

(Hints)

Name: _____

Student ID: _____

Answer ALL questions. (Full marks: 40)

Total Marks: _____

Q1: _____ Q2: _____ Q3: _____

Question 1 (10 marks)

Suppose $z^3 - 4yz + 4x^2 = 1$ determines the function $z = z(x, y)$ as a function of x, y locally at $(x, y, z) = (1, 1, 1)$.

- (i) Find the linear approximation of z at $(x, y, z) = (1, 1, 1)$. (6 marks)
- (ii) Find the quadratic surface approximation of z at $(x, y, z) = (1, 1, 1)$. (4 marks)

$$(i) \quad Z(x, y) \approx Z(1, 1) + Z_x(1, 1)(x-1) + Z_y(1, 1)(y-1)$$

$$(ii) \quad Z(x, y) \approx Z(1, 1) + Z_x(1, 1)(x-1) + Z_y(1, 1)(y-1) \\ + \frac{1}{2!} [Z_{xx}(1, 1)(x-1)^2 + 2Z_{xy}(1, 1)(x-1)(y-1) + Z_{yy}(1, 1)(y-1)^2]$$

Question 2 (15 marks)

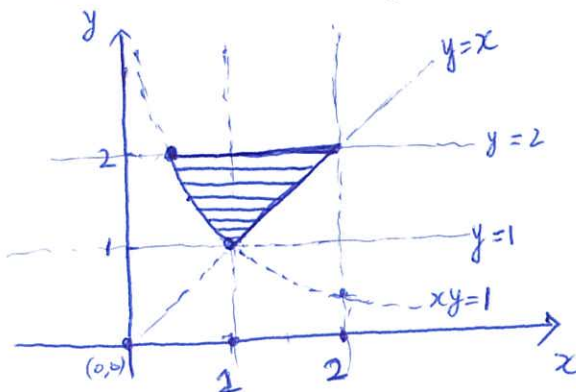
Evaluate the integral

$$\int_1^2 \int_{1/y}^y \sqrt{\frac{y}{x}} e^{\sqrt{xy}} dx dy$$

by the following steps:

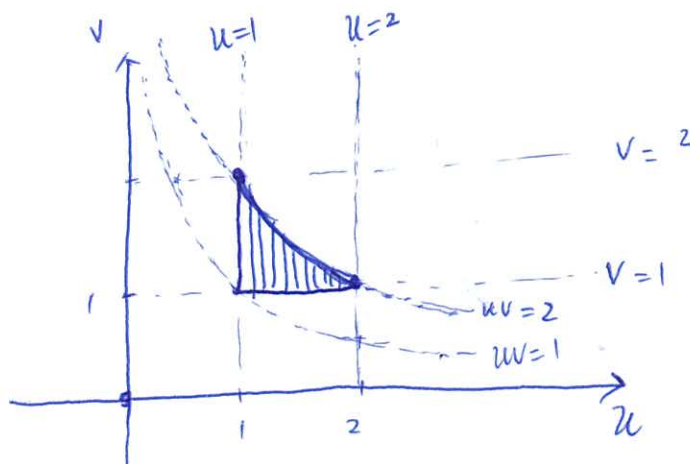
- (i) Sketch the region R of integration. (2 marks)
- (ii) Let $u = \sqrt{xy}$ and $v = \sqrt{\frac{y}{x}}$, calculate the Jacobian $J(u, v) = \frac{\partial(x, y)}{\partial(u, v)}$. (3 marks)
- (iii) Sketch the new region R^* in the new coordinate system (u, v) . (2 marks)
- (iv) Find the integration under the new coordinate system. (8 marks)

(i)



R

(iii)



R^*

Question 3 (15 marks)

- (i) Show that, under the spherical polar coordinate, $\rho = 2\cos \varphi$ is the sphere $x^2 + y^2 + (z - 1)^2 = 1$, and $\varphi = \frac{\pi}{4}$ is the cone $z = \sqrt{x^2 + y^2}$. (5 marks)
- (ii) Use spherical polar coordinate to calculate the volume of the solid V that is bounded below by the sphere $x^2 + y^2 + (z - 1)^2 = 1$ and above by the cone $z = \sqrt{x^2 + y^2}$. (10 marks)

