

Block A Unit 3 outline

- One port network
 - › Thevenin equivalent circuit
 - › Norton equivalent circuit
 - › Source transformation
- Dependent sources
 - › Dependent voltage source
 - › Dependent current source
- Maximum power transfer

Alexander & Sadiku, “Fundamentals of Electric Circuits” 5th Edition Ch 4.4 – 4.6, 4.8

One-Port Network

Introduction

- A **one port network** is simply a **two terminal device** (See Fig below)

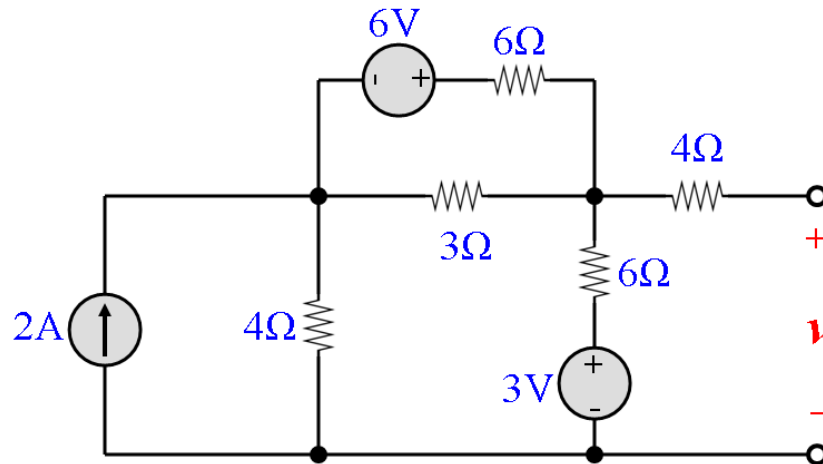


Fig 1: An example of a one port network

- Given a number of different resistor loads, if you were asked to find the current and voltage at the terminals for each resistor, you would have to recalculate the whole circuit
- A different load will give you different output current & voltage

One-Port Network

Simpler analysis

- Transform any 2 terminal circuit into a circuit that is as simple as having 1 resistor and 1 source (see Fig 2)

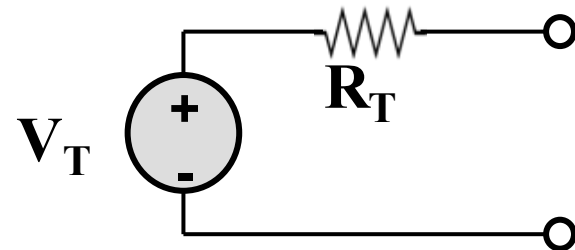
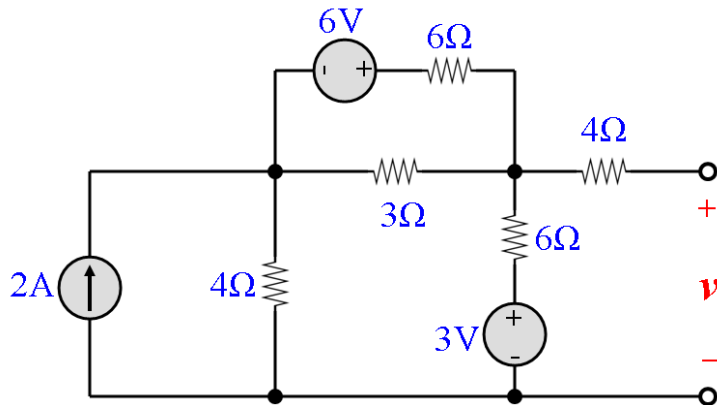


Fig 2: Transformed Thevenin equivalent circuit

The first part of this unit is organized into the following 3 sections:

- Thevenin equivalent
- Norton equivalent
- Source transformation

The last part of this unit will cover all concepts in Block A and apply them to dependent sources.

I. Thevenin

Definition

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **voltage source** V_T in **series** with an equivalent **resistance** R_T

Deriving the Thevenin equivalent circuit

Step 1: Remove the load from the rest of the one port network. This is the first most fundamental step.

Step 2: Find the equivalent resistance R_T . Disable all ideal sources then find the resistance across the terminals.

For voltage source – replace with short circuit

For current source – replace with open circuit

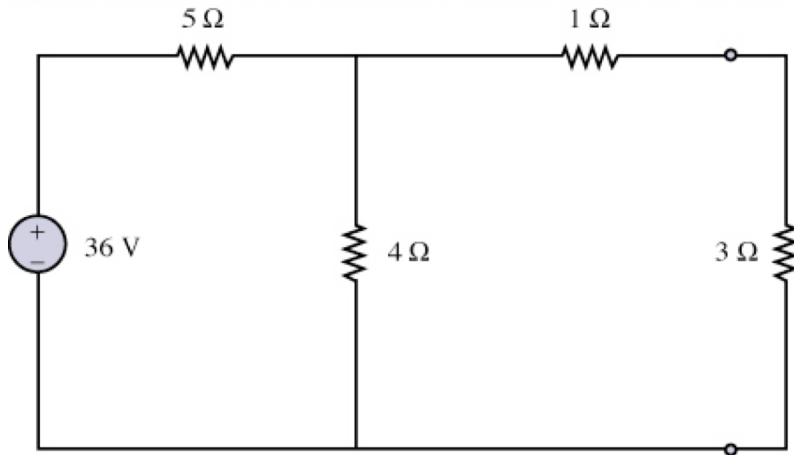
Step 3: Find the Thevenin voltage source. The Thevenin voltage source is equal to the open circuit voltage seen across the terminals (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

Worked Example on Thevenin 1

Derive the Thevenin equivalent circuit seen by the 3Ω load.

Answer: $R_{th} = 3.22\ \Omega$, $V_{th} = 16\ V$

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Steps:

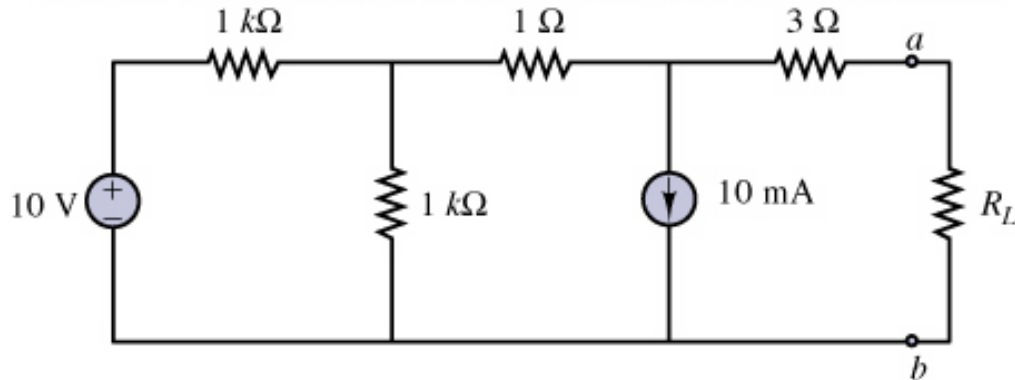
1. Remove the 3Ω load
2. Find R_T (disable the voltage source)
3. Find V_T (with no load)
4. Draw the Thevenin circuit

Worked Example on Thevenin 2

Derive the Thevenin equivalent circuit as seen by the resistor R_L .

Answer: $R_{th} = 504 \Omega$, $V_{th} = -10 \text{ mV}$

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Steps:

1. Remove the load
2. Find R_T (disable the voltage and current sources)
3. Find V_T (with no load)
4. Draw the Thevenin circuit

II. Norton

Definition

- Any one-port network composed of ideal voltage and current sources, and linear resistors, can be represented by an equivalent circuit consisting of an ideal **current source** I_N in **parallel** with an equivalent **resistance** R_N

Deriving the Norton equivalent circuit

Steps 1 and 2 for deriving the Norton equivalent circuit are exactly the same as that for Thevenin.

Step 3: Find the Norton current source. The Norton current source is equal to the short circuit current seen across the terminals (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

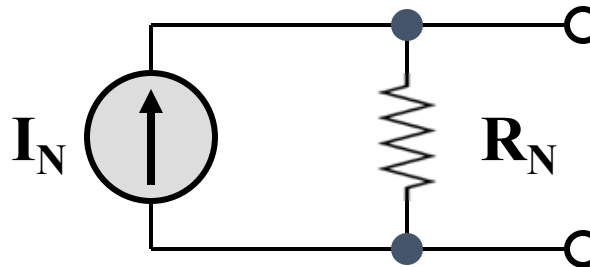


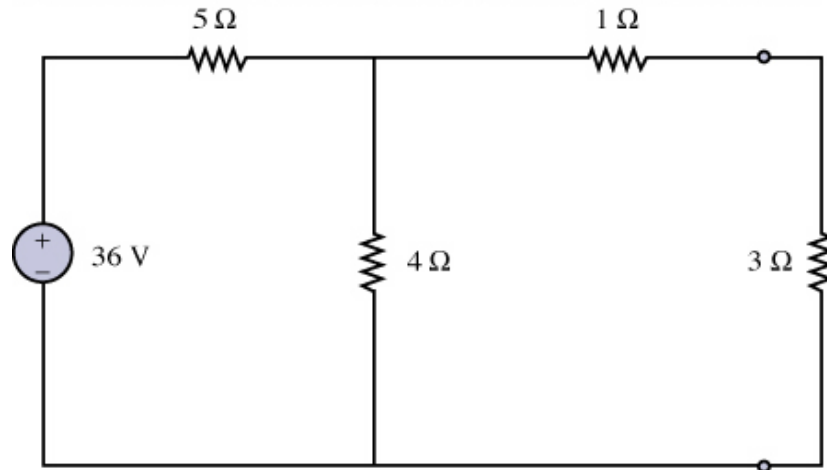
Fig 3: Transformed Norton equivalent circuit

Worked Example on Norton 1

Derive the Norton equivalent circuit seen by the 3Ω load.

Answer: $R_N = 3.22\ \Omega$, $I_N = 4.96\ \text{A}$

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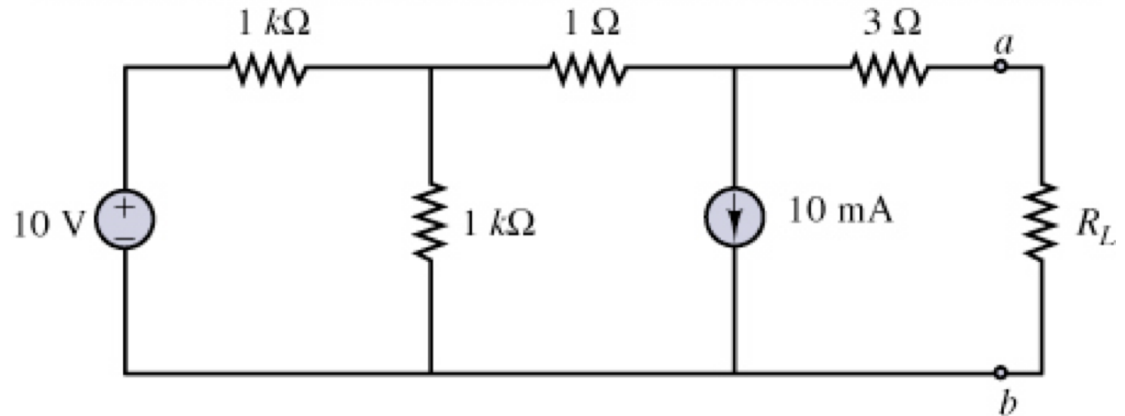


Worked Example on Norton 2

Derive the Norton equivalent circuit seen by the load R_L .

Answer: $R_N = 504 \Omega$, $I_N = -19.8 \mu\text{A}$

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III. Source Transformation

Definition

- This trick rests on the fact that the Thevenin and Norton forms are equivalent to each other and therefore are also interchangeable

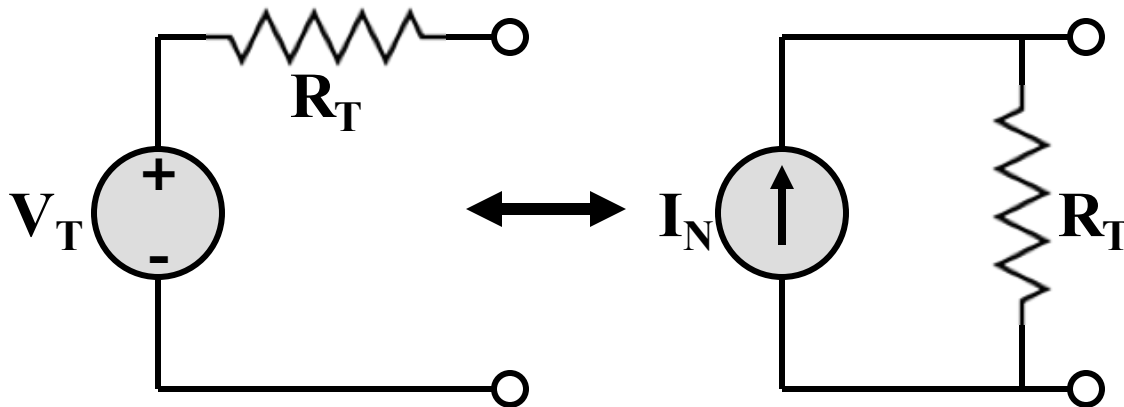


Fig 4a: Thevenin equivalent circuit

Fig 4b: Norton equivalent circuit

We can transform between the two equivalent circuits, observing each time that:

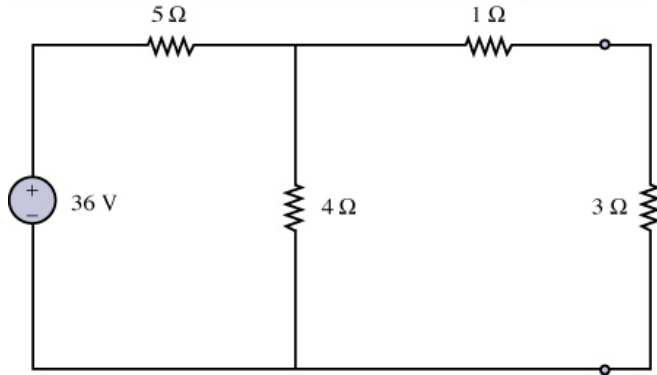
$$V_T = I_N R_T$$

- Rather than transform the whole circuit in one go to Thevenin or Norton equivalent forms, we can instead transform part of the circuit
- The process of merge transform and merge again can be repeated

Worked Example on Source Transformation 1

Derive either of the equivalent circuit forms seen by the 3Ω load.

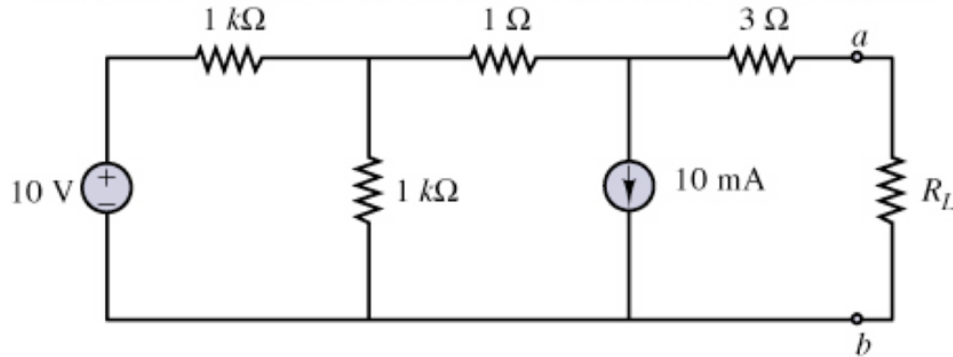
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Worked Example on Source Transformation 2

Derive either of the equivalent circuit forms seen by the load R_L .

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III. Dependent Sources

- All the sources we have come across are independent source
 - > For a voltage source, the voltage maintained across the source is fixed
 - > For a current source, the current through the source is fixed
- There also exist dependent sources in circuit theory
- Unlike independent sources, the value of a dependent source is not predetermined, but is set by the current or voltage through a specific branch

Symbol

- The symbol for an independent source is a circle
- For a dependent source, the symbol is a diamond

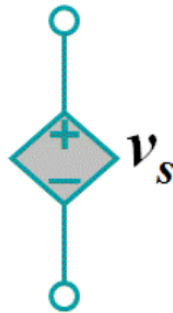


Fig 5a: Dependent voltage source

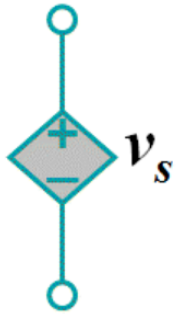


Fig 5b: Dependent current source

Dependent Sources

Dependent Voltage Source

v_S is the voltage source across the terminals.



Current Controlled Voltage Source

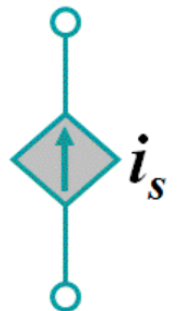
For example, $v_S = 5i_X$. The current i_X is not the current through the source but belongs to another branch.

Voltage Controlled Voltage Source

For example, $v_S = 7v_X$. The voltage v_X is not the source voltage. As v_X changes, so also v_S according to the above relations.

Dependent Current Source

i_S is the current source through the terminals.



Current Controlled Current Source

For example, $i_S = 5i_X$. The current i_X is not the current through the source but belongs to another branch.

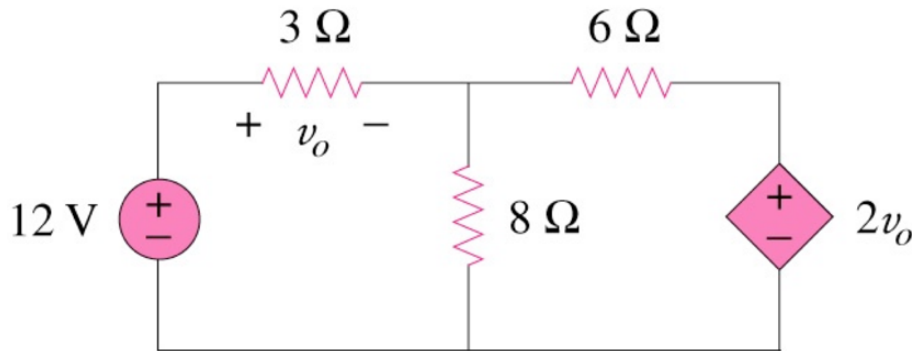
Voltage Controlled Current Source

For example, $i_S = 7v_X$. The voltage v_X is not the source voltage. As v_X changes, so also i_S according to the above relations.

Worked Example on Dependent Source 1

Calculate v_o in the circuit.

Answer: $v_o = 3.65 \text{ V}$



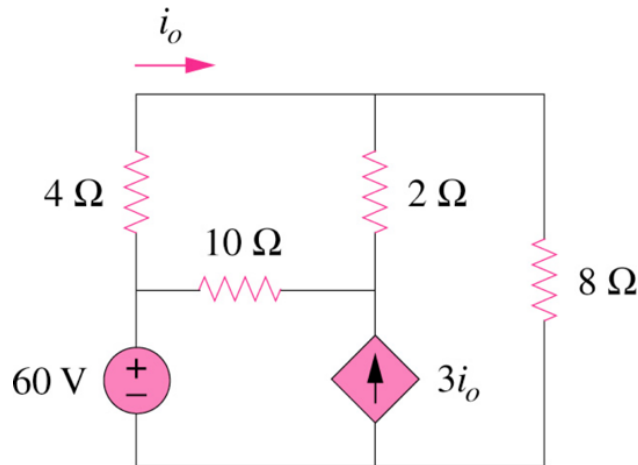
Hints:

1. The dependent source here is a voltage controlled voltage source
2. Apply KCL

Worked Example on Dependent Source 2

Calculate i_o in the circuit.

Answer: $i_o = 1.73 \text{ A}$

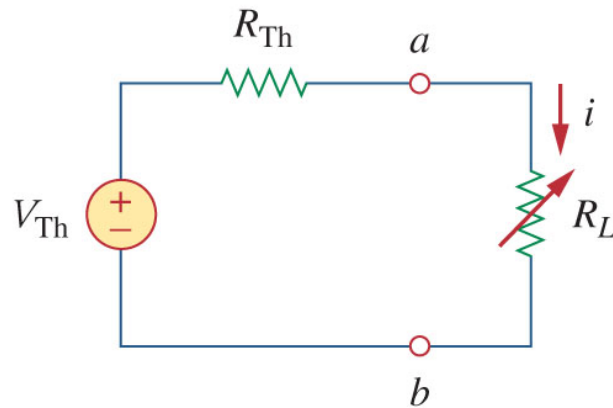


Hints:

1. The dependent source here is a current controlled current source
2. Apply KCL

Maximum Power Transfer

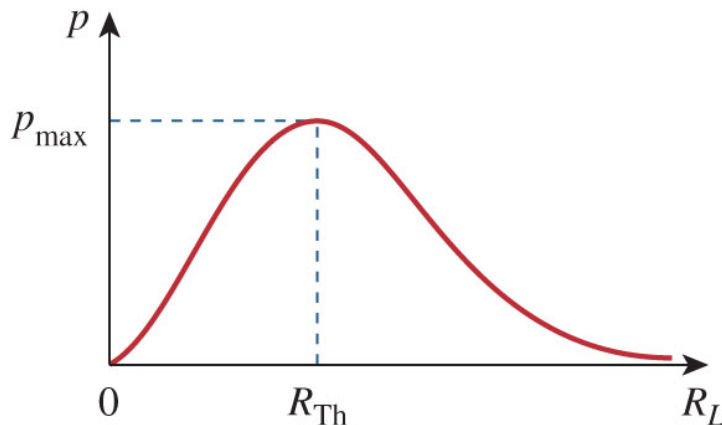
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$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

$$\frac{dp}{dR_L} = 0 \longrightarrow R_L = R_{Th}$$

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$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$