

EE3114 Tutorial 3 : Transfer Function and State-Space Model

1.

Convert the state-space model:

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} V_{c1} \\ V_{c2} \end{bmatrix} + \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} i$$

$$V_{R2} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} V_{c1} \\ V_{c2} \end{bmatrix} + \frac{1}{3} i$$

into transfer function. Express the state-space model in controllable or observable canonical form.

2. Find and sketch the impulse response of a system with a transfer function between input and output given by

$$H(s) = \frac{2s + 1}{s^2 + 3s + 2}$$

Hints: Impulse response of a system is the inverse Laplace Transform of the transfer function

- (i) Partial-fraction expansion of $H(s)$
- (ii) Use the Laplace Transform table, find the inverse Laplace Transform

To sketch the function

- (iii) Find the y-intercept:
 - a. Substitute $t=0$ to the output function found in (ii), or
 - b. Use Initial Value Theorem, $h(0) = \lim_{s \rightarrow \infty} sH(s)$.
- (iv) Find the value that the impulse response converges to
 - a. Substitute $t = \infty$ to the solution in (ii), or
 - b. Use Final Value Theorem, $h(\infty) = \lim_{s \rightarrow 0} sH(s)$
- (v) Others: x-intercept(s), local min./max.?

Note: You can validation your results by Matlab

num = [2 1]; den = [1 3 2]; impulse(num,den);

3. Transfer Function from the State Description

Find the transfer function of the system described by

$$\dot{x}_1 = -7x_1 - 12x_2 + u$$

$$\dot{x}_2 = x_1$$

$$y = x_1 + 2x_2$$

Hints:

(i) What are the state-variable description matrices of the system: A, B, C, D

(ii) $G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$