
EE1001

Foundations of Digital Techniques

Logic

Exercise 1

Q+A

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Question 1

- $\mathbf{A} = \{4, 13, 14, 66, 70\}$, $\exists x \in \mathbf{A}$ such that x is an odd number. Determine whether the statement is T or F.

Ans:
True

Question 2

- $\mathbf{A} = \{1, 2, 3\}$, $p: \forall x \in \mathbf{A}, x < 4$. Determine whether the statement is T or F.

Ans:
True

Question 3

Write the negations of following statements.

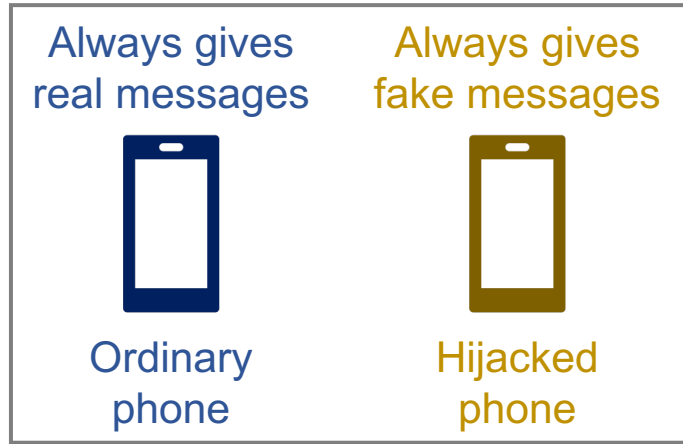
- i) $\forall n \in \mathbf{N}, n+1 > 2$
- ii) $\forall x \in \mathbf{N}, x^2 + x$ is an even number

Ans:

- i) $\sim(\forall n \in \mathbf{N}, n+1 > 2)$
 $= \exists n \in \mathbf{N}, n+1 \leq 2$
- ii) $\sim(\forall x \in \mathbf{N}, x^2 + x \text{ is an even number})$
 $= \exists x \in \mathbf{N}, x^2 + x \text{ is not an even number}$

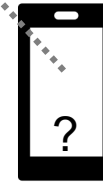
Question 4

There are two types of phones:



The Message given by A:

I am ordinary or hijacked.



A

Question:

Is A ordinary or hijacked? Use truth table to justify.

Ans:

Let p = "A is an ordinary phone"

The statement "A is ordinary or hijacked" can be formulated as " $p \vee \sim p$ "

The condition is satisfied only when the truth value of the phone and the truth value of the phone's message are the same (= if and only if (iff)), i.e.,

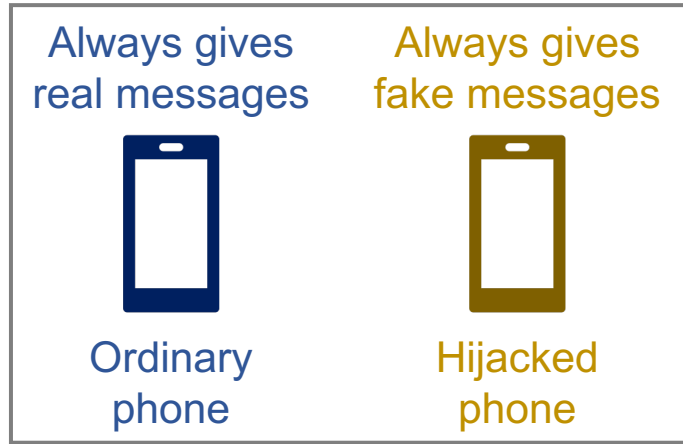
$$p \leftrightarrow (p \vee \sim p) = \text{True}$$

p	$\sim p$	$p \vee \sim p$	$p \leftrightarrow (p \vee \sim p)$
T	F	T	T
F	T	T	F

$\therefore$ A is an ordinary phone.

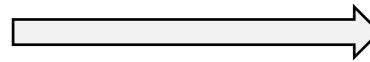
Question 5

There are two types of phones:



Input Message:

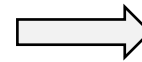
A is ordinary or hijacked.



A (hijacked)

Output Message:

?



Question:

A is a hijacked phone, and my input message is
“A is ordinary or hijacked”.

What message will A output?

Ans:

Let p = “A is an ordinary phone”

The statement “A is ordinary or hijacked” can be formulated as “ $p \vee \sim p$ ”

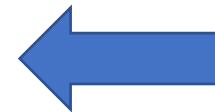
Given A is a hijacked phone, its output message should be the **negation** of the input message, i.e.,

Output message = $\sim(p \vee \sim p)$

= $\sim p \wedge \sim(\sim p)$ (De Morgan's laws)

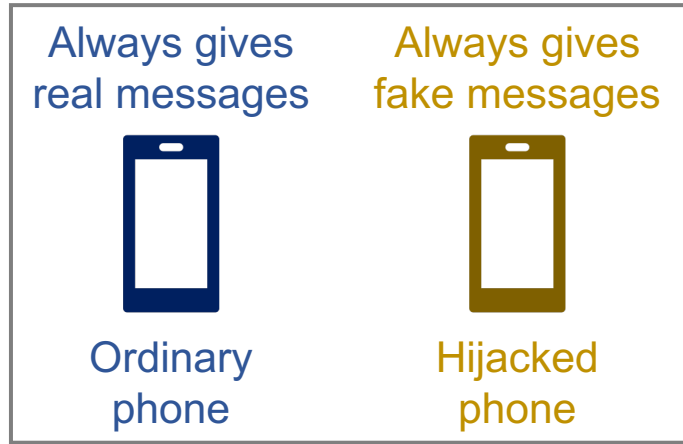
= $\sim p \wedge p$

= **A is hijacked and ordinary**

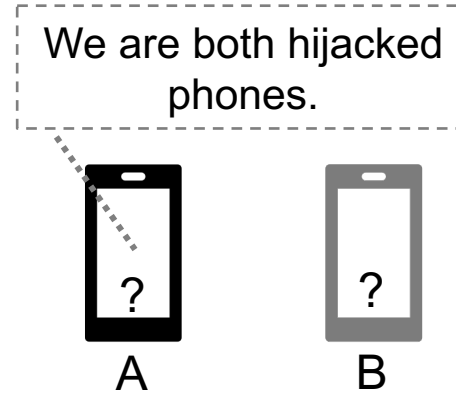


Question 6

There are two types of phones:



The Message given by A:



Question:

Are A and B ordinary or hijacked? Use truth table to justify.

Ans:

Let p = "A is an ordinary phone", and q = "B is an ordinary phone".

Therefore, the statement "A and B are hijacked" can be formulated as " $\sim p \wedge \sim q$ "

The condition is satisfied only when $p \leftrightarrow (\sim p \wedge \sim q) = \text{True}$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \leftrightarrow (\sim p \wedge \sim q)$
T	T	F	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	F

$\therefore$ The only possible solution is "A is hijacked and B is ordinary."

Question 7

Given two types of devices: normal and hijacked devices. The normal devices always give real (i.e., true) messages, and the hijacked devices always give fake (i.e., false) messages.

There are THREE (3) devices, and their messages are as follows.

Device A: “If Device *B* is hijacked, then Device *C* is also hijacked.”

Device B: “Devices *A* and *C* are hijacked.”

Device C: “This device could be normal or hijacked.”

Let *a*, *b*, and *c* be

a = “Device *A* is normal”

b = “Device *B* is normal”

c = “Device *C* is normal”

- (i) Formulate the messages of the devices using propositional logic.
- (ii) Using truth table to determine whether the devices *A*, *B*, and *C* are normal or hijacked.

Ans:

(i)

The message on the box:

A’s message: “ $\sim b \rightarrow \sim c$ ”

B’s message: “ $\sim a \wedge \sim c$ ”

C’s message: “ $c \vee \sim c$ ” = **t** (tautology)

(ii)

The condition is satisfied only when the truth values of the boxes’ statuses and the truth values of the boxes’ messages are the same, i.e.,

$[a \leftrightarrow (\sim b \rightarrow \sim c)] \wedge [b \leftrightarrow (\sim a \wedge \sim c)] \wedge [c \leftrightarrow (c \vee \sim c)]$ must be true

<i>a</i>	<i>b</i>	<i>c</i>	$\sim b \rightarrow \sim c$	$\sim a \wedge \sim c$	$c \vee \sim c$	$(a \leftrightarrow \sim b \rightarrow \sim c) \wedge (b \leftrightarrow \sim a \wedge \sim c) \wedge (c \leftrightarrow c \vee \sim c)$
T	T	T	T	F	T	F
T	T	F	T	F	T	F
T	F	T	F	F	T	F
T	F	F	T	F	T	F
F	T	T	T	F	T	F
F	T	F	T	T	T	F
F	F	T	F	F	T	T
F	F	F	T	T	T	F

Therefore, device C is normal, and devices A and B are hijacked.

Question 8

Determine the validity of the following arguments and brief the reasons.

- (i) If it rains today, we would not go swimming. We go swimming today. Therefore, it did not rain today.
- (ii) Alex is a history major. Thus, Alex is either an engineering major or a history major.
- (iii) If Smith plays football, then he will stay outside too long. If Smith stays outside too long, then he will get heatstroke. Therefore, if Smith plays football, he will get heatstroke.
- (iv) If I am tired and if I do not spend time on the mobile phone, then I will sleep early. I didn't sleep early. Therefore, I was not tired and I spent time on the mobile phone.
- (v) If Kate doesn't work hard, then she will fail in this course. If Kate doesn't fail in this course, then she will get the scholarship. Kate gets the scholarship. Therefore, Kate worked hard.

Ans:

- (i) **Valid.** (Inference Rules – Modus Tollens)
- (ii) **Valid.** (Inference Rules – Addition)
- (iii) **Valid.** (Inference Rules – Hypothetical Syllogism)
- (iv) **Invalid.**

Let p = "I am tired"; q = "I spend time on the mobile phone"; r = "I sleep early".

- 1. $p \wedge \sim q \rightarrow r$ (premise)
- 2. $\sim r$ (premise)
- 3. $\sim(p \wedge \sim q)$ (MT 1,2)
- 4. $\sim p \vee q$ (De Morgan's law)

Therefore, to make the argument valid, the conclusion should be "I was not tired or I spent time on the mobile phone

(v) **Invalid.**

Let a = "Kate works hard"; b = "Kate fails in this course"; c = "Kate gets the scholarship".

- 1. $\sim a \rightarrow b$ (premise)
- 2. $\sim b \rightarrow c$ (premise)
- 3. c (premise)

These 3 premises have no logical relation with each other. (Can be further verified using truth table)