

## EE1004 Assignment 1 - 2022-23 Semester A

1. John was tested positive for COVID-19. We assume that a person gets COVID-19 with 2% of the chance. When a person actually gets COVID-19, the test incorrectly predicts negative 80% of the time. When a person doesn't get COVID-19, the test incorrectly predicts positive 85% of the time. What is the probability that John gets COVID-19? [30 marks]

1. *Solution:* The sample space is defined by two mutually-exclusive events – a person gets COVID-19, or a person does not get COVID-19. Additionally, a third event occurs when the test predicts positive for COVID-19. Notation for these events appears below.

- Event  $A_1$ . A person gets COVID-19.
- Event  $A_2$ . A person does not get COVID-19.
- Event  $B$ . The test predicts positive for COVID-19.

In terms of probabilities, we know the following:

- $P(A_1) = 0.02$
- $P(A_2) = 0.98$
- $P(B | A_1) = 0.2$  [When a person actually gets COVID-19, the test correctly predicts positive 20% of the time.]
- $P(B | A_2) = 0.85$  [When a person doesn't get COVID-19, the test incorrectly predicts positive 85% of the time.]

We want to know  $P(A_1 | B)$ , the probability that John gets COVID-19, given that he was tested positive for COVID-19. The answer can be determined from Bayes' theorem, as shown below.

$$\begin{aligned} P(A_1 | B) &= [P(A_1) P(B | A_1)] / [P(A_1) P(B | A_1) + P(A_2) P(B | A_2)] \\ &= [0.02 \times 0.2] / [0.02 \times 0.2 + 0.98 \times 0.85] \\ &= [0.004] / [0.837] \\ &= 0.00478 \end{aligned}$$

2. The following are scores on IQ tests of a random sample of 9 students at a large university.

130, 122, 119, 142, 136, 127, 120, 152, 181

Construct 95 and 99 percent confidence interval estimates of the average IQ score of all students at the university, respectively. Note that the number of significant digits after the decimal point should be limited to four during the calculation. [40 marks]

2 *Solution.* Use the **t Distribution Calculator** to find the critical t statistic.

The sample standard deviation = 19.9130

The sample mean = 136.5556

degrees of freedom = 9 - 1 = 8

a) For a 95 percent two-sided confidence interval:

$$\text{critical probability} = 1 - (1-0.95)/2 = 0.975$$

$$\text{critical t statistic} = 2.306$$

$$\text{confidence interval} = 136.5556 \pm 2.306 \times (19.9130)/\sqrt{9} = 136.5556 \pm 15.3065$$

b) For a 99 percent two-sided confidence interval:

$$\text{critical probability} = 1 - (1-0.99)/2 = 0.995$$

$$\text{critical t statistic} = 3.355$$

$$\text{confidence interval} = 136.5556 \pm 3.355 \times (19.9130)/\sqrt{9} = 136.5556 \pm 22.2694$$

3. A producer specifies that the mean lifetime of a certain type of battery is at least 245 hours.

A sample of 8 such batteries yielded the following data.

237, 242, 232, 242, 248, 230, 244, 243

Assuming that the life of the batteries is approximately normally distributed with population standard deviation 6, do the data indicate that the specifications are not being met at the  $\alpha = 0.15$  level of significance? What is the conclusion if the mean lifetime of a certain type of battery is at least 245 hours? [30 marks]

3. Solution.

- Null hypothesis:  $\mu \geq 245$
- Alternative hypothesis:  $\mu < 245$

The sample mean  $\bar{x} = 239.75$  and the population standard deviation  $\delta = 6$

- Calculate the  $s$  score from the data set:  $s = \frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}} = \frac{239.75 - 245}{\frac{6}{\sqrt{8}}} = -2.4749$

- Apply **Normal Distribution Calculator** to obtain the cumulative probability

$$P(T < t) = 0.00666.$$

- This is a **one-tailed** test so the  $P$ -value is obtained from

$$P = P(T < t) = 0.00666$$

Since  $P < \alpha = 0.15$ , we reject the null hypothesis, so the data indicate that the specifications are not being met at the  $\alpha = 0.15$  level of significance, i.e., the mean lifetime of a certain type of battery is less than 245 hours.