

Session:

Name:

Student ID:

MA 1201 Semester B 2018/19

Assignment 3 — Due at 5 pm, 18/4/2019 (Thursday) in tutorial classes

Instructions:

- Please show your work. Unsupported answers will receive **NO** credits.
- Make sure you write down the correct lecture session (A/B/C/D/E/F/G/H) you have registered for, together with your full name and student ID on the front page of your answer script. Hand your solution to your TA in tutorial class before the deadline.
- In case your tutorial class is scheduled on days other than Thursday, you may choose to hand your solution to your TA in one of the tutorial classes scheduled on Thursday.
- **NO** late homework will be accepted. Homework submitted to wrong tutorial sessions will **NOT** be graded and will receive **0 POINTS**.

1. (15 points) Find the area of the surface generated by revolving the curve $y = \sqrt{2x - x^2}$, $1/2 \leq x \leq 3/2$, about the x -axis.

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2. (10 points) Suppose a complex number z satisfies the equation

$$\frac{1+z}{1-z} = e^{i\theta},$$

for some $\theta \in (\pi, 2\pi)$. Find the complex number z and express the result in Euler's form with principal argument.

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3. (10 points) Suppose a complex number z satisfies the equation

$$2z^2 - (3 + 8i)z - (m + 4i) = 0.$$

Given that m is a real constant and that one of the two solutions is real, find the two solutions of the above complex quadratic equation. (Hint: The real solution is easy to find. To find the other solution, use the fact that the two solutions z_1, z_2 of the quadratic equation $az^2 + bz + c = 0$ satisfy $z_1 + z_2 = -b/a$.)

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4. (15 points) Find all the complex solutions of the equation

$$z^4 + (\sqrt{3} - 3i)(1 + z)^4 = 0.$$

Express the results in Cartesian form (you may express your answer as a function of k , e.g. $z = e^{i(k\pi/2)} = \cos(k\pi/2) + i \sin(k\pi/2)$, $k = 0, 1, 2, 3$, without explicitly evaluating the expression for each k).

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5. (15 points) Let

$$A = \begin{pmatrix} 3 & 1 & -2 \\ -2 & 2 & 2 \\ 0 & 1 & 1 \end{pmatrix}.$$

- (a) (5 points) Evaluate the determinant of A by the cofactor expansion and compute $\det(A^{-2})$.
- (b) (10 points) Find all values of λ such that $\det(A - \lambda I_3) = 0$, where I_3 is the 3×3 identity matrix.

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6. (15 points) Consider the linear system

$$x_1 - 2x_2 + 3x_3 = 1$$

$$2x_1 - x_2 + 2cx_3 = 2$$

$$3x_1 - 3x_2 + c^2x_3 = c$$

Find all possible values of c such that the system

- (a) (5 points) has a unique solution.
- (b) (5 points) has infinitely many solutions.
- (c) (5 points) has no solutions.

7. (20 points) Let

$$A = \begin{pmatrix} 0 & -1 & 5 & -3 \\ 1 & -2 & 3 & -4 \\ 2 & -3 & 1 & -5 \end{pmatrix}, \quad b = \begin{pmatrix} 8 \\ 5 \\ 2 \end{pmatrix}.$$

- (a) (15 points) Solve the linear system $Ax = b$ by Gaussian elimination and express the general solution in vector form.
- (b) (5 points) Write down the corresponding homogenous system $Ax = 0$ explicitly and determine all non-trivial solutions from (a) without resolving the system.