$

$$(ii) H = (j\hat{x} - \hat{y})e^{-jkz}$$

$$(iii) \vec{E} = (\hat{x} + \sqrt{3}\hat{y})e^{-jkz}$$

$$(iv) \vec{E} = (\hat{x} + 2e^{\frac{j\pi}{2}}\hat{y})e^{-jkz}$$

(i) x-polarized, propagating in the $-z$ direction

$$(ii) \quad \vec{H}(x, y, z, t) = -\hat{x} \sin(\omega t - kz) - \hat{y} \cos(\omega t - kz)$$

$$\begin{aligned} \vec{E}(x, y, z, t) &= \eta_0 \vec{H}(x, y, z, t) \times \hat{z} \\ &= \hat{y} \eta_0 \sin(\omega t - kz) - \hat{x} \eta_0 \cos(\omega t - kz) \end{aligned}$$

left circular polarization, $+z$ propagation

(iii) Linear polarization at an angle 60 degree from the x-axis, $+z$ propagation

$$(iv) \quad \vec{E}(x, y, z, t) = -\hat{x} \sin(\omega t - kz) - \hat{y} 2 \sin(\omega t - kz + \frac{\pi}{2})$$

Left elliptical polarization, $+z$ propagation

Q8. Two x-polarized uniform plane waves of different strength are traveling in opposite directions and are superimposed :

$$\vec{E} = \hat{x}(e^{-jkz} + ae^{jkz})$$

What average power (W/m²) is associated with each wave considered separately (in the absence of the other), and what average power is associated with the combination?

The wave traveling in the +z direction has an average power given by

$$\frac{1}{2} \text{Re}\{\bar{S}\} = \hat{z} \frac{1}{2\eta_0} |\bar{E}|^2 = \hat{z} \frac{1}{2 \times 377} = \hat{z} 1.3 \times 10^{-3} (W / m^2)$$

The wave traveling in the -z direction has an average power

$$\frac{1}{2} \text{Re}\{\bar{S}\} = -\hat{z} 1.3 \times 10^{-3} a^2 (W / m^2)$$

Both waves superimposed have a time-averaged power

$$\begin{aligned} \frac{1}{2} \text{Re}\{\bar{S}\} &= \frac{1}{2} \text{Re}\{\bar{E} \times \bar{H}^*\} \\ &= \frac{1}{2} \text{Re}\left\{ \hat{x}(e^{-jkz} + ae^{jkz}) \times \frac{1}{\eta_0} \hat{y}(e^{-jkz} - ae^{jkz})^* \right\} \\ &= \hat{z} \frac{1}{2\eta_0} (1 - |a|^2) W / m^2 \end{aligned}$$