

EE2104 Introduction to Electromagnetics
Assignment 1 Solution

Deadline (for uploading): 7:00 pm, 19 Sep 2020
Please put down your name and university ID.

1. Given the vectors $\mathbf{M} = -10\hat{x} + 4\hat{y} - 8\hat{z}$ and $\mathbf{N} = 8\hat{x} + 7\hat{y} - 2\hat{z}$. Find:
- (a) the unit vector pointing from $\mathbf{M}$ to $\mathbf{N}$;
 - (b) $\mathbf{M} \cdot \mathbf{N}$;
 - (c) the angle between $\mathbf{M}$ and $\mathbf{N}$;
 - (d) $\mathbf{M} \times \mathbf{N}$; and
 - (e) an unit vector perpendicular to both $\mathbf{M}$ and $\mathbf{N}$.

Solution:

- (a) The vector pointing from $\mathbf{M}$ to $\mathbf{N}$ can be expressed as

$$\mathbf{R} = \mathbf{N} - \mathbf{M} = 18\hat{x} + 3\hat{y} + 6\hat{z}$$

We have

$$|\mathbf{R}| = \sqrt{18^2 + 3^2 + 6^2} = 3\sqrt{41}$$

Therefore, the unit vector pointing from $\mathbf{M}$ to $\mathbf{N}$ is

$$\frac{\mathbf{R}}{|\mathbf{R}|} = \frac{6\sqrt{41}}{41}\hat{x} + \frac{\sqrt{41}}{41}\hat{y} + \frac{2\sqrt{41}}{41}\hat{z} = 0.94\hat{x} + 0.16\hat{y} + 0.31\hat{z}$$

- (b)

$$\mathbf{M} \cdot \mathbf{N} = (-10\hat{x} + 4\hat{y} - 8\hat{z}) \cdot (8\hat{x} + 7\hat{y} - 2\hat{z}) = -36$$

- (c) The angle between $\mathbf{M}$ and $\mathbf{N}$ is

$$\theta = \cos^{-1}\left(\frac{\mathbf{M} \cdot \mathbf{N}}{|\mathbf{M}||\mathbf{N}|}\right) = \cos^{-1}\left(\frac{-36}{\sqrt{(-10)^2 + 4^2 + (-8)^2} \times \sqrt{8^2 + 7^2 + (-2)^2}}\right) = 1.82 \text{ (rad)}$$

- (d)

$$\mathbf{M} \times \mathbf{N} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -10 & 4 & -8 \\ 8 & 7 & -2 \end{vmatrix} = 48\hat{x} - 84\hat{y} - 102\hat{z}$$

- (e) The unit vector perpendicular to both $\mathbf{M}$ and $\mathbf{N}$ is

$$\frac{\mathbf{M} \times \mathbf{N}}{|\mathbf{M} \times \mathbf{N}|} = \frac{48\hat{x} - 84\hat{y} - 102\hat{z}}{\sqrt{48^2 + (-84)^2 + (-102)^2}} = 0.34\hat{x} - 0.60\hat{y} - 0.73\hat{z}$$

2. A field is given as

$$\mathbf{G}(x, y) = 7 \frac{x\hat{x} + y\hat{y}}{x^2 + y^2}$$

Find:

- (a) the unit vector in the direction of $\mathbf{G}$ at $P(4, -2)$ and
- (b) the angle between $\mathbf{G}(x, y)$ and $\hat{x}$ at P .

Solution:

- (a) At point $P(4, -2)$, we have

$$\mathbf{G} = 7 \frac{4\hat{x} - 2\hat{y}}{4^2 + (-2)^2} = 1.4\hat{x} - 0.7\hat{y}, \quad |\mathbf{G}| = \sqrt{1.4^2 + (-0.7)^2} = 1.57$$

Therefore, the unit vector in the direction of $\mathbf{G}$ at $P(4, -2)$ is

$$\frac{\mathbf{G}}{|\mathbf{G}|} = 0.89\hat{x} - 0.45\hat{y}$$

- (b) At point $P(4, -2)$, the angle between $\mathbf{G}$ and $\hat{x}$ is

$$\theta = \cos^{-1} \left(\frac{\mathbf{G} \cdot \hat{x}}{|\mathbf{G}| |\hat{x}|} \right) = \cos^{-1} \left(\frac{(1.4\hat{x} - 0.7\hat{y}) \cdot \hat{x}}{1.57 \times 1} \right) = 0.47 \text{ (rad)}$$

3. Show that the two vector fields

$$\begin{aligned}\mathbf{A}(\rho, \phi, z) &= \rho^{\frac{3}{2}} \cos\phi \hat{\boldsymbol{\rho}} + \rho^{\frac{3}{2}} \sin\phi \hat{\boldsymbol{\phi}} + \rho \hat{\mathbf{z}} \\ \mathbf{B}(\rho, \phi, z) &= \rho^{\frac{1}{2}} \cos\phi \hat{\boldsymbol{\rho}} + \rho^{\frac{1}{2}} \sin\phi \hat{\boldsymbol{\phi}} - \rho \hat{\mathbf{z}}\end{aligned}$$

are everywhere perpendicular to each other.

Solution:

We find

$$\mathbf{A} \cdot \mathbf{B} = (\rho^2 \cos^2\phi + \rho^2 \sin^2\phi) - \rho^2 = 0 = |\mathbf{A}||\mathbf{B}|\cos\theta$$

where θ is the angle between $\mathbf{A}$ and $\mathbf{B}$.

Therefore, we have

$$\cos\theta = 0 \Rightarrow \theta = 90^\circ$$

So these two vector fields are everywhere perpendicular to each other.

4. Consider a line at $y = L$ to $2L$. It contains electric charge of density $\rho_L = \rho_{L0}(y/L)^{-2}$ for some constant ρ_{L0} in C/m. Find the total charge of this line.

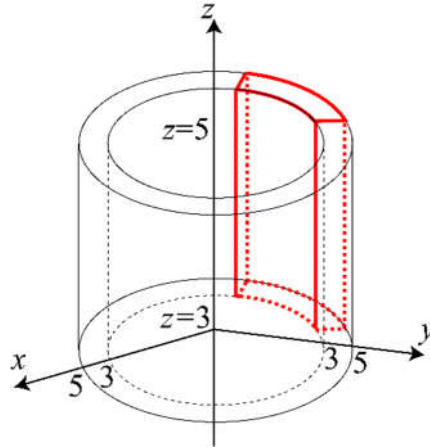
Solution:

The total charge is

$$\begin{aligned} Q &= \int_L^{2L} \rho_L dy \\ &= \int_L^{2L} \rho_{L0} (y/L)^{-2} dy \\ &= \rho_{L0} \cdot L^2 \int_L^{2L} \frac{1}{y^2} dy \\ &= \frac{L \cdot \rho_{L0}}{2} \text{ (C)} \end{aligned}$$

5. In the cylindrical coordinate systems, the six surfaces: $\rho = 3$, $\rho = 5$, $\phi = 100^\circ$, $\phi = 170^\circ$, $z = 3$, and $z = 5$ define a closed surface. Find:
- the enclosed volume;
 - the total area of the enclosing surface; and
 - the total length of the twelve edges of the surfaces.

Solution:



The closed surface is shown with the red edges in the figure.

- (a) The enclosed volume is

$$\text{Vol} = \int_{\rho=3}^5 \int_{\phi=100^\circ}^{170^\circ} \int_{z=3}^5 \rho d\rho d\phi dz = 1.22 \times 2 \times 8 = 19.55$$

- (b) The total area of the enclosing surface is

$$\begin{aligned} \text{Area} &= 2 \int_{\phi=100^\circ}^{170^\circ} \int_{\rho=3}^5 \rho d\rho d\phi + \int_{z=3}^5 \int_{\phi=100^\circ}^{170^\circ} 3 d\phi dz \\ &\quad + \int_{z=3}^5 \int_{\phi=100^\circ}^{170^\circ} 5 d\phi dz + 2 \int_{z=3}^5 \int_{\rho=3}^5 d\rho dz = 47.10 \end{aligned}$$

- (c) The total length of the twelve edges of the surfaces is

$$\text{Length} = 4 \times 2 + 4 \times 2 + 2 \times \left(\frac{70^\circ}{360^\circ} \times 2\pi \times 3 + \frac{70^\circ}{360^\circ} \times 2\pi \times 5 \right) = 35.55$$