

EE3210 Signals and Systems

Semester B 2023-2024

Assignment 1 Solution

1. (a) $x(t)$ is not a periodic signal. For $t < 0$, $x(t) = 0$. Therefore, one cannot find a T to satisfy $x(t+T) = x(t)$.

(b) $x(t)$ is a power signal since the energy $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \infty$. Or $|E_x|^2$ occupy infinite area, so $x(t)$ is a power signal not an energy signal.

$$\begin{aligned}\text{The power } P_x &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^0 |x(t)|^2 dt + \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T/2} |x(t)|^2 dt \\ &= \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T/2} [1 - \cos(\pi t)] dt \\ &= \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T/2} 1 dt - \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T/2} \cos(\pi t) dt \\ &= \frac{1}{4}\end{aligned}$$

2. Suppose $x(t) = x_1(t)x_2(t)$

(a) if both $x_1(t)$ and $x_2(t)$ are even signals, then

$x(-t) = x_1(-t)x_2(-t) = x_1(t)x_2(t) = x(t)$. Therefore, $x(t)$ is an even signal.

if both $x_1(t)$ and $x_2(t)$ are odd signals, then

$x(-t) = x_1(-t)x_2(-t) = -x_1(t) \cdot [-x_2(t)] = x(t)$. Therefore, $x(t)$ is an even signal.

(b) if $x_1(t)$ is an even signal, and $x_2(t)$ is an odd signal, then

$x(-t) = x_1(-t)x_2(-t) = x_1(t) \cdot [-x_2(t)] = -x(t)$. Therefore, $x(t)$ is an odd signal.

3. (1) $y(0) = x(-2) + x(2)$, the system output depends on previous and future output.

Therefore, the system is not memoryless.

(2) Give a counter example to show the system is non invertible: if input signal is $x(t) = t$, output $y(t) = t - 2 + 2 - t = 0$. Any inputs will lead to zero output. Therefore, the system is not invertible.

(3) if $|x(t)| < A$ for all t , we have $|x(t-2)| < A$ and $|x(2-t)| < A$. Therefore, $|y(t)| < 2A$. Therefore, the system is stable.

(4) $y(0) = x(-2) + x(2)$, the system output depends on previous and future output. Therefore, the system is not causal.

(5) Suppose $y_1(t) = x_1(t-2) + x_1(2-t)$, and $y_2(t) = x_2(t-2) + x_2(2-t)$

Assigning $x_3(t) = ax_1(t) + bx_2(t)$, we have

$$y_3(t) = x_3(t-2) + x_3(2-t)$$

$$\begin{aligned}
&= ax_1(t-2) + bx_2(t-2) + ax_1(2-t) + bx_2(2-t) \\
&= ay_1(t) + by_2(t)
\end{aligned}$$

Therefore, the system is linear.

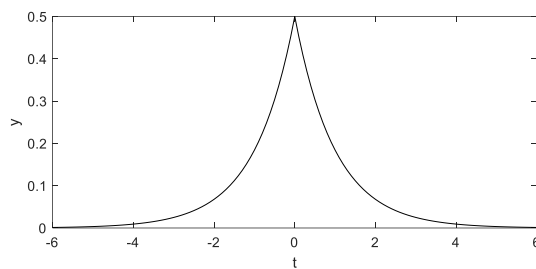
(6) Assigning $x_1(t) = x(t-t_0)$, we have

$$\begin{aligned}
y_1(t) &= x_1(t-2) + x_1(2-t) \\
&= x(t-2-t_0) + x(2-t-t_0) \\
y(t-t_0) &= x(t-2-t_0) + x(2-(t-t_0)) \\
&= x(t-2-t_0) + x(2-t+t_0)
\end{aligned}$$

Therefore, $y_1(t) \neq y(t-t_0)$. The system is not time invariance.

4.

$$\begin{aligned}
y(t) &= x(t) \otimes h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} e^{\tau}u(-\tau)e^{-(t-\tau)}u(t-\tau)d\tau \\
&= \int_{-\infty}^0 e^{2\tau-t}u(t-\tau)d\tau \\
&= \begin{cases} \int_{-\infty}^0 e^{2\tau-t}d\tau, t > 0 \\ \int_{-\infty}^t e^{2\tau-t}d\tau, t \leq 0 \end{cases} \\
&= \begin{cases} \frac{1}{2}e^{-t}, t > 0 \\ \frac{1}{2}e^t, t \leq 0 \end{cases}
\end{aligned}$$



5.
$$x(t) \otimes h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Assign $t-\tau = \lambda$, we have $d\tau = -d\lambda$, and

$$\begin{aligned}
x(t) \otimes h(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{\infty}^{-\infty} x(t-\lambda)h(\lambda)d(-\lambda) \\
&= \int_{-\infty}^{\infty} x(t-\lambda)h(\lambda)d\lambda = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda = h(t) \otimes x(t)
\end{aligned}$$

6.

$$y[n] = x[n] \otimes h[n]$$

$$\begin{aligned}
&= \sum_{m=-\infty}^{\infty} x[m]h[n-m] \\
&= \sum_{m=-\infty}^{\infty} \left(\frac{1}{2}\right)^{m-2} u[m-2]u[m+2-m] \\
&= \sum_{m=2}^{\infty} \left(\frac{1}{2}\right)^{m-2} u[m+2-m] \\
&= \begin{cases} \sum_{m=2}^{n+2} \left(\frac{1}{2}\right)^{m-2}, n \geq 0 \\ 0, n < 0 \end{cases}
\end{aligned}$$

Assign $k = m - 2$, we have

$$y[n] = x[n] \otimes h[n]$$

$$= \begin{cases} \sum_{k=0}^n \left(\frac{1}{2}\right)^k, n \geq 0 \\ 0, n < 0 \end{cases}$$

$$= \left[2 - \left(\frac{1}{2}\right)^n\right] u[n]$$

Since $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |u[n+2]| = \infty$, the system is not stable.

Since $h[-2] = h[-1] = 1 \neq 0$, the system is not causal.

7.

There are many possible answers.

One example is a software that computes the moving average of stock prices. In this software system, the input is the close price of a stock to be investigated. The output is the moving average based on the previous close prices including the current one. It is used for analysis the stock, particularly to see if it is on the rising or falling trend.

Consider N -day moving average where $x[n]$ is the input and $y[n]$ is the output, the input-output relationship is then:

$$y[n] = \frac{1}{N} (x[n] + x[n-1] + \cdots + x[n-N+1])$$

It is a difference equation, and hence the system is both linear and time-invariant.