

EE3114 Tutorial – State-Space Design

1. Control Law for a Second-Order System

Suppose you have an undamped oscillator with frequency ω_0 and a state-space description given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Find the control law, $K = [K_1 \ K_2]$, that places the closed-loop poles of the system so that they are both at $-2\omega_0$. In other words, you wish to double the natural frequency and increase the damping ratio from 0 to 1.

Hints:

i) What is the desired characteristic equation?

ii) Let $K = [K_1 \ K_2]$. Find the characteristic equation of the closed-loop system with control:

$$\det[sI - (A - BK)] = 0$$

iii) Equate the coefficients with like powers of s in (i) and (ii) and find the control law K

2. How Zero Location Can Affect the Control Law

A specific system is described by

$$A = \begin{bmatrix} -7 & 1 \\ -12 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -z_0 \end{bmatrix}, C = [1 \ 0], D = 0$$

With a zero at $s = z_0$. Find the state feedback gains necessary for placing the poles of this system at the roots of $s^2 + 2\zeta\omega_n s + \omega_n^2$.

Hints:

i) Let $K = [K_1 \ K_2]$. Find the characteristic equation of the closed-loop system with control:

$$\det[sI - (A - BK)] = 0$$

ii) Equate the equation in (i) to the desired characteristic equation. Express K_1 and K_2 in terms of z_0 .

iii) What will happen when z_0 approaches to -3 or -4?

3. An Estimator Design for a 2nd Order System

Design an estimator for a simple pendulum with equations of motions:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \ 0]x$$

Compute the estimator gain matrix that will place both the estimator error poles at $-10\omega_n$.

Hints:

i) What is the desired characteristic equation?

ii) Let $H = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$. Express the new characteristic equation in h_1 and h_2 , i.e. $\det[sI - (A - HC)]$.

ii) Compare the coefficients.