

EF2452: Mathematics for Economics and Finance

Lecture 5

Department of Economics and Finance
City University of Hong Kong
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Main Topics

- Efficient firm in a perfectly competitive market
- Startup and breakeven points
- Determining the supply curve of an efficient, perfectly competitive firm
- Relations between marginal cost (MC), average cost (AC), average variable cost (AVC)

Price Taker

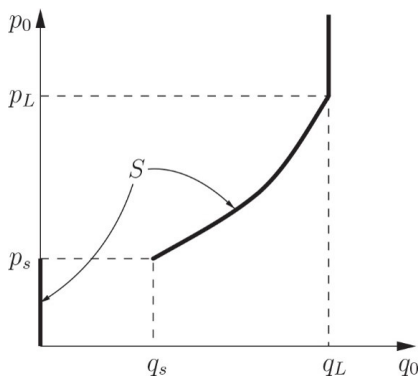
- Consider a firm in a perfectly competitive market. It's a small firm so that the price of the firm's product in the market is not affected by its own level of production.
- In other words, there is a given market price p_0 which is not under the firm's control: the firm is a **price taker**.

Efficient Small Firm

- Suppose that the cost function C for the firm is known.
- Then the firm is said to be **efficient** if it behaves in the following way. When the given market price is p_0 , it will produce the amount q_0 for which profit is maximized.

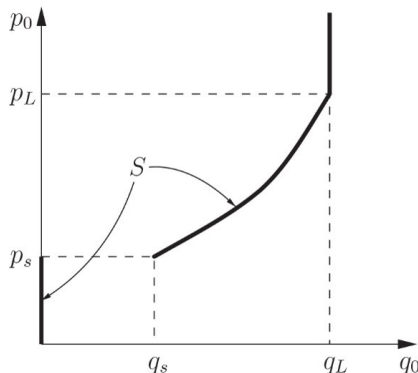
A typical supply set for an efficient small firm

- When price is too low, p_0 is below a certain level p_s , the firm will produce nothing.
- When p_0 reaches p_s , the firm will 'start up' with a production level q_s .



A typical supply set for an efficient small firm

- Production will then increase as the price increases, until p_0 reaches the level p_L at which the firm is at the upper limit of its production, q_L .
- When p_0 exceeds p_L , the firm still produces its maximum limit, q_L .



A typical supply set for an efficient small firm

- We shall show that, under certain assumptions, a supply set S resembling the one in the figure can be derived from the firm's cost function C .
- Suppose the firm were to produce a quantity q when the market price is p_0 . Then its revenue $R(q)$ would be

$$R(q) = p_0 q$$

- The firm's cost is $C(q)$. Therefore, its profit is

$$\Pi(q) = p_0 q - C(q)$$

‘Price equals marginal cost’ condition

- Since the firm is efficient, the amount q_0 which it actually produces will be such that q_0 maximizes profit. We know that, the first-order derivative must be equal to zero at this q_0 . Therefore,

$$\Pi'(q_0) = p_0 - C'(q_0) = 0 \implies p_0 = C'(q_0)$$

- In other words, at a price of p_0 , the quantity that the efficient firm will supply is given by the condition that ‘price equals marginal cost’.
- Thus, a firm’s inverse supply function is essentially equal to its marginal cost function.
- In terms of the earlier figure, we have shown that the ‘sloping’ part of the supply set is just the graph of C' , marginal cost.

Fixed Costs

- When the market price p is given, and provided it lies within a certain range, the firm will produce a quantity q satisfying the rule “price equals marginal cost”, i.e., $p = C'(q)$.
- In this range, the profit from producing q is

$$\Pi(q) = qp - C(q) = qC'(q) - C(q)$$

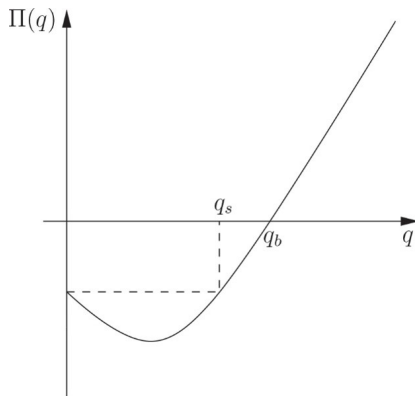
- What if the market price p is small? In this situation, it may be in the firm's best interest to produce nothing. The profit when zero units are produced is

$$\Pi(0) = -C(0)$$

- In general, $C(0)$ is known as the **fixed cost** of production. In the short-run, it is positive: you cannot escape the fixed costs in the short-run even if you produce nothing; these are your overheads, machinery rent, etc.; it's a loss!

A typical profit function

- When q is just greater than zero, it is reasonable to expect that the loss will exceed the fixed cost, because the return from producing a small amount does not justify the expense.



Startup point

- The **startup point** is the production level q_s where the loss becomes equal to the fixed cost, so that it is worthwhile to start production, at least in the short run.
- Thus q_s can be found by solving the equation

$$\Pi(q_s) = \Pi(0)$$

$$\text{or, } q_s C'(q_s) - C(q_s) = -C(0)$$

- The corresponding value of the market price, p_s , is determined by the fact that $p_s = C'(q_s)$. So, we have

$$q_s p_s - C(q_s) = -C(0)$$

$$\implies p_s = C'(q_s) = \frac{C(q_s) - C(0)}{q_s}$$

Startup point

- $C(q) - C(0)$ represents the total cost less the fixed cost, and is known as the **variable cost**, VC .
- Average variable cost, $AVC = VC/q$.
- Therefore, $C'(q_s) = \frac{C(q_s) - C(0)}{q_s}$ means that, at the startup point, q_s , **the marginal cost equals the average variable cost**, $MC = AVC$.

Breakeven point

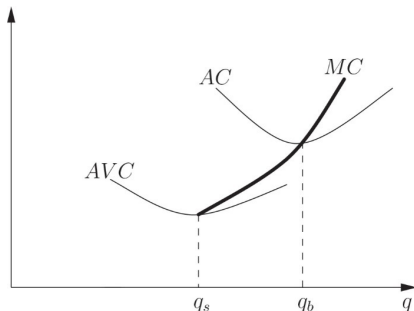
- When does $\Pi(q)$ become positive?
- We define the **breakeven point** q_b by the equation $\Pi(q) = 0$. Using the formula for Π we get

$$q_b C'(q_b) - C(q_b) = 0; \quad \text{that is} \quad C'(q_b) = \frac{C(q_b)}{q_b}$$

- At the breakeven point, **the marginal cost is equal to the average cost**, $MC = AC$.

Relations between MC , AC and AVC

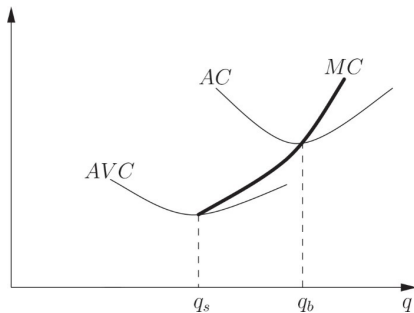
- The MC curve is the graph of the firm's inverse supply function for $q \geq q_s$.



- MC curve intersects the AVC curve at q_s .
- MC curve intersects the AC curve at q_b .

Relations between MC , AC and AVC

- At the startup point, the derivative of the AVC is zero.
- At the breakeven point, the derivative of the AC is zero.



References

Following chapter of the textbook:

10 The Derivative in Economics II