

EE3122: Analogue Circuit Fundamentals

Current Mirror

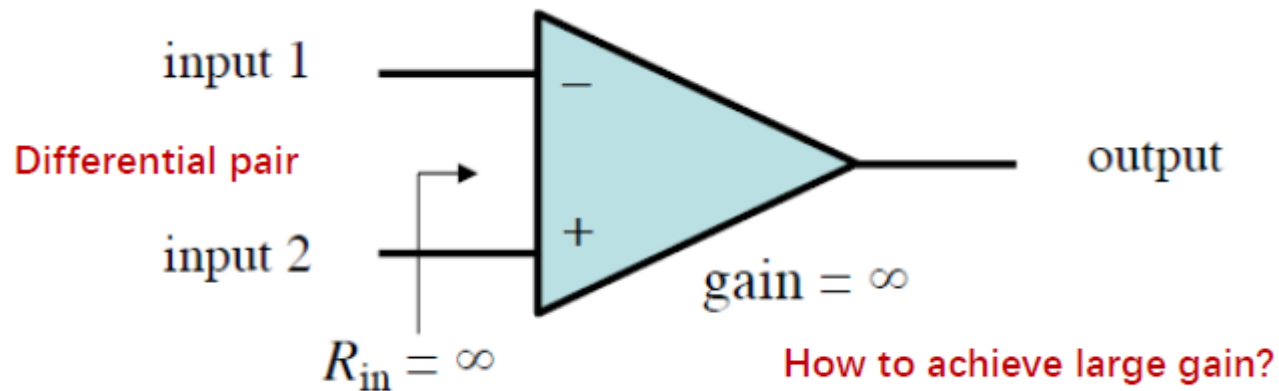
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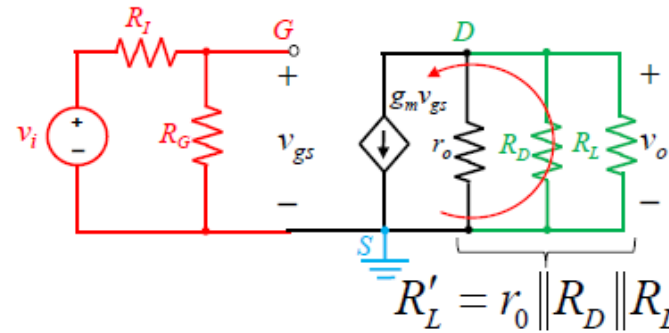
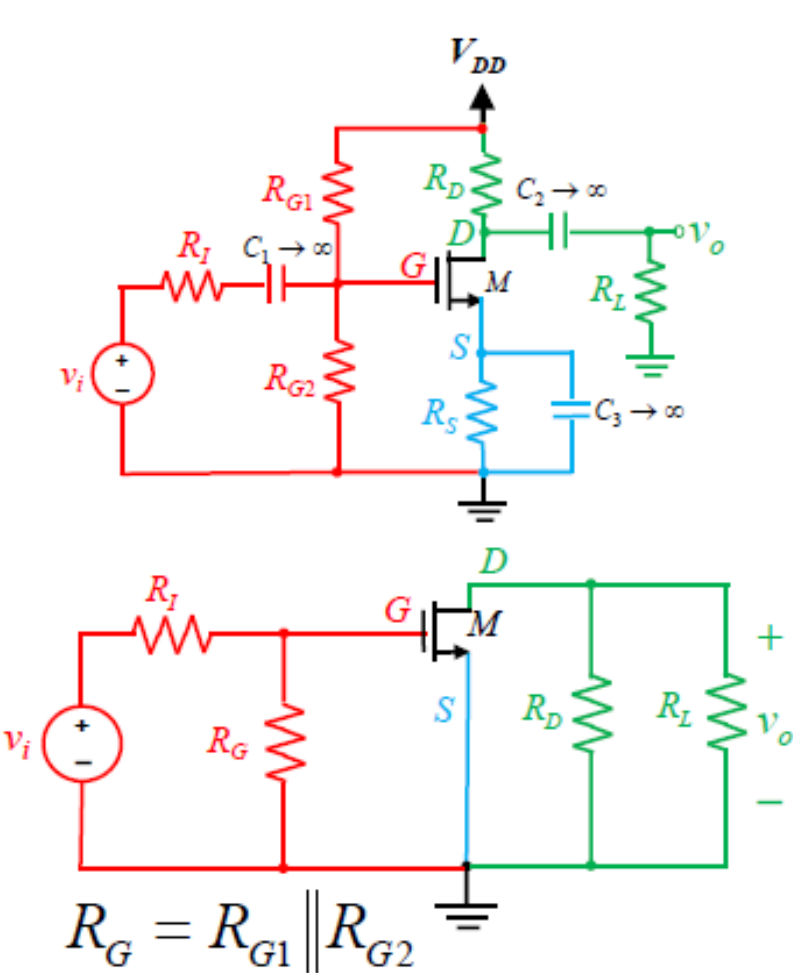
Prof. Arindam BASU

Motivation

- How to achieve very high gain?
- How to achieve very high CMRR?



Voltage gain in fully bypassed CS Amplifier



Terminal voltage gain between gate and drain is:

$$A_{vt} = \frac{v_d}{v_g} = \frac{-g_m v_{gs} R'_L}{v_{gs}} = -g_m R'_L$$

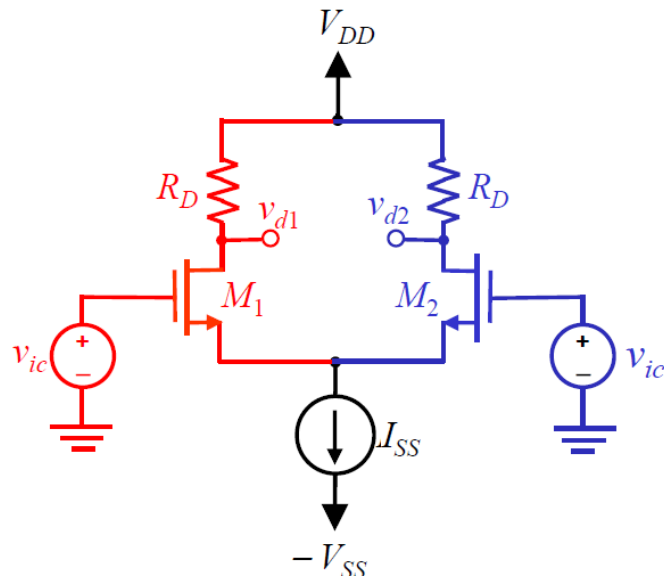
Overall voltage gain from source v_i to output voltage v_o across R_L is:

$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{v_g}{v_i}$$

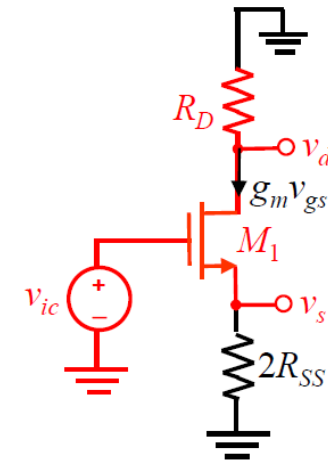
$$= -g_m R'_L \left(\frac{R_G}{R_I + R_G} \right)$$

- Can increase gain by increasing R_D & R_L .
- But DC bias issue if increase $R_D \rightarrow$ voltage headroom limited due to V_{DD}
 - E.g.: If $I_{bias} = 1\text{mA}$ & $R_D = 100\text{ k}\Omega$, need $V_{DD} = 100\text{ V}!!$

CMRR in Differential Amplifier



Common-mode half-circuit

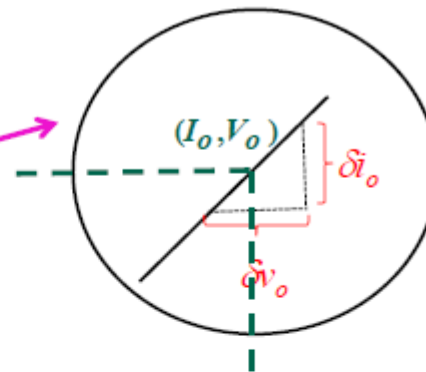
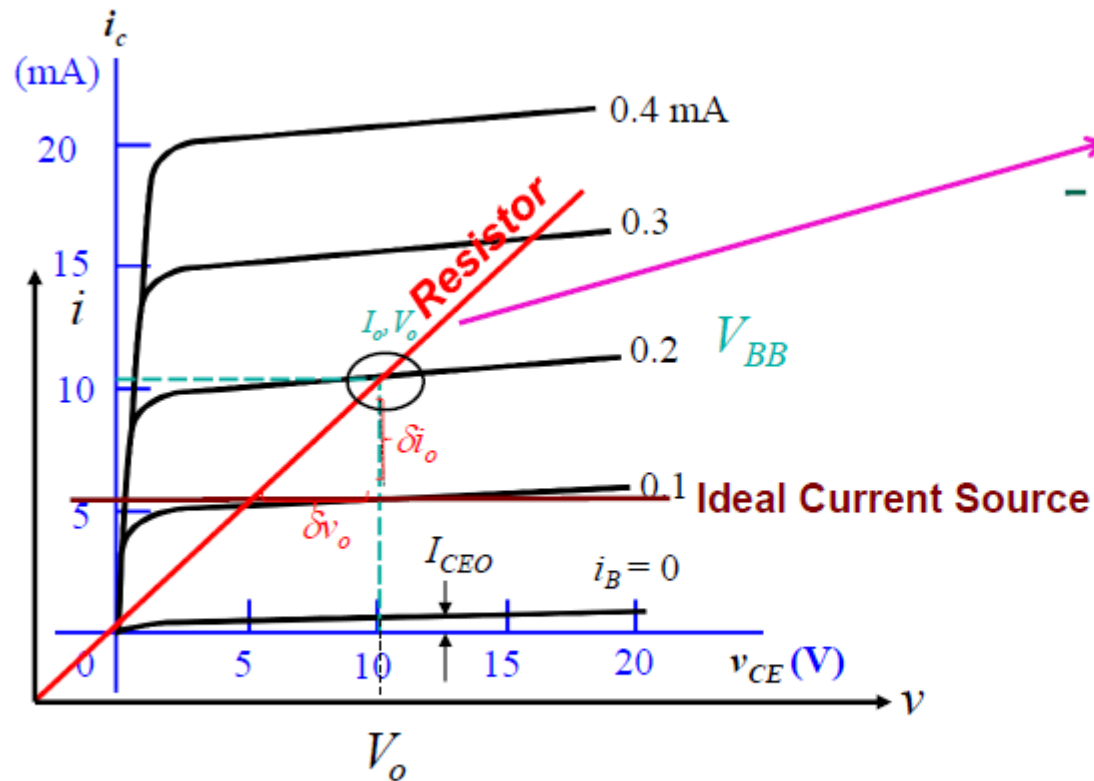


- For single-ended output, $CMRR = \left| \frac{A_{dm-se}}{A_{cm-se}} \right| = \frac{\frac{g_m R_D}{2}}{\frac{R_D}{2R_{SS}}} = g_m R_{SS}$

- If use normal resistor, increase $R_{SS} \rightarrow$ DC power supply limit due to $I_{SS} R_{SS}$ drop!

Active load

Need resistor with different AC & DC resistance $\rightarrow$ transistor as solution!

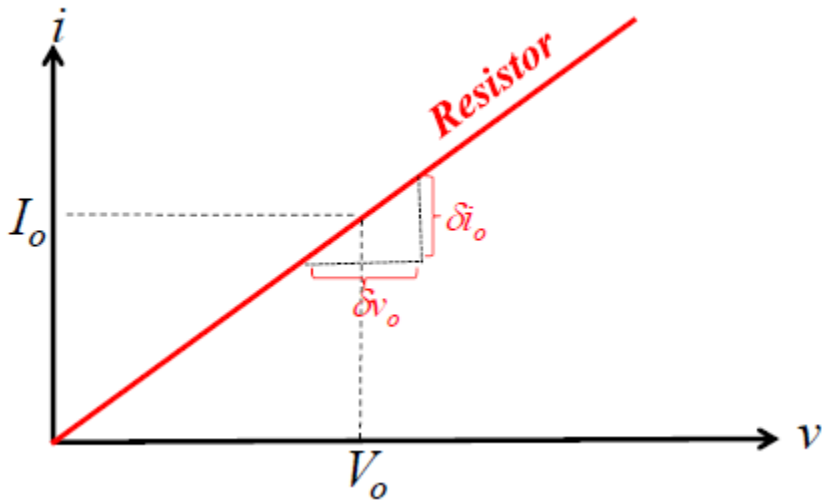


$$R_{DC} = V_o/I_o$$

$$R_{AC} = \delta V_o/\delta I_o$$

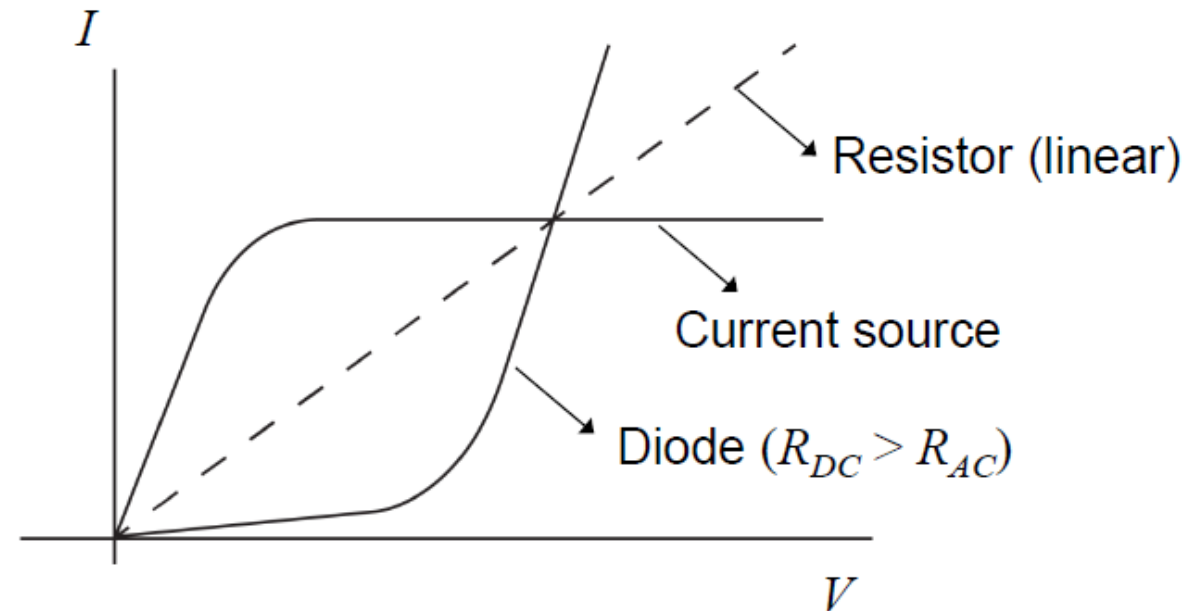
Current mirrors allow R_{DC} to be different from R_{AC} .

DC & AC resistances

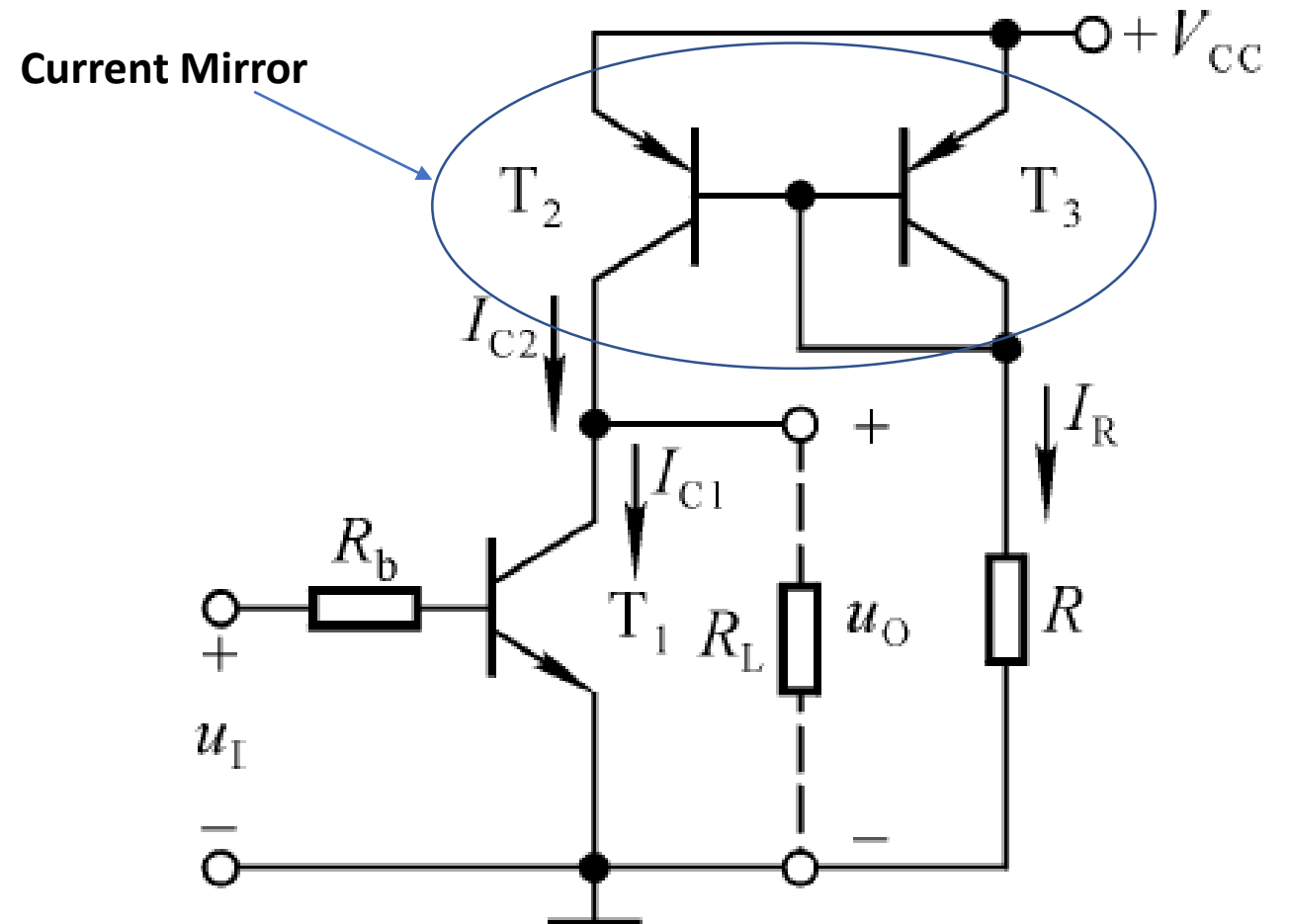
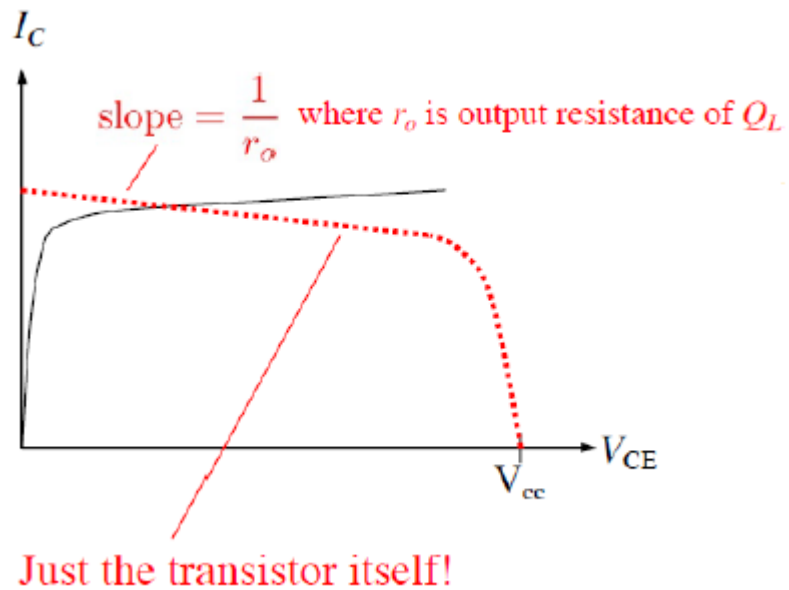


What happens in a resistor?

$$\frac{\delta v_o}{\delta i_o} = \frac{V_o}{I_o} = R$$

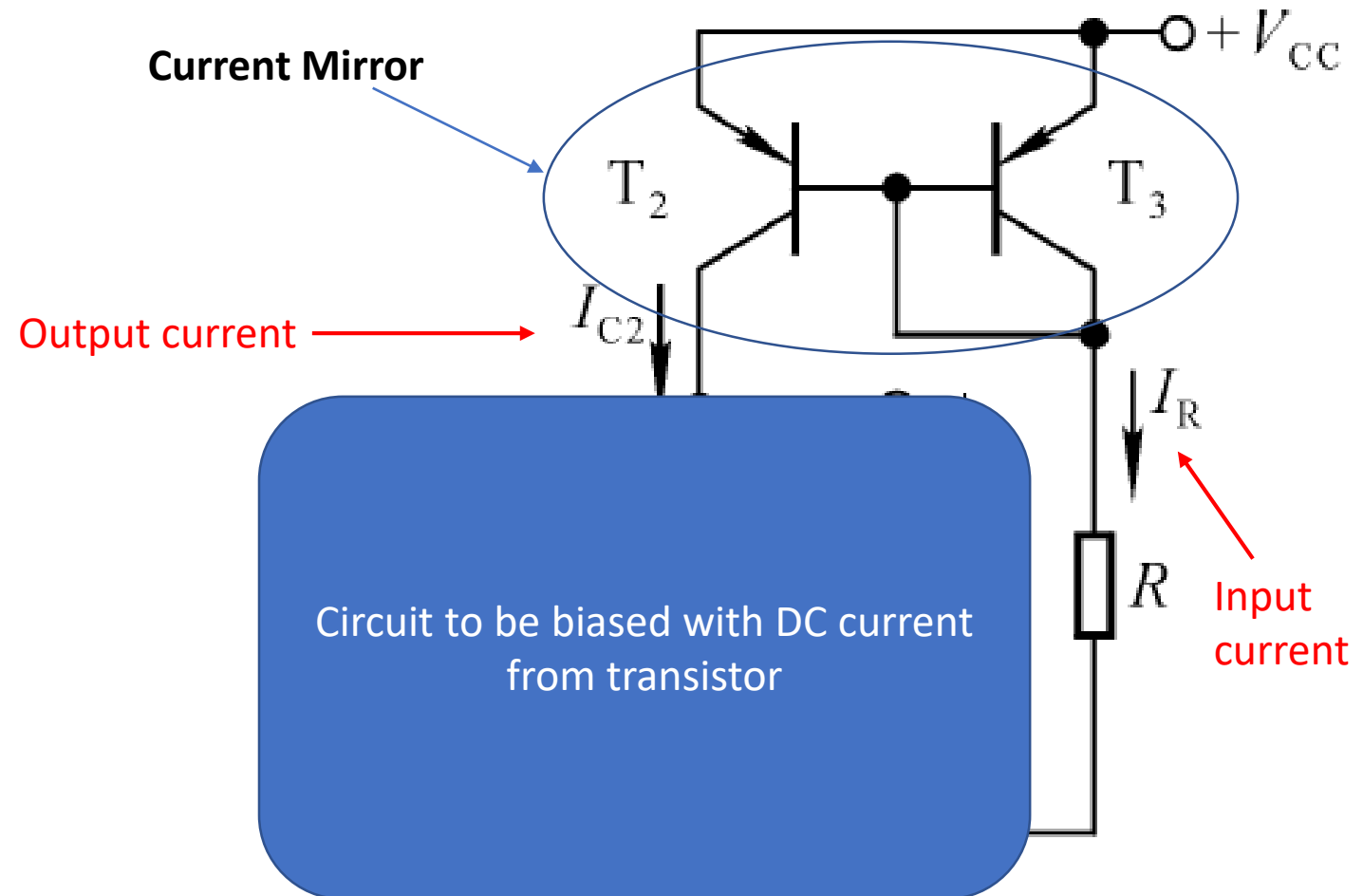


BJT as Active Load

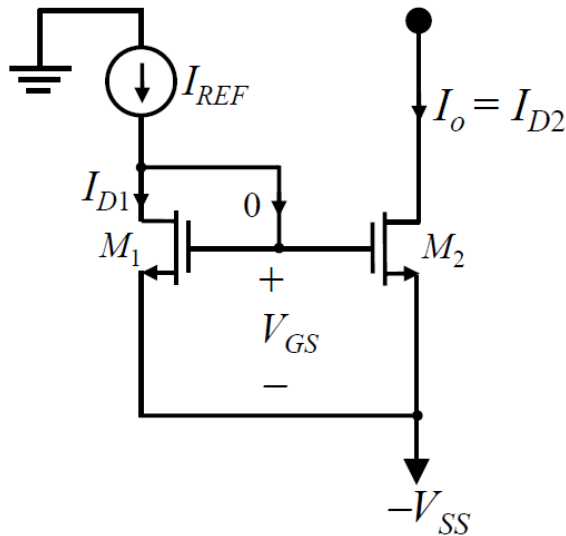


Current Mirror

- Current in the MOS/BJT current source is fixed by the Current Mirror
- Also has advantage of making many copies with arbitrary gain!



MOS Current Mirror: DC Analysis



M_1 and M_2 are matched with identical V_{TN} and K_n .

I_{REF} provides operating bias to the current mirror.

$$I_{REF} = I_{D1} = \frac{K_n}{2} (V_{GS1} - V_{TN})^2$$

$$I_O = I_{D2} = \frac{K_n}{2} (V_{GS2} - V_{TN})^2$$

$$\therefore V_{DS1} = V_{GS1} = V_{GS2} = V_{GS}$$

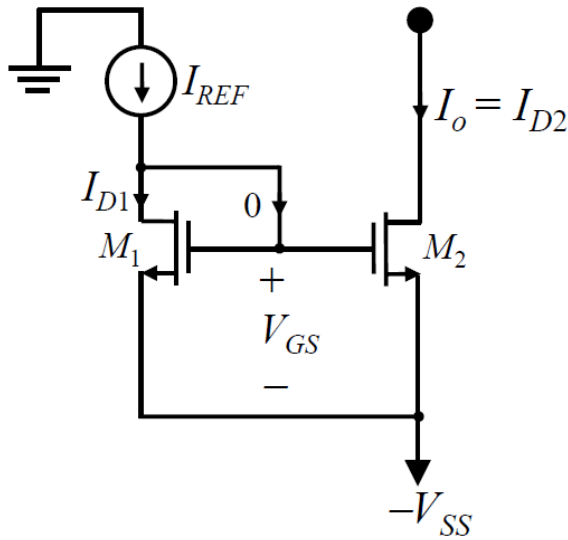
$$\text{Mirror Ratio, } MR = \frac{I_o}{I_{REF}}$$

The current mirror is designed to have $I_o = I_{REF}$ with $MR = 1$

But $\rightarrow$ (See next page)

Let's start from the ideal devices, i.e., no early effect.

MOS Current Mirror: DC Analysis



M_1 and M_2 are matched with identical V_{TN} , K_n , and λ .

I_{REF} provides operating bias to the current mirror.

$$I_{REF} = I_{D1} = \frac{K_n}{2} (V_{GS1} - V_{TN})^2 (1 + \lambda V_{DS1})$$

$$I_O = I_{D2} = \frac{K_n}{2} (V_{GS2} - V_{TN})^2 (1 + \lambda V_{DS2})$$

$$\therefore V_{DS1} = V_{GS1} = V_{GS2} = V_{GS}$$

$$\text{Mirror Ratio, } MR = \frac{I_O}{I_{REF}} = \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}} = \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{GS}}$$

$$\text{where } V_{GS} = V_{TN} + \sqrt{\frac{2I_{REF}}{K_n(1 + \lambda V_{GS})}} \approx V_{TN} + \sqrt{\frac{2I_{REF}}{K_n}}$$

The current mirror is designed to have $I_O = I_{REF}$ with $MR = 1$, but mismatch in V_{DS} causes error in MR : V_{DS1} is fixed by I_{REF} , while V_{DS2} is floating; they are not guaranteed to be the same!

Real transistors always have an early effect!

Early Effect & Connection with Small-signal model

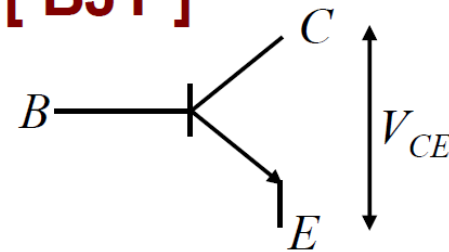
$$I_C = I_o e^{V_{BE}/V_T} \left(1 + \frac{V_{CE}}{V_A} \right)$$

$$I_D = K_n (V_{GS} - V_{th})^2 (1 + \lambda V_{DS})$$

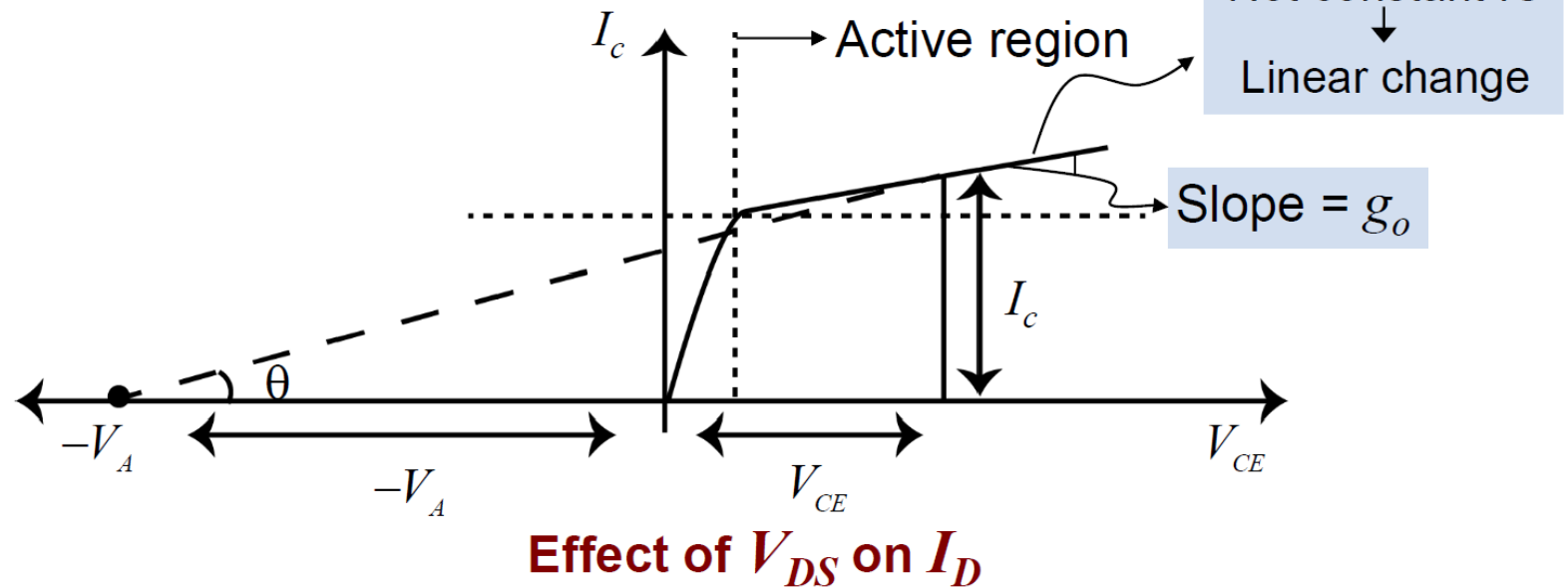
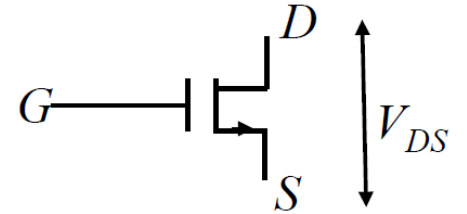
Concept:

- Effect of V_{CE} on I_C
- Effect of V_{DS} on I_D

[BJT]



[MOS]



Early Effect & Connection with Small-signal model

Linear Change of I_C with V_{CE} :

$$I_C = \underbrace{I_o e^{V_{BE}/V_T}}_K \left(1 + \frac{V_{CE}}{V_A} \right) = K \left(1 + \frac{V_{CE}}{V_A} \right)$$

$$I_C = 0 \Rightarrow V_{CE} = -V_A$$

$$y = mx + c$$
$$I_C = \frac{KV_{CE}}{V_A} + K$$

Slope:

$$\frac{\partial I_C}{\partial V_{CE}} = g_o \quad (\text{output conductance})$$

AC
resistance
related to
slope

Output resistance, $r_o = \frac{1}{g_o}$

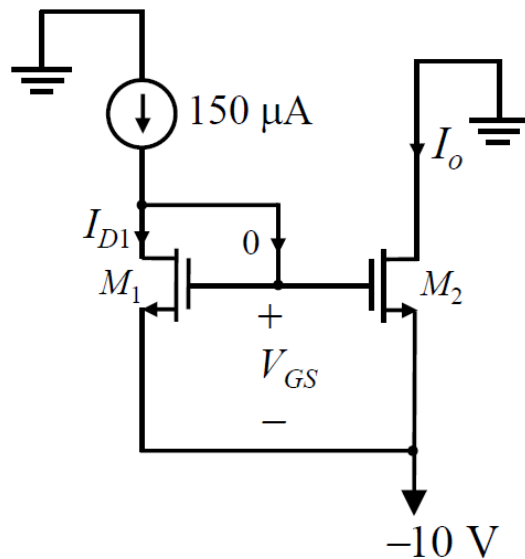
Slope = $\tan \theta$

$$= \frac{I_C}{V_{CE} - (-V_A)}$$
$$= \frac{I_C}{V_{CE} + V_A} = g_o$$

$$r_o = \frac{V_{CE} + V_A}{I_C}$$

MOS Current Mirror: Example

Calculate the output current for the given current mirror, if it is given that $V_{TN} = 1 \text{ V}$, $K_n = 250 \mu\text{A}/\text{V}^2$, and $\lambda = 0.0133 \text{ V}^{-1}$.



Let's first try to neglect $1 + \lambda V_{DS1}$ term to simplify calculation.

$$V_{GS} = V_{Tn} + \sqrt{\frac{2I_{ref}}{K_n}}$$

$$V_{DS1} = V_{GS} = 1 + \sqrt{\frac{2 \times 150 \times 10^{-6}}{250 \times 10^{-6}}} = 2.10 \text{ V}$$

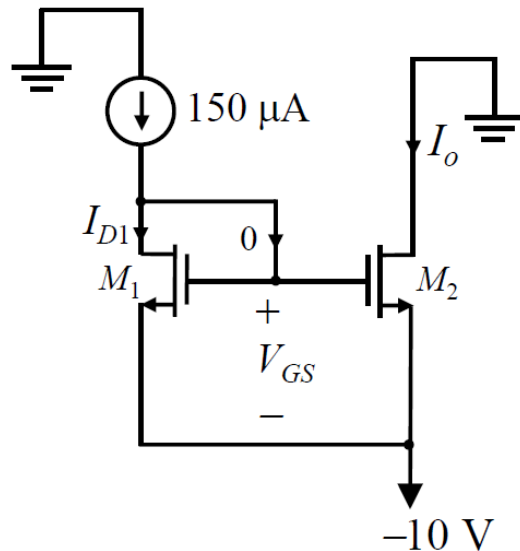
$$MR = \frac{1 + 0.0133 \times 10}{1 + 0.0133 \times 2.1} = 1.1$$

$$I_o = 1.1 \times 150 \times 10^{-6} = 165 \mu\text{A}$$

Actual current is found to be mismatched by approximately 10% from the ideal output current of $150 \mu\text{A}$.

MOS Current Mirror: Example

Calculate the output current for the given current mirror, if it is given that $V_{TN} = 1$ V, $K_n = 250$ $\mu\text{A}/\text{V}^2$, and $\lambda = 0.0133$ V^{-1} .



If you do not neglect $1 + \lambda V_{DS1}$, it is difficult to do manual calculation

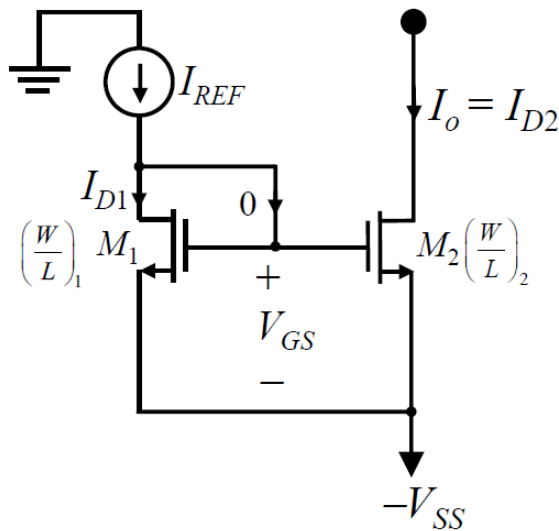
$$V_{DS1} = 1 + \sqrt{\frac{2 \times 150 \times 10^{-6}}{250 \times 10^{-6} (1 + 0.0133 V_{DS1})}}$$
$$= 2.08 \text{ V (Iterative Solution)}$$

$$MR \approx 1.1$$

$$I_o = 1.1 \times 150 \times 10^{-6} = 165 \text{ } \mu\text{A}$$

You can note very minimal error, compared to the previous page, $\Rightarrow (1 + \lambda V_{DS1})$ can always be ignored during calculation of bias V_{GS} .

MOS Current Mirror: Gain ($MR \neq 1$)



Mirror ratio can be changed by modifying W/L ratios of the mirror transistors.

$$K_n = K'_n \left(\frac{W}{L} \right) \text{ where } K'_n = \mu_n C_{ox}$$

$$\therefore K_{n1} = K'_n \left(\frac{W}{L} \right)_1 \text{ and } K_{n2} = K'_n \left(\frac{W}{L} \right)_2$$

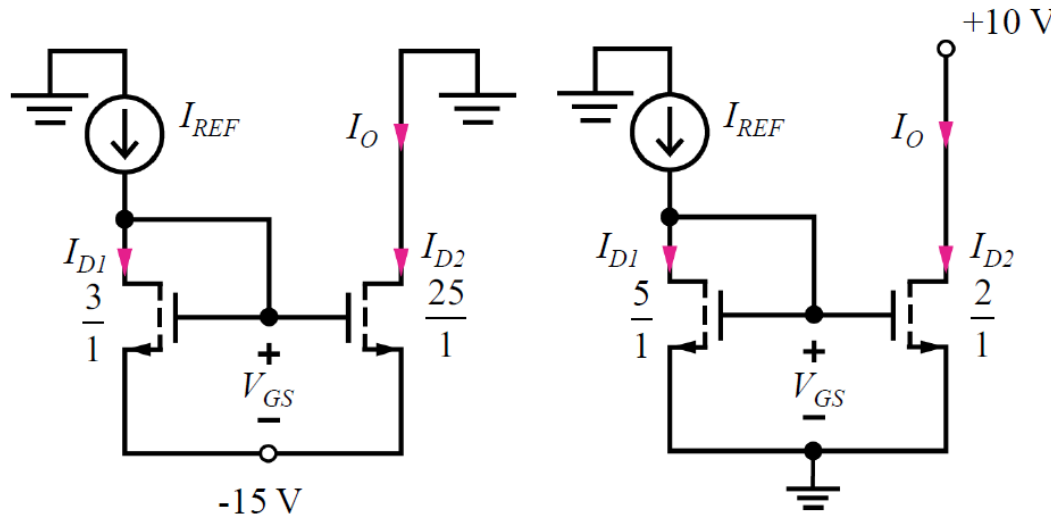
$$I_{REF} = I_{D1} = \frac{K'_n}{2} \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{TN})^2 (1 + \lambda V_{DS1})$$

$$I_O = I_{D2} = \frac{K'_n}{2} \left(\frac{W}{L} \right)_2 (V_{GS2} - V_{TN})^2 (1 + \lambda V_{DS2})$$

$$\because V_{DS1} = V_{GS1} = V_{GS2} = V_{GS}$$

$$MR = \frac{I_O}{I_{REF}} = \frac{\left(\frac{W}{L} \right)_2 (1 + \lambda V_{DS2})}{\left(\frac{W}{L} \right)_1 (1 + \lambda V_{DS1})} = \frac{\left(\frac{W}{L} \right)_2 (1 + \lambda V_{DS2})}{\left(\frac{W}{L} \right)_1 (1 + \lambda V_{GS})}$$

MOS Current Mirror: Example



(a) Calculate mirror ratio for $\lambda = 0$

(b) $\lambda = 0.02 \text{ V}^{-1}$

$V_{Tn} = 1 \text{ V}$, $K_n' = 25 \mu\text{A/V}^2$, and $I_{ref} = 50 \mu\text{A}$

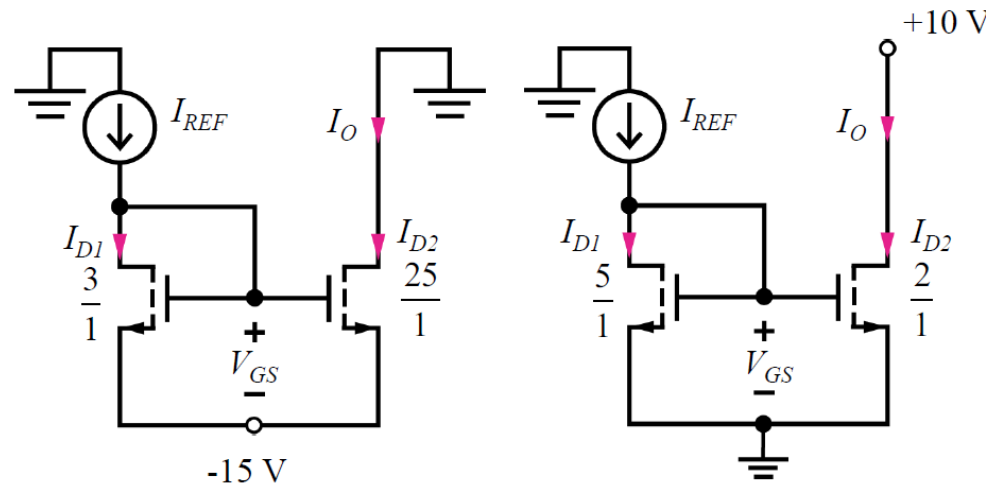
$$MR = \frac{I_O}{I_{REF}} = \frac{\left(\frac{W}{L}\right)_2 (1 + \lambda V_{DS2})}{\left(\frac{W}{L}\right)_1 (1 + \lambda V_{DS1})}$$

$$= \frac{\left(\frac{W}{L}\right)_2 (1 + \lambda V_{DS2})}{\left(\frac{W}{L}\right)_1 (1 + \lambda V_{GS})}$$

$$V_{GS} = V_{Tn} + \sqrt{\frac{2I_{ref}}{K_n}}$$

MOS Current Mirror: Example

(Cont.)



(a)

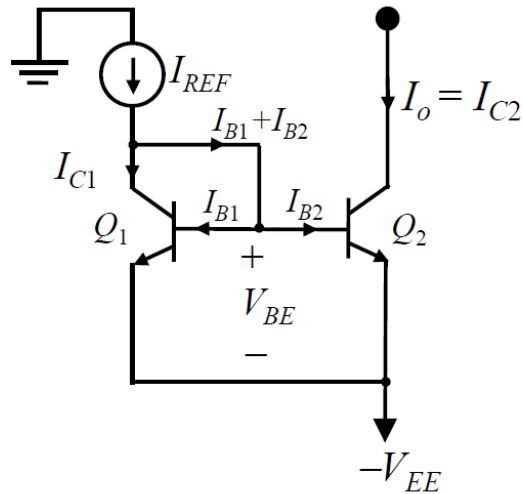
$$MR = \frac{25/1}{3/1} = 8.33$$

(b)

$$V_{DS1} = 1 \text{ V} + \sqrt{\frac{2(50 \mu\text{A})}{3(25 \mu\text{A}/\text{V}^2)}} = 2.16 \text{ V} \quad \left| \quad MR = 8.33 \frac{1 + 0.02(15)}{1 + 0.02(2.16)} = 10.4 \right.$$

$$MR = \frac{2/1}{5/1} = 0.400 \quad \left| \quad V_{DS1} = 1 \text{ V} + \sqrt{\frac{2(50 \mu\text{A})}{5(25 \mu\text{A}/\text{V}^2)}} = 1.89 \text{ V} \quad \left| \quad MR = 0.4 \frac{1 + 0.02(10)}{1 + 0.02(1.89)} = 0.463 \right.$$

BJT Current Mirror



Q_1 and Q_2 are assumed to have identical I_S and β .

$$I_{REF} = I_{C1} + I_{B1} + I_{B2}$$

$$V_{CE1} = V_{BE1} = V_{BE2} = V_{BE}$$

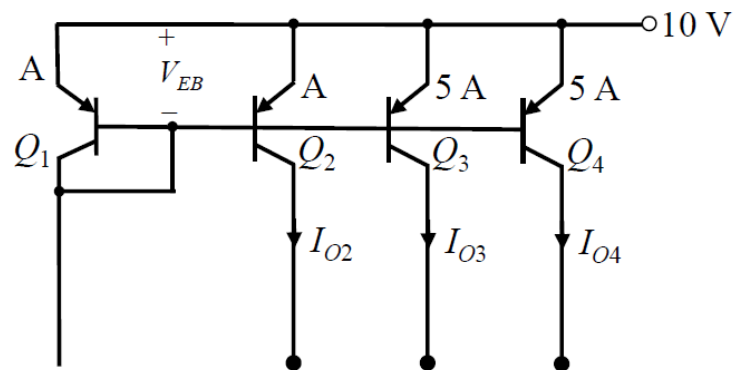
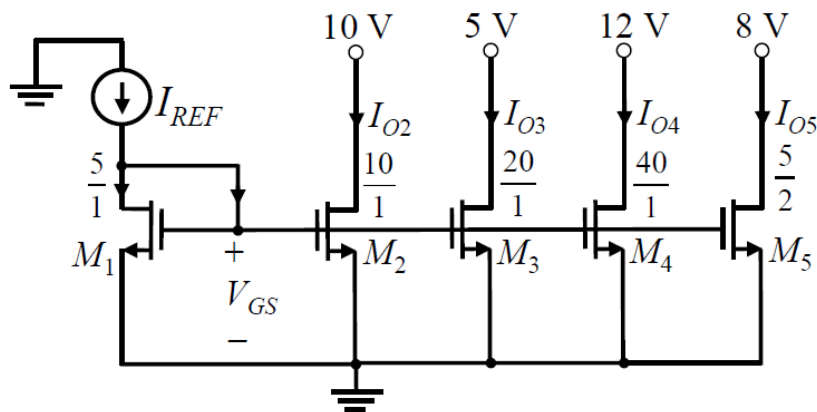
$$I_{C1} = I_S e^{\frac{V_{BE}}{V_T}}, I_O = I_{C2} = I_S e^{\frac{V_{BE}}{V_T}}$$
$$I_{B1} = \frac{I_{C1}}{\beta_1} = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}}, I_{B2} = \frac{I_{C2}}{\beta_2} = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}}$$
$$I_{REF} = I_S e^{\frac{V_{BE}}{V_T}} + \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}} + \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}} = \left(1 + \frac{2}{\beta}\right) I_S e^{\frac{V_{BE}}{V_T}}$$

$$MR = \frac{I_O}{I_{REF}} = \frac{I_S e^{\frac{V_{BE}}{V_T}}}{\left(1 + \frac{2}{\beta}\right) I_S e^{\frac{V_{BE}}{V_T}}} = \frac{1}{1 + \frac{2}{\beta}}$$

Similarly, if the BJT transistors are ideal
 $\Rightarrow$ without **Early Effect**, life will be much easier.

- Even in ideal case, BJT current mirror has $MR < 1$ due to base currents!!

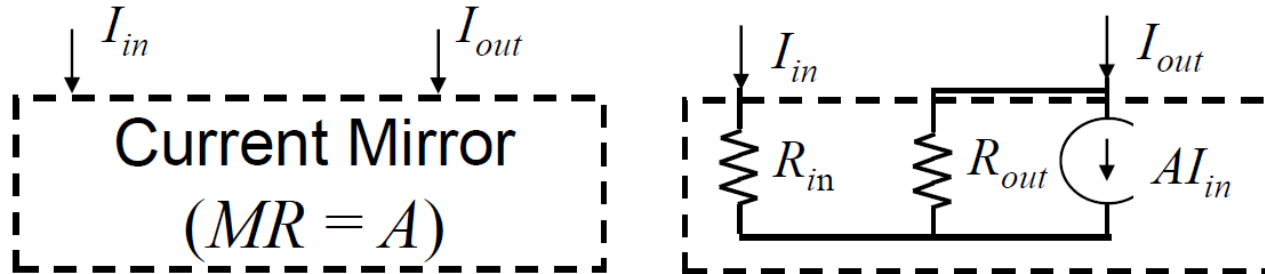
Multiple Current Sources



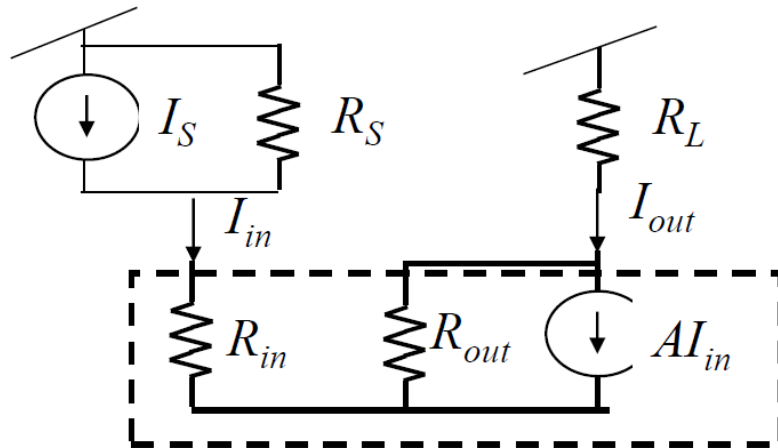
- Reference current enters diode-connected transistor M_1 establishing gate-source voltage to bias M_2 through M_5 , each with different W/L ratio.
- Absence of current gain defect permits large number of MOSFETs to be driven by one reference transistor.
- Similar multiple bipolar sources can be built from one reference BJT.
- As base current error term worsens when more BJTs are added, number of outputs of basic bipolar mirror are limited.

Terminal Resistances of Current Mirrors

What do we ideally want in terms of input and output resistances?



Model of a current mirror including terminal impedances

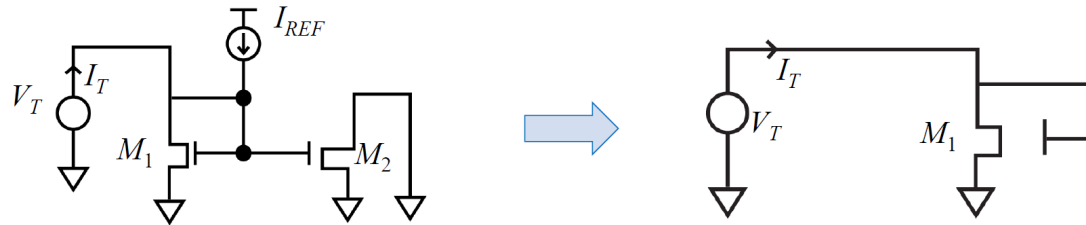


We want:

- $I_{out} = AI_S$
- All of I_S to go to $I_{in} \rightarrow R_{in} \sim 0$
- All of AI_{in} to go to $I_{out} \rightarrow R_{out} \sim \infty$

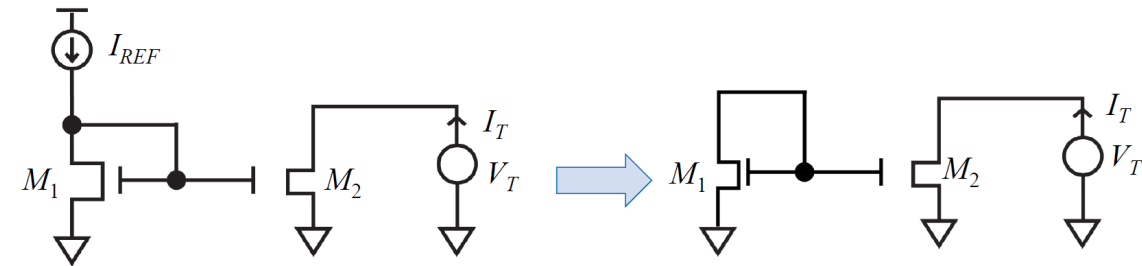
MOS Current Mirror: AC Analysis

R_{in}

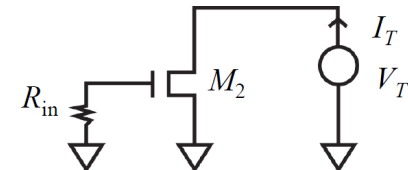


$$R_{in} = \frac{1}{g_{m1}}$$

R_{out}

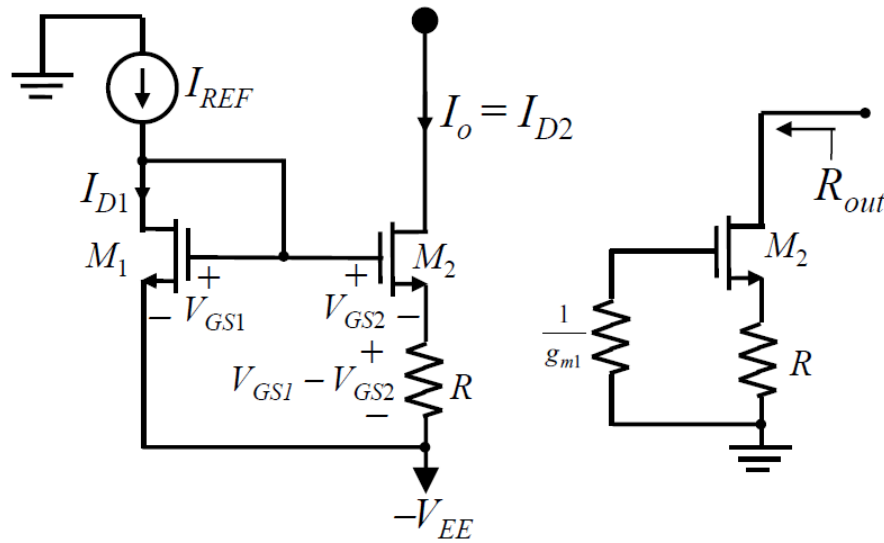


$$R_{out} = r_{o2}$$



Advanced Current Mirror: Increasing R_{out}

MOS Widlar current mirror



Current through R is given by:

$$I_O = \frac{V_{GS1} - V_{GS2}}{R} = \frac{1}{R} \left(\sqrt{\frac{2I_{REF}}{K_{n1}}} - \sqrt{\frac{2I_O}{K_{n2}}} \right)$$

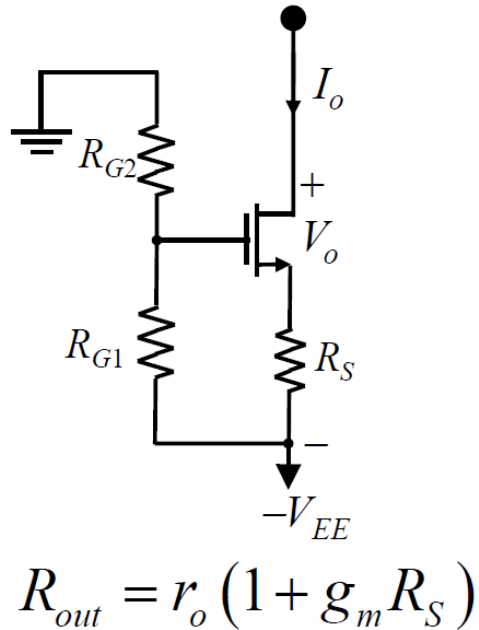
$$= \frac{1}{R} \sqrt{\frac{2I_{REF}}{K_{n1}}} \left(1 - \sqrt{\frac{I_O (W/L)_1}{I_{REF} (W/L)_2}} \right)$$

If I_{REF} is known, I_O can be directly calculated – **quadratic equation!**

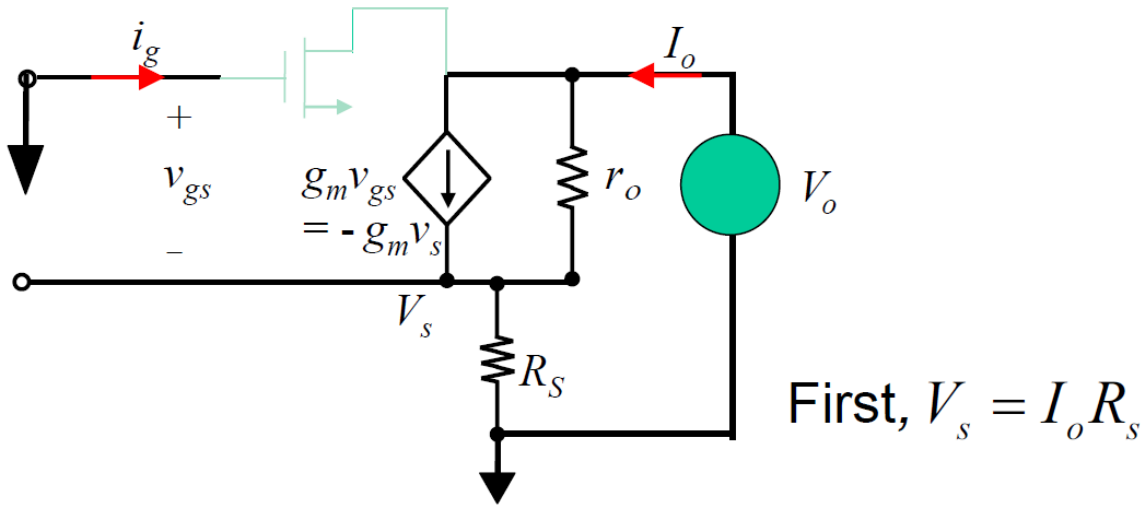
Small-signal model for MOS Widlar source represents a C-S stage with resistor R in its source.

$$\therefore R_{out} \approx r_{o2} (1 + g_{m2} R)$$

Advanced Current Mirror: Increasing R_{out}



Since $I_G = 0$, the gate is effectively shorted to ground.



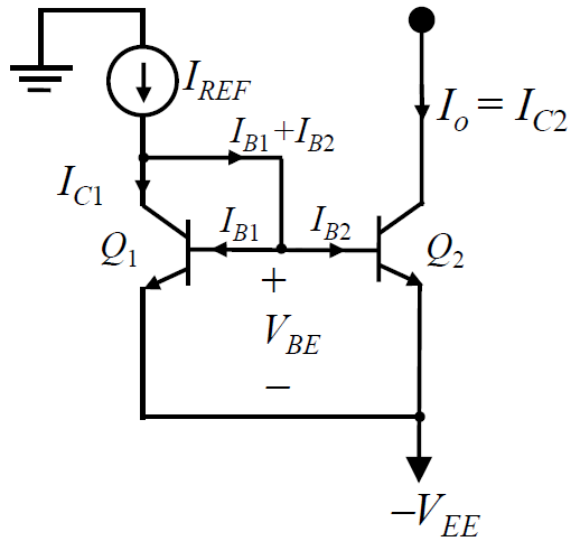
$$\begin{aligned} \text{By KCL at output, } I_o &= -g_m V_s + (V_o - V_s) / r_o = -(g_m + g_o) V_s + g_o V_o \\ &= -(g_m g_o) I_o R_S + g_o V_o \end{aligned}$$

$$\text{Hence, } I_o (1 + (g_m + g_o) R_S) = V_o / R_o$$

$$\text{Therefore, } R_{out} = V_o / I_o = r_o (1 + (g_m + g_o) R_S) \approx r_o (1 + g_m R_S)$$

Appendix

BJT Current Mirror: DC Analysis



Q_1 and Q_2 are assumed to have identical I_S , V_A , and β .

$$I_{REF} = I_{C1} + I_{B1} + I_{B2}$$

$$V_{CE1} = V_{BE1} = V_{BE2} = V_{BE}$$

However, real transistors always have an **Early Effect!**

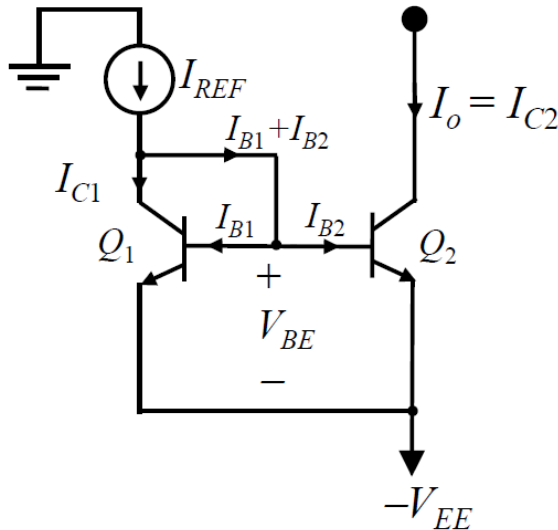
$$I_{C1} = I_S e^{\frac{V_{BE}}{V_T}}, I_O = I_{C2} = I_S e^{\frac{V_{BE}}{V_T}}$$

$$I_{B1} = \frac{I_{C1}}{\beta_1} = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}}, I_{B2} = \frac{I_{C2}}{\beta_2} = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}}$$

$$I_{REF} = I_S e^{\frac{V_{BE}}{V_T}} + \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}} + \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}} = \left(1 + \frac{2}{\beta}\right) I_S e^{\frac{V_{BE}}{V_T}}$$

$$MR = \frac{I_O}{I_{REF}} = \frac{I_S e^{\frac{V_{BE}}{V_T}}}{\left(1 + \frac{2}{\beta}\right) I_S e^{\frac{V_{BE}}{V_T}}} = \frac{1}{1 + \frac{2}{\beta}}$$

BJT Current Mirror: DC Analysis



Q_1 and Q_2 are assumed to have identical I_S , V_A , and β .

$$I_{REF} = I_{C1} + I_{B1} + I_{B2}$$

$$V_{CE1} = V_{BE1} = V_{BE2} = V_{BE}$$

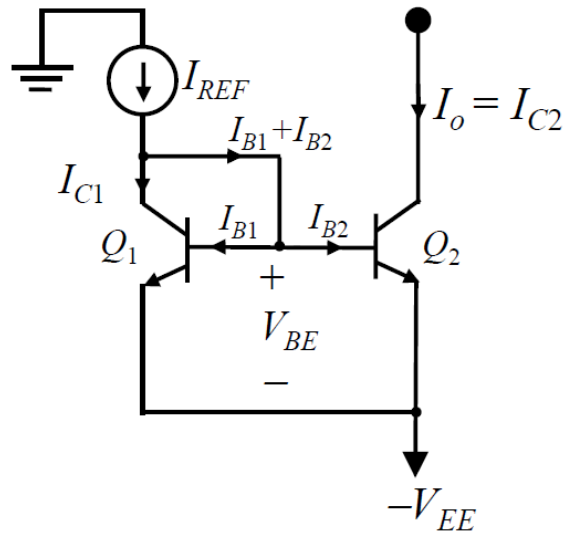
However, real transistors always have an **Early Effect**!

$$I_{C1} = I_S \left(1 + \frac{V_{CE1}}{V_A} \right) e^{\frac{V_{BE}}{V_T}}, \quad I_{C2} = I_S \left(1 + \frac{V_{CE2}}{V_A} \right) e^{\frac{V_{BE}}{V_T}}$$

$$I_{B1} = \frac{I_{C1}}{\beta_1} = I_S \left(1 + \frac{V_{CE1}}{V_A} \right) e^{\frac{V_{BE}}{V_T}} / \left(\beta \left(1 + \frac{V_{CE1}}{V_A} \right) \right) = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}}$$

New definition of β including **Early Effect**

BJT Current Mirror: DC Analysis



Q_1 and Q_2 are assumed to have identical I_S , V_A , and β .

$$I_{REF} = I_{C1} + I_{B1} + I_{B2}$$

$$V_{CE1} = V_{BE1} = V_{BE2} = V_{BE}$$

However, real transistors always have an **Early Effect!**

$$I_{C1} = I_S \left(1 + \frac{V_{CE1}}{V_A} \right) e^{\frac{V_{BE}}{V_T}}, I_{C2} = I_S \left(1 + \frac{V_{CE2}}{V_A} \right) e^{\frac{V_{BE}}{V_T}}$$

$$I_{B1} = \frac{I_{C1}}{\beta_1} = I_S \left(1 + \frac{V_{CE1}}{V_A} \right) e^{\frac{V_{BE}}{V_T}} / \beta_{FO} \left(1 + \frac{V_{CE1}}{V_A} \right) = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}}$$

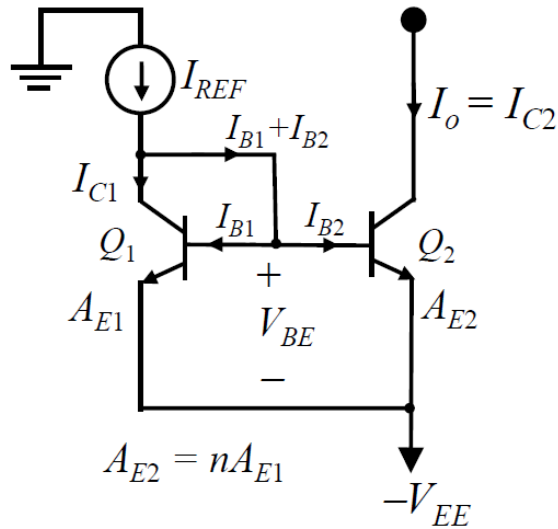
$$I_{B2} = \frac{I_{C2}}{\beta_2} = I_S \left(1 + \frac{V_{CE2}}{V_A} \right) e^{\frac{V_{BE}}{V_T}} / \beta_{FO} \left(1 + \frac{V_{CE2}}{V_A} \right) = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}}$$

$$MR = \frac{I_O}{I_{REF}} = \frac{1 + \frac{V_{CE2}}{V_A}}{1 + \frac{V_{CE1}}{V_A} + \frac{2}{\beta}} = \frac{1 + \frac{V_{CE2}}{V_A}}{1 + \frac{V_{BE}}{V_A} + \frac{2}{\beta}}$$

If $V_A = \infty$,

$$MR = \frac{1}{1 + \frac{2}{\beta_{FO}}}$$

BJT Current Mirror: Changing Gain



Mirror ratio can be changed by modifying the emitter.

$$I_S = I_{SO} \left(\frac{A_E}{A} \right)$$

where I_{SO}/A is the saturation current per unit emitter area.

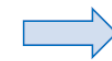
Again, if the transistors are ideal:

$$I_{C1} = I_{SO} \left(\frac{A_{E1}}{A} \right) e^{\frac{V_{BE}}{V_T}}$$

$$I_{C2} = I_{SO} \left(\frac{nA_{E1}}{A} \right) e^{\frac{V_{BE}}{V_T}} \quad \left(\text{where } n = \frac{A_{E2}}{A_{E1}} \right)$$

$$I_{B1} = \frac{I_{C1}}{\beta_1} = \frac{I_{SO}}{\beta} \left(\frac{A_{E1}}{A} \right) e^{\frac{V_{BE}}{V_T}}, \quad I_{B2} = \frac{I_{C2}}{\beta_2} = \frac{I_{SO}}{\beta} \left(\frac{nA_{E1}}{A} \right) e^{\frac{V_{BE}}{V_T}}$$

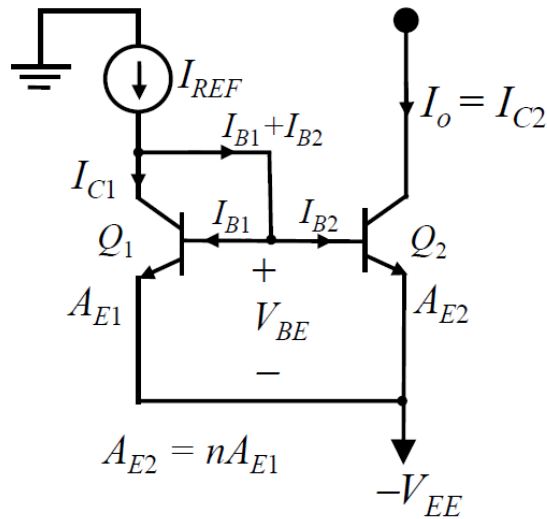
$$MR = \frac{I_O}{I_{REF}} = \frac{n}{1 + \frac{1+n}{\beta}}$$



Ideally, $MR = n$, but for finite β ,

$$MR = \frac{n}{1 + \frac{n+1}{\beta}}$$

BJT Current Mirror: Changing Gain



Mirror ratio can be changed by modifying the emitter.

$$I_S = I_{SO} \left(\frac{A_E}{A} \right)$$

where I_{SO}/A is the saturation current per unit emitter area.

Real transistors always have Early Effect
 $\Rightarrow$ complicated:

$$I_{C1} = I_{SO} \left(\frac{A_{E1}}{A} \right) \left(1 + \frac{V_{CE1}}{V_A} \right) e^{\frac{V_{BE}}{V_T}}$$

$$I_{C2} = I_{SO} \left(\frac{nA_{E1}}{A} \right) \left(1 + \frac{V_{CE2}}{V_A} \right) e^{\frac{V_{BE}}{V_T}} \quad \left(\text{where } n = \frac{A_{E2}}{A_{E1}} \right)$$

$$I_{B1} = \frac{I_{C1}}{\beta_1} = \frac{I_{SO}}{\beta} \left(\frac{A_{E1}}{A} \right) e^{\frac{V_{BE}}{V_T}}, \quad I_{B2} = \frac{I_{C2}}{\beta_2} = \frac{I_{SO}}{\beta} \left(\frac{nA_{E1}}{A} \right) e^{\frac{V_{BE}}{V_T}}$$

$$MR = \frac{I_O}{I_{REF}} = \frac{n \left(1 + \frac{V_{CE2}}{V_A} \right)}{1 + \frac{V_{BE}}{V_A} + \frac{1+n}{\beta}} \Rightarrow \text{Ideally, } MR = n, \text{ but for finite } \beta, \quad MR = \frac{n}{1 + \frac{n+1}{\beta}}$$