

Marks:

/20

Name: _____ SID: _____

MA2001 (CA1)

Quiz 1

Semester A, 2023-24

Answer all questions. Write your answers in the space provided.

1. State whether the following statements are true (T) or false (F). [4 marks]

(a) If B is a real symmetric matrix, then B is diagonalizable.

Answer: T

(b) If matrix C is invertible, then C is diagonalizable.

Answer: F

(c) If $\det(M) = 0$, then one of the eigenvalues of M is 0.

Answer: T

(d) The quadratic form $q = 2x_1^2 + x_2^2 + 4x_3^2 - 4x_2x_3$ is positive definite.

Answer: F

$$= 2x_1^2 + (x_2 - 2x_3)^2$$

2. If A is a 2×2 matrix with $A \begin{pmatrix} 6 \\ -1 \end{pmatrix} = \begin{pmatrix} -18 \\ 3 \end{pmatrix}$ and $A \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

(a) Find the eigenvalues and the corresponding eigenvectors of A . [4 marks]

$$A \begin{pmatrix} 6 \\ -1 \end{pmatrix} = \begin{pmatrix} -18 \\ 3 \end{pmatrix} = -3 \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

$$\therefore \lambda_1 = -3, \quad \vec{x}_1 = k \begin{pmatrix} 6 \\ -1 \end{pmatrix}, \quad k \neq 0$$

$$A \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore \lambda_2 = 4, \quad \vec{x}_2 = k \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad k \neq 0$$

(b) Write down an invertible matrix P and a diagonal matrix D such that $AP = PD$. [4 marks]

$$P = \begin{pmatrix} 6 & 1 \\ -1 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} -3 & 0 \\ 0 & 4 \end{pmatrix}$$

(c) Find the matrix A .

[4 marks]

$$AP = PD$$

$$\Rightarrow A = PDP^{-1}$$

$$= \begin{pmatrix} 6 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -3 & 0 \\ 0 & 4 \end{pmatrix} \frac{1}{7} \begin{pmatrix} 1 & -1 \\ 1 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} -18 & 4 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 6 \end{pmatrix} \frac{1}{7}$$

$$= \begin{pmatrix} -14 & 42 \\ 7 & 21 \end{pmatrix} \frac{1}{7}$$

$$= \begin{pmatrix} -2 & 6 \\ 1 & 3 \end{pmatrix}$$

(d) Find the eigenvalues of $A^5 - 2A$.

[2 marks]

$$\lambda_1 = (-3)^5 - 2(-3) = -237$$

$$\lambda_2 = 4^5 - 2(4) = 1016$$

(e) Is the quadratic form $q = [x_1 \ x_2] A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ positive definite, negative definite, positive semidefinite, negative semidefinite or indefinite? Show how you obtain your answer.

[2 marks]

$$q = -2x_1^2 + 7x_1x_2 + 3x_2^2$$

$$= -2\left(x_1^2 - \frac{7}{2}x_1x_2\right) + 3x_2^2$$

$$= \underbrace{-2}_{<0} \left(x_1 - \frac{7}{4}x_2\right)^2 + \underbrace{\frac{73}{8}}_{>0} x_2^2$$

$\therefore q$ is indefinite.

- End -