

EE 4211 Computer Vision

Lecture 4B: Morphology

Semester A, 2020-2021

Lecture Outline

- Morphological Algorithms
 - Hit or Miss Transform
 - Boundary Extraction
 - Hole Filling
 - Connected Components
 - Skeletons

Hit-or-Miss Transform

- Hit-or-Miss Transform is a powerful method for finding shapes, and their locations in images
- Can be defined entirely in terms of erosion only

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

$$B = (B_1, B_2)$$

$$B_1 : \text{object} \quad B_2 : \text{background}$$

- Useful for **detecting specific shapes** that are intended to extract, e.g. squares, triangles, ridges, corners, junctions, etc.

Hit-or-Miss Transform

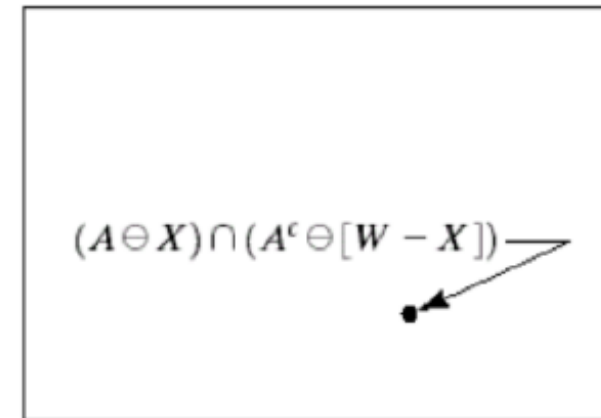
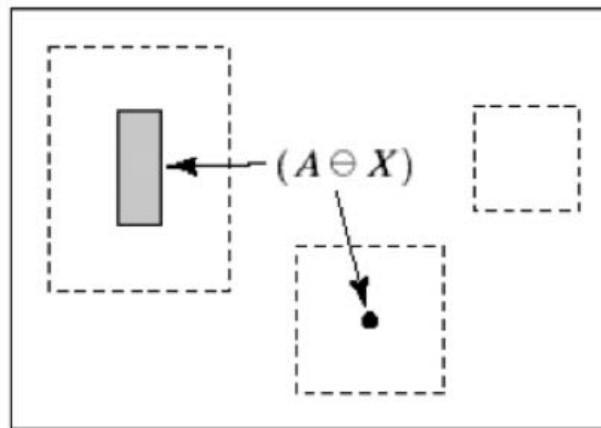
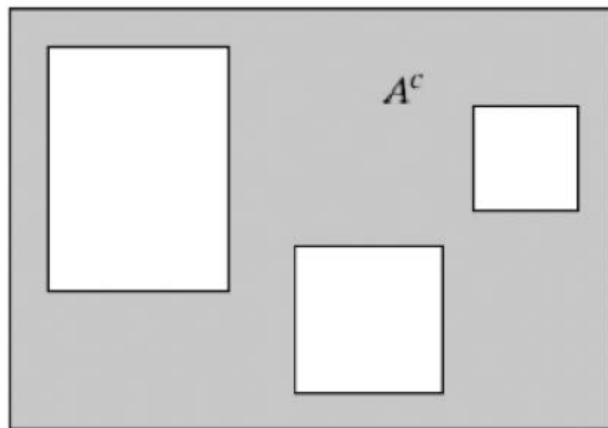
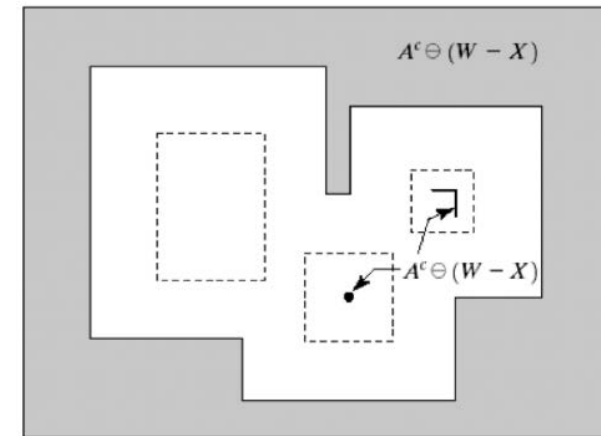
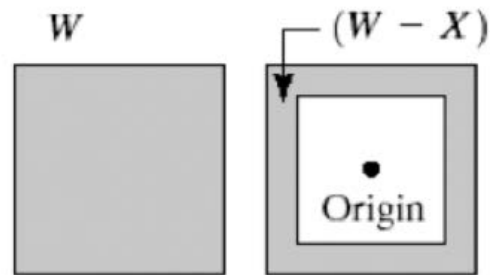
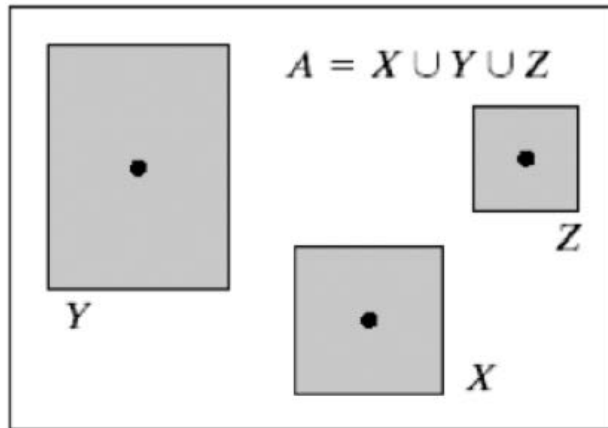
- Steps

Perform an erosion $A \ominus B_1$ with B_1 being the SE shape that we intend to find.

Next, erode the complement of A with B_2 , a SE that is the border that encloses around the shape B_1 .

The **intersection of the two erosion operations** would produce just one pixel at the center position of the found, shape, resulting in a “**hit**”. Other parts of set A which did not return anything are considered “**miss**”.

Hit-or-Miss Transform



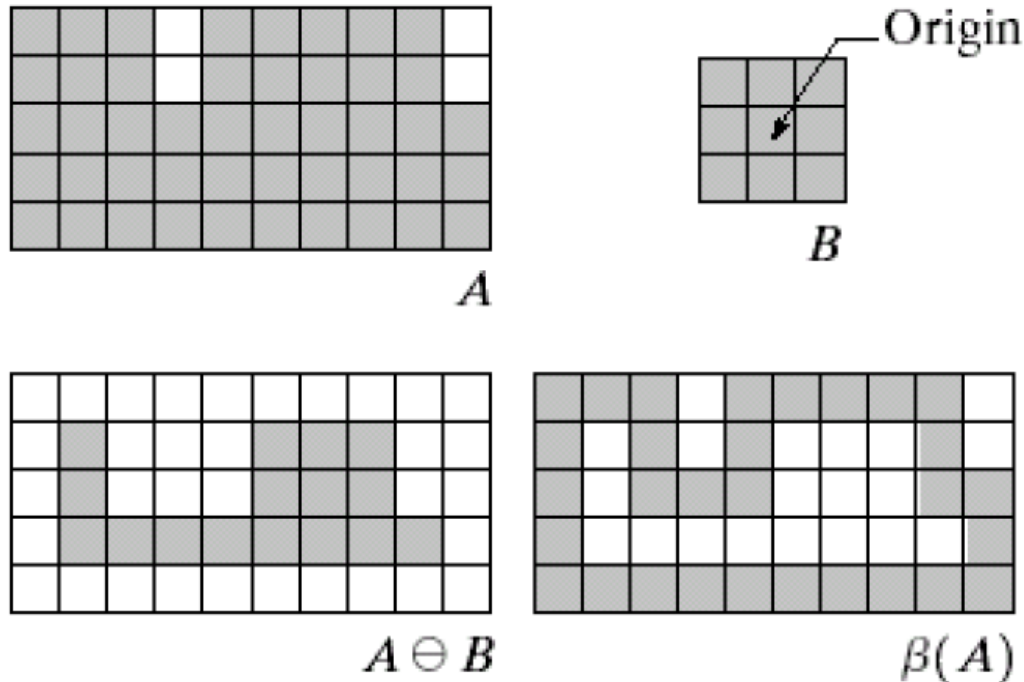
Lecture Outline

- Morphological Algorithms
 - Hit or Miss Transform
 - **Boundary Extraction**
 - Hole Filling
 - Connected Components
 - Skeletons

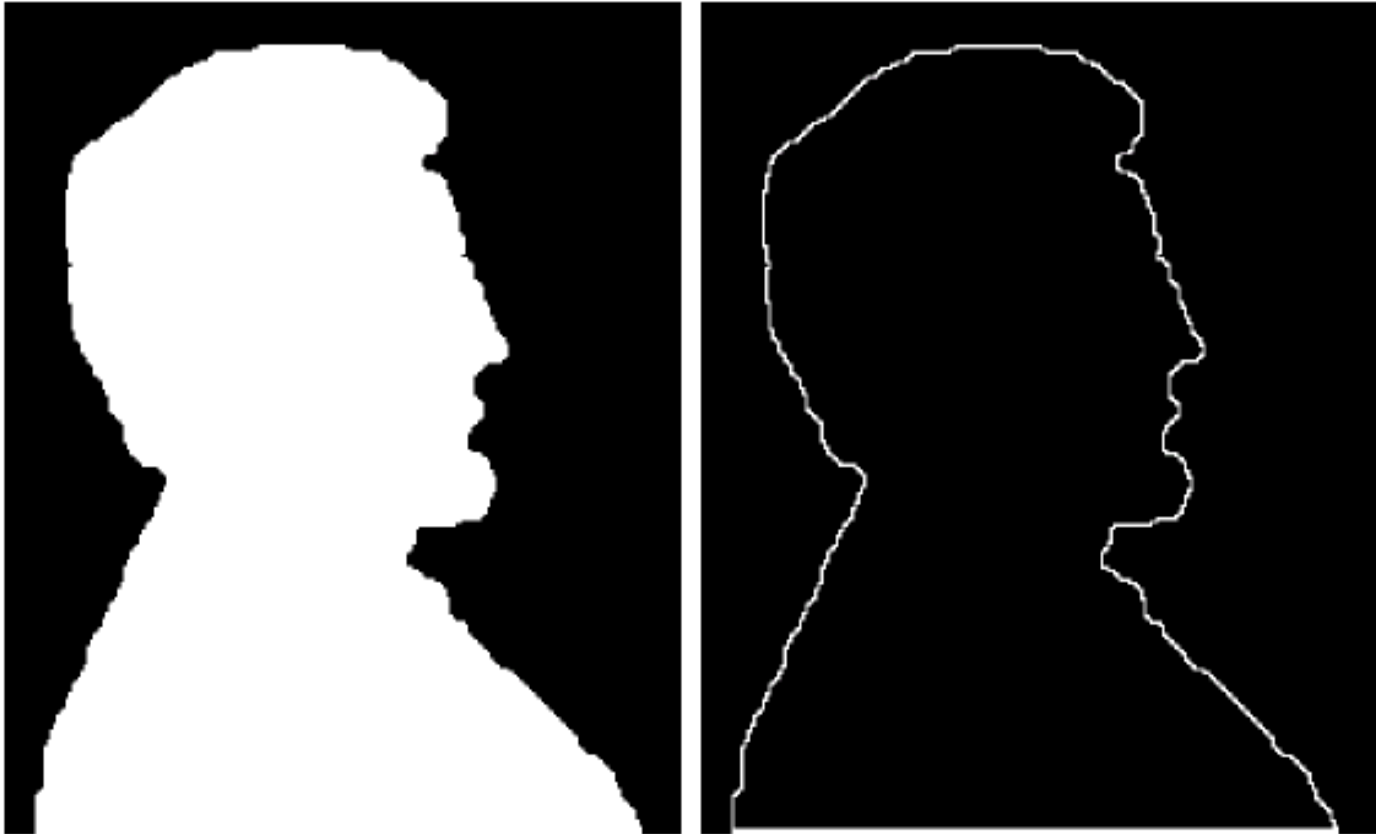
Boundary Extraction

- A boundary of a set A , denoted by $\beta(A)$, is obtained by eroding A by B , then perform the set difference between A and its erosion:

$$\beta(A) = A - (A \ominus B)$$



Boundary Extraction



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Hole Filling

- Sometimes also referred to as Region Filling
- **Hole**: A background region surrounded by a connected border of foreground pixels
- Let A denote a set whose elements are 8-connected boundaries – each boundary encloses a background region.
- **Objective**: to fill all holes in set A with 1s

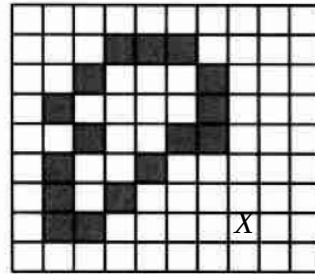
Region Filling

- Start with a point inside the region
- Repeatedly dilate
- At each step, set to zero the points corresponding to the region

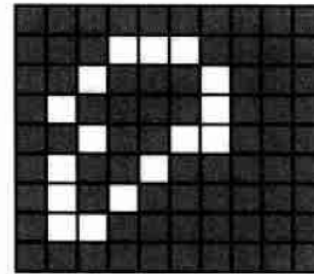
$$X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \dots$$

- A^c is the complement of A
- Stop when no more changes

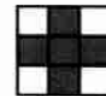
Region Filling



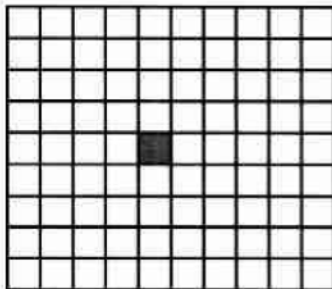
A



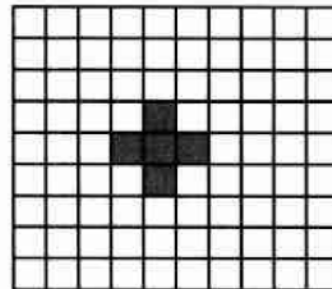
A^c



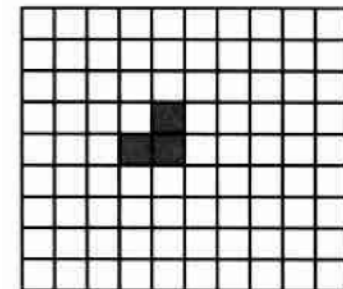
B



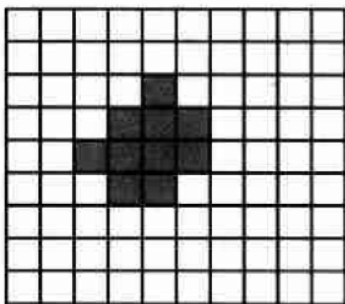
X_0



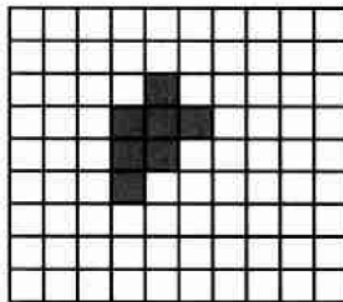
$X_0 \oplus B$



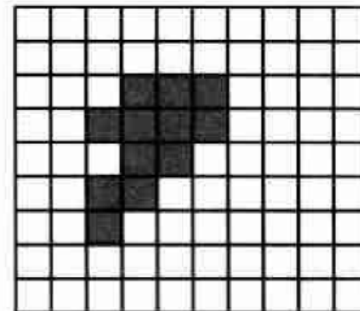
X_1



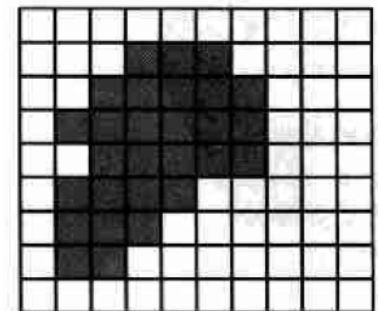
$X_1 \oplus B$



X_2

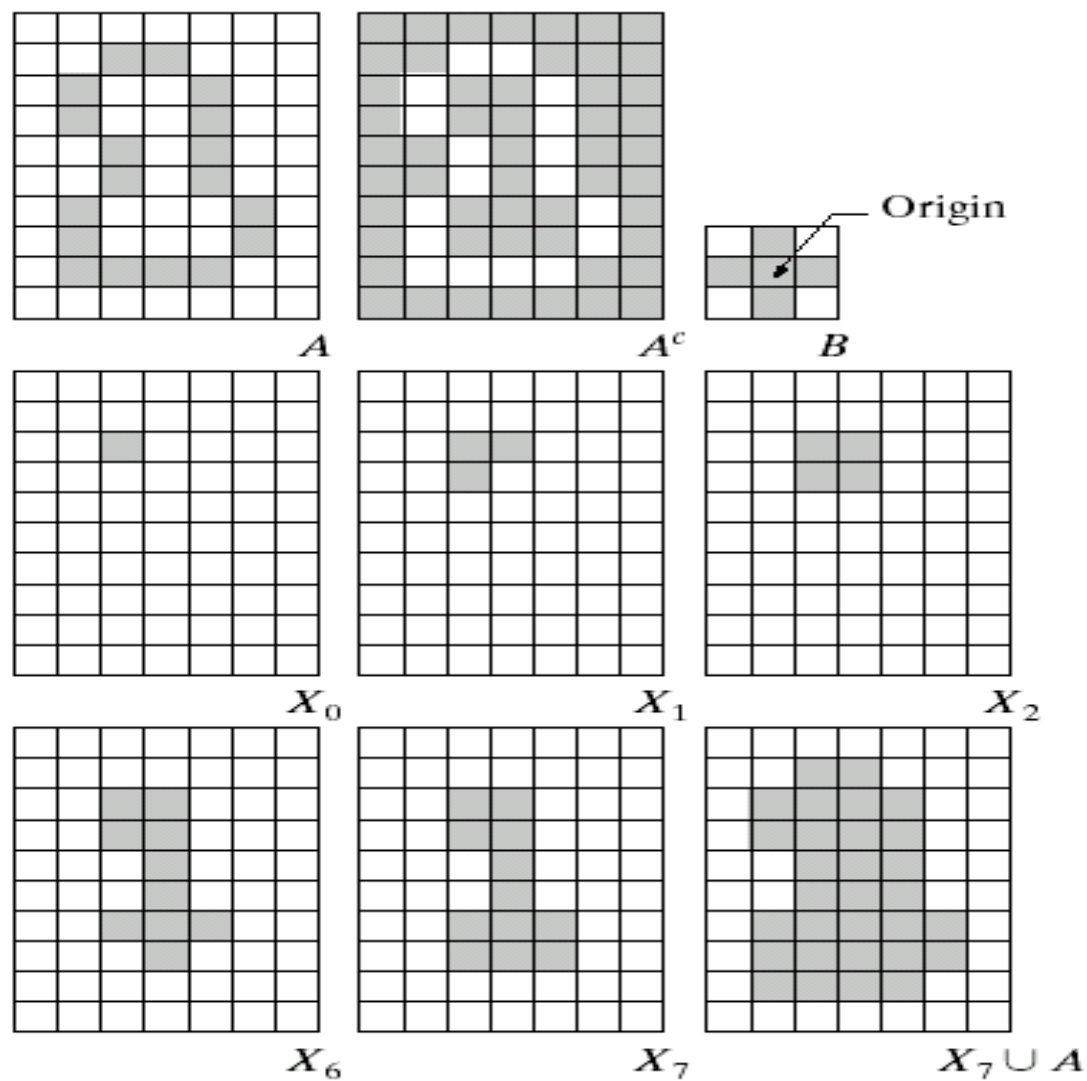


$X_2 \oplus B$



Final

Region Filling



Region Filling

Form an array, X_0 of 0s (same size as array containing A), except at the locations in X_0 corresponding to the points in each hole, which are set to 1. The following procedure fills all the holes with 1s.

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \dots$$

where B is a symmetric SE.

The algorithm terminates at iteration step k if $X_k = X_{k-1}$.

Set X_k contains all the filled holes.

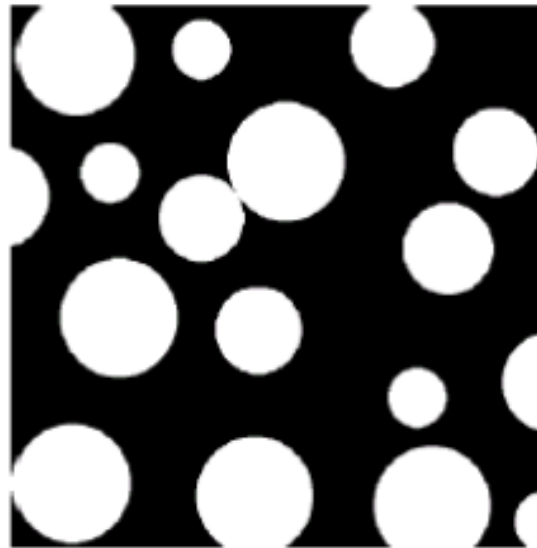
The set union, $X_k \cup A$ contains all filled holes and their boundaries.

Lecture Outline

- Morphological Algorithms
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 - Hole Filling
 - Connected Components
 - Skeletons

Extraction of Connected Components

- Process that is central to many automated image analysis applications
- Connected components require connectivity to be specified (4-connected, 8-connected)
- Labelling: How many “connected components” are there in this image?



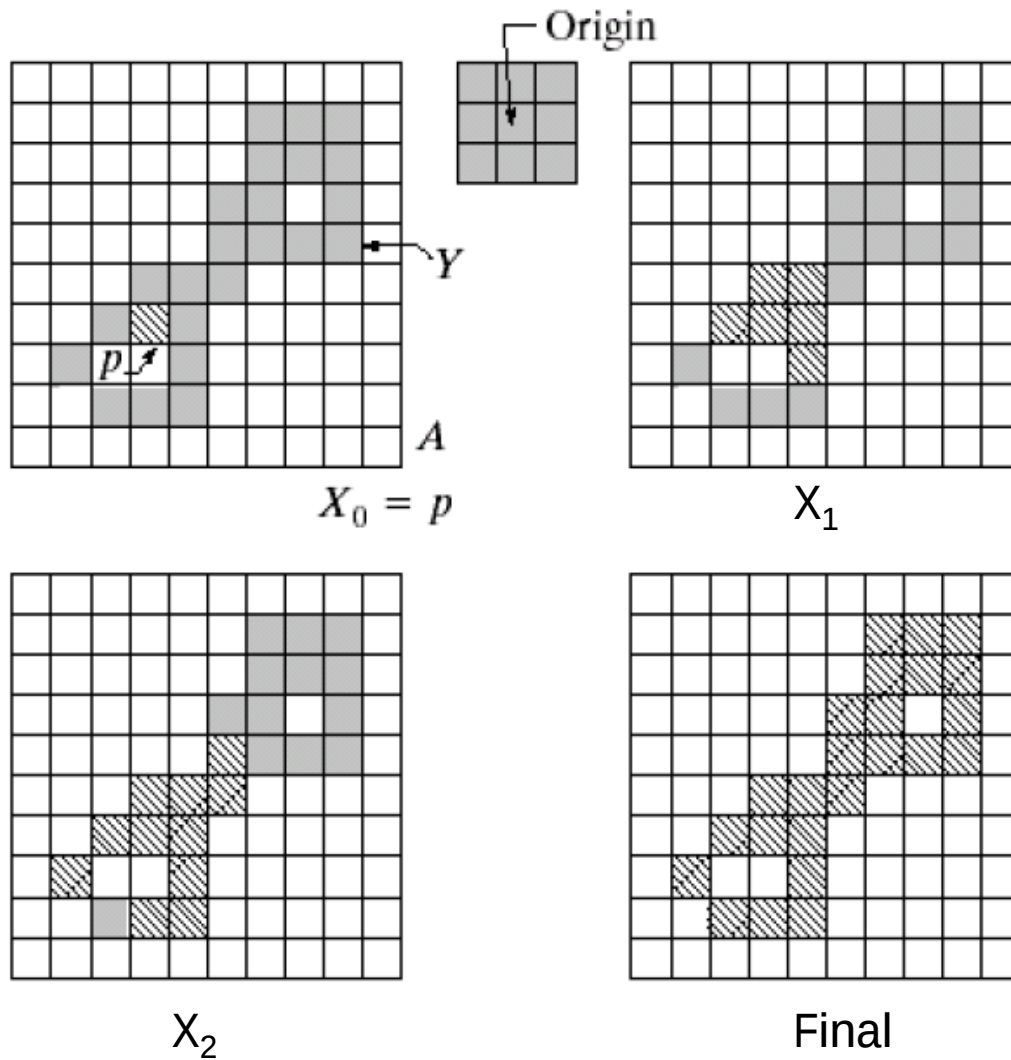
Extraction of Connected Components

- Let A be the set of 8 connected boundary points of a region
- Start with a point inside the region, Repeatedly dilate
- At each step, set to zero the points corresponding to the region boundary

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \dots$$

- Stop when no more changes
- Note its similarity to Hole Filling algorithm

Extraction of Connected Components



Y : connected component in set A ,
 p : a known point in Y

$$X_0 = p$$

$$X_k = (X_{k-1} \oplus B) \cap A$$

if $X_k = X_{k-1}$

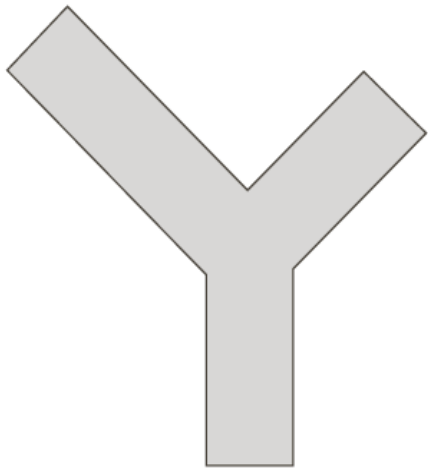
then $Y = X_k$

Lecture Outline

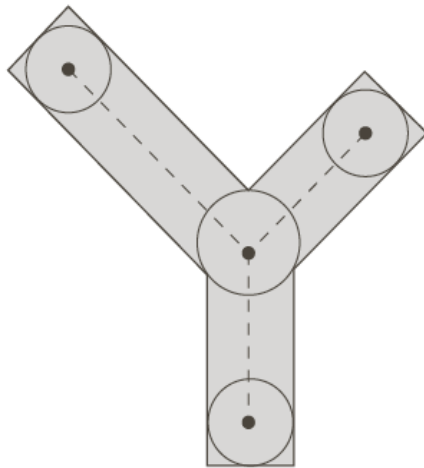
- Morphological Algorithms
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 - Connected Components
 - **Skeletons**

Skeletons (Skeletonizing)

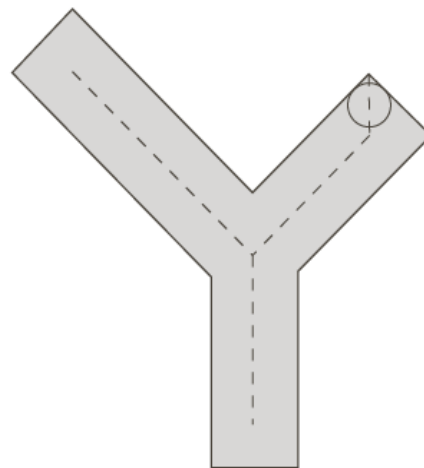
The notion of a **skeleton**, $S(A)$, of a set A :



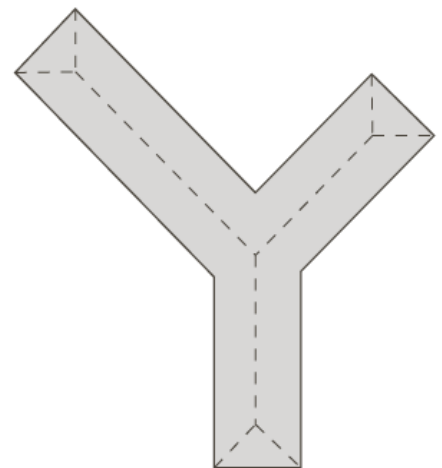
(a)



(b)



(c)



(d)

(a) Set A

(b) Various positions of maximum disks with centers on the skeleton of A

(c) Another maximum disk on a different segment of the skeleton of A

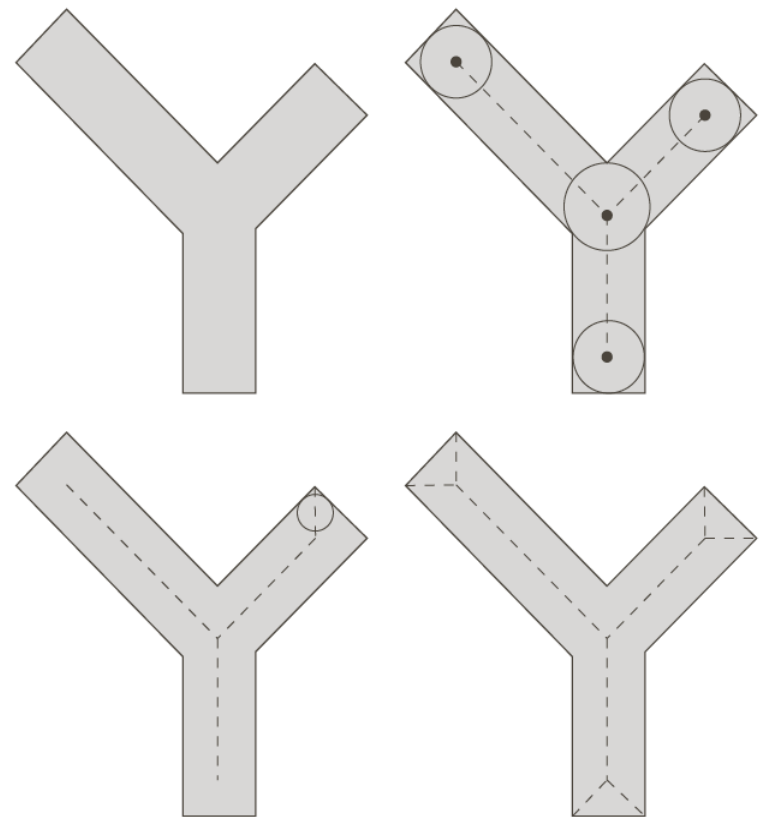
(d) Complete Skeleton

Skeletons (Skeletonizing)

The notion of a **skeleton**, $S(A)$, of a set A :

If z is a point of $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A , one cannot find a larger disk (not necessarily centered at z) containing $(D)_z$ and included in A . The disk $(D)_z$ is a maximum disk.

The disk $(D)_z$ touches the boundary of A at two or more different places



Skeletons (Skeletonizing)

The skeleton of A can be expressed in terms of erosions and openings:

$$S(A) = \bigcup_{k=0}^K S_k(A)$$

where

$$S_k(A) = (A \ominus k B) - [(A \ominus k B) \circ B]$$

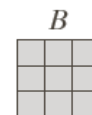
where B is the SE, and $(A \ominus k B)$ indicates k successive erosions of A:

$$(A \ominus k B) = (((...((A \ominus B) \ominus B) \ominus ...) \ominus B)$$

k times, and K is the last iterative step before A erodes to an empty set.

Skeletons (Skeletonizing)

k	$A \ominus kB$	$(A \ominus kB) \circ B$	$S_k(A)$	$\bigcup_{k=0}^K S_k(A)$
0				
1				
2				



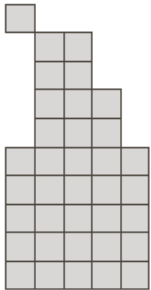
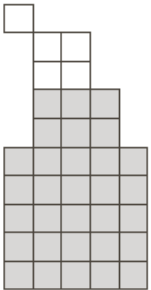
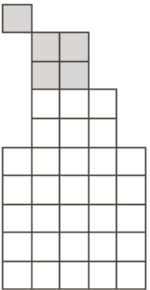
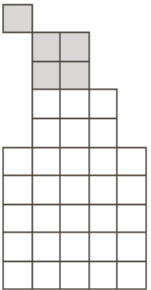
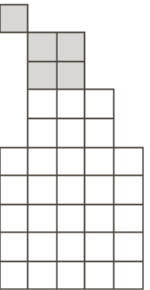
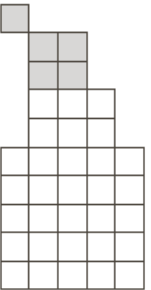
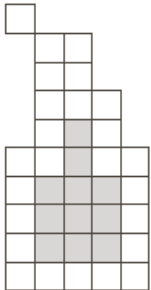
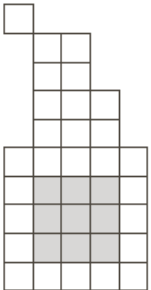
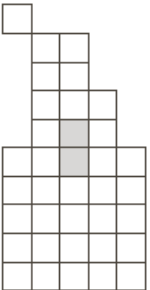
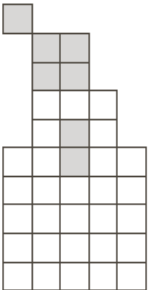
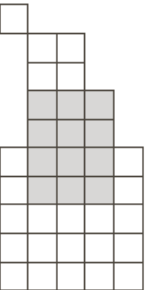
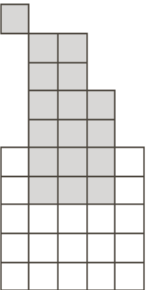
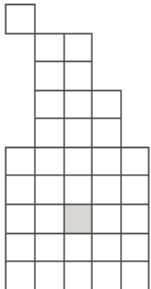
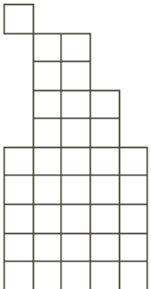
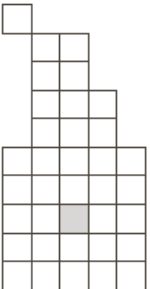
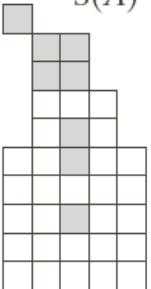
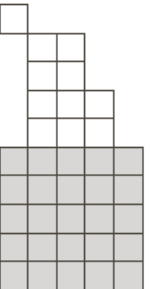
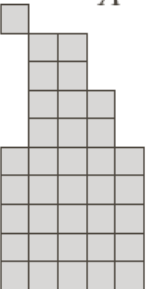
Skeletons (Skeletonizing)

A can be reconstructed from the subsets by using

$$A = \bigcup_{k=0}^K (S_k(A) \oplus kB)$$

where $S_k(A) \oplus kB$ denotes k successive dilations of A .

$$(S_k(A) \oplus kB) = (((S_k(A) \oplus B) \oplus B) \dots \oplus B)$$

k	$A \ominus kB$	$(A \ominus kB) \circ B$	$S_k(A)$	$\bigcup_{k=0}^K S_k(A)$	$S_k(A) \oplus kB$	$\bigcup_{k=0}^K S_k(A) \oplus kB$
0						
1						
2						



Skeleton Reconstructed
set