

EE2302 Foundations of Information and Data Engineering

Assignment 7

Due: Nov 2 (**Fri**)

Full Mark: 20 points

1. (6 points) Determine whether each of the following scalar-valued functions n -vectors is linear. If it is linear, give its inner product representation, i.e., an n -vector a for which $f(x) = a^T x$ for all x . If it is not linear, give specific x, y, α, β for which superposition fails, i.e.,

$$f(\alpha x + \beta y) \neq \alpha f(x) + \beta f(y).$$

- a) $f(x) = \max_k x_k$
- b) $f(x) = x_n - x_1$
2. (4 points) What 3 by 3 matrices represent the transformations that
- a) reflect every vector through the x - y plane?
- b) rotate the x - y plane through 90° , leaving the z -axis alone?
3. (4 points) Consider the 2-dimensional space and the projection of b on the line through a , where $a = (2, 1)$.
- a) Determine the corresponding projection matrix.
- b) Suppose $b = (5, 5)$. Determine the result after the projection.
4. (6 points) A business sells n products, and is considering changing the price of one of the products to increase its total profits. A business analyst develops a regression model that (reasonably accurately) predicts the total profit when the product prices are changed, given by $\hat{P} = \beta^T x + P$, where the n -vector x denotes the fractional change in the product prices, $x_i = (p_i^{new} - p_i)/p_i$. Here P is the profit with the current prices, $\hat{P}$ is the predicted profit with the changed prices, p_i is the current (positive) price of product i , and p_i^{new} is the new price of product i .
- a) What does it mean if $\beta_3 < 0$? (And yes, this can occur.)
- b) Suppose that you are given permission to change the price of *one* product, by up to 1%, to increase total profit. Which product would you choose, and would you increase or decrease the price? By how much?
- c) Repeat (b), assuming you are allowed to change the price of two products, each by up to 1%.