

1. Let θ be an angle which lies between 180° and 270° , and $\cos \theta = -\frac{7}{25}$. Without using a calculator, find the values of
 - (a) $4 \tan \theta + 25 \sin \theta$
 - (b) $\cos \frac{\theta}{2}$.
2. It is given that $\sin A = \frac{4}{5}$ and $\cos B = \frac{5}{13}$. Without using a calculator, show that one possible value of $\sin(A+B)$ is $\frac{56}{65}$, and find all other possible values.
(Hint: $\sin(A+B) = \sin A \cos B + \cos A \sin B$)
3. If $\cos(\alpha - \beta) = -\frac{3}{5}$, $\cos(\alpha + \beta) = \frac{3}{5}$, where $\frac{\pi}{2} < \alpha - \beta < \pi$ and $\frac{3\pi}{2} < \alpha + \beta < 2\pi$, find $\cos 2\beta$.
(Hint: $2\beta = (\alpha + \beta) - (\alpha - \beta)$)
4. If $\sin \alpha + \sin \beta = p$, $\cos \alpha + \cos \beta = q$, where $q \neq 0$ and $p \neq q$, find $\tan(\alpha + \beta)$.
(Hint: Use the sum-to-product formulae and $\tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}$)
5. Find all solutions of the equation $-2 \cos^2 \theta = \sin 2\theta$ for which $0 < \theta < 2\pi$.
(Hint: $\sin 2\theta = 2 \sin \theta \cos \theta$)
6.
 - (a) Starting from the formula $\sin(A+B) = \sin A \cos B + \cos A \sin B$, prove that $\sin 3x = 3 \sin x - 4 \sin^3 x$.
 - (b) Solve the equation $\sin 3x = 2 \sin x$, giving all solutions for x such that $0 \leq x \leq 2\pi$.
7.
 - (a) Prove that $\tan x + \cot x = 2 \operatorname{cosec} 2x$.
 - (b) Find the general solution, in radians, of the equation $1 + 2 \operatorname{cosec} 2x = \cot x$.
8.
 - (a) Express $\cos x - \sqrt{3} \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
(Hint: $\cos(A+B) = \cos A \cos B - \sin A \sin B$)
 - (b) Hence, or otherwise, find the general solution of the equation $\cos x - \sqrt{3} \sin x + \sqrt{2} = 0$.

Answers:

1. (a) $-\frac{72}{7}$

(b) $-\frac{3}{5}$

2. Since $\sin A > 0$, we have $0^\circ < A < 90^\circ$ or $90^\circ < A < 180^\circ$.

Since $\cos B > 0$, we have $0^\circ < B < 90^\circ$ or $270^\circ < B < 360^\circ$.

Case I: For $0^\circ < A < 90^\circ$ and $0^\circ < B < 90^\circ$,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B = \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) = \frac{56}{65}$$

Case II: For $0^\circ < A < 90^\circ$ and $270^\circ < B < 360^\circ$,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B = \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right) = -\frac{16}{65}$$

Case III: For $90^\circ < A < 180^\circ$ and $0^\circ < B < 90^\circ$,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B = \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(-\frac{3}{5}\right)\left(\frac{12}{13}\right) = -\frac{16}{65}$$

Case IV: For $90^\circ < A < 180^\circ$ and $270^\circ < B < 360^\circ$,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B = \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(-\frac{3}{5}\right)\left(-\frac{12}{13}\right) = \frac{56}{65}$$

3. -1

4. $\frac{2pq}{q^2 - p^2}$

5. $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}$

6. (b) $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

7. (b) $x = n\pi - \frac{\pi}{4}$, for $n = 0, \pm 1, \pm 2, \pm 3, \dots$

8. (a) $\cos x - \sqrt{3} \sin x = 2 \cos\left(x + \frac{\pi}{3}\right)$

(b) $x = 2n\pi + \frac{5\pi}{12}$ or $x = 2n\pi - \frac{13\pi}{12}$, for $n = 0, \pm 1, \pm 2, \pm 3, \dots$