

Block B

Operational Amplifiers

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Summary

If an output of a sensor was 1mV, which is the smallest your DMM could measure, you will need to amplify the 1mV voltage signal for it to be detectable by the DMM. The effect you want to achieve is illustrated in Fig 1. You can see that through the amplifier, the signal is now stronger.

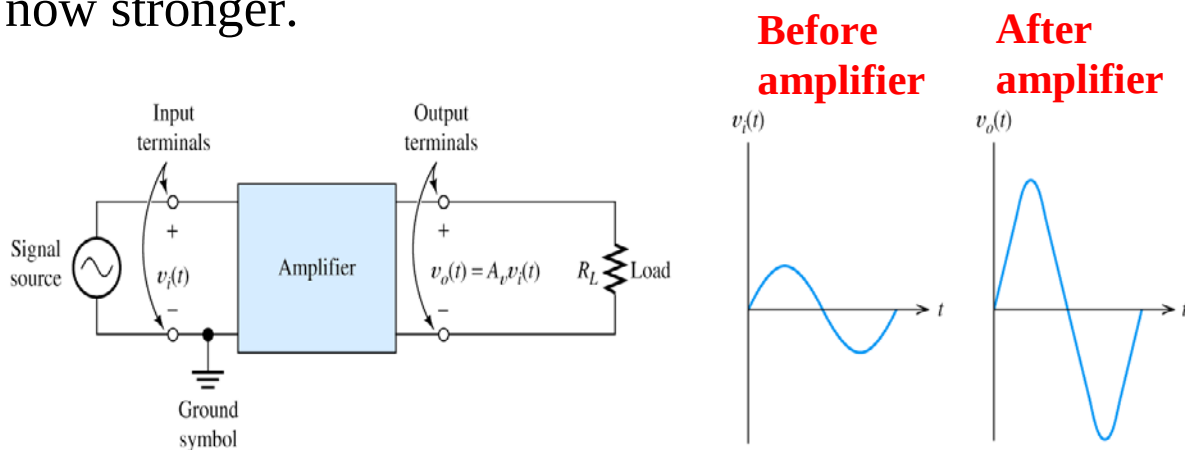


Figure 11.1 Electronic amplifier.

This entire block is dedicated to building circuits for amplifying weak signals. These circuits are based upon operational amplifiers (Op amps for short). If you are interested to know what is inside an op amp, this will have to wait till Block D. In this block, we focus on how to use the op amp and apply them to build practical circuits first.

I. The Ideal Operational Amplifier

Terminals

The ideal op amp has 3 terminals that you should notice. Fig 1 shows the symbol for an op amp. The 3 terminals to notice are:

- Non-inverting input terminal (+)
- Inverting input terminal (-)
- Output terminal (V_{out})

In other words, it has two inputs and one output.

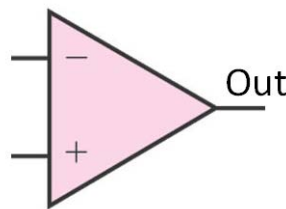


Figure 1: Symbol of an op amp

Open loop gain (A)

The op amp sees the voltage difference across the two inputs and multiplies it by a factor A. This factor A is known as the open loop gain:

$$V_{out} = A(V_{+} - V_{-}) \quad (*)$$

Input resistance (R_{in})

In an op amp, there is a resistance that lies between the two input terminals. This resistance usually designed to be very large and is known as the input resistance.

Output resistance (R_{out})

To see the effect of the output resistance, we refer to the model for the op amp shown in Fig 2. It contains a voltage controlled voltage source. Only when R_{out} is 0, do we obtain the relationship for V_{out} in equation (*).

Characteristics of an ideal op amp

- Infinite open loop gain ($A \rightarrow \infty$)
- Infinite input resistance ($R_i \rightarrow \infty$)
- Zero output resistance ($R_o = 0$)

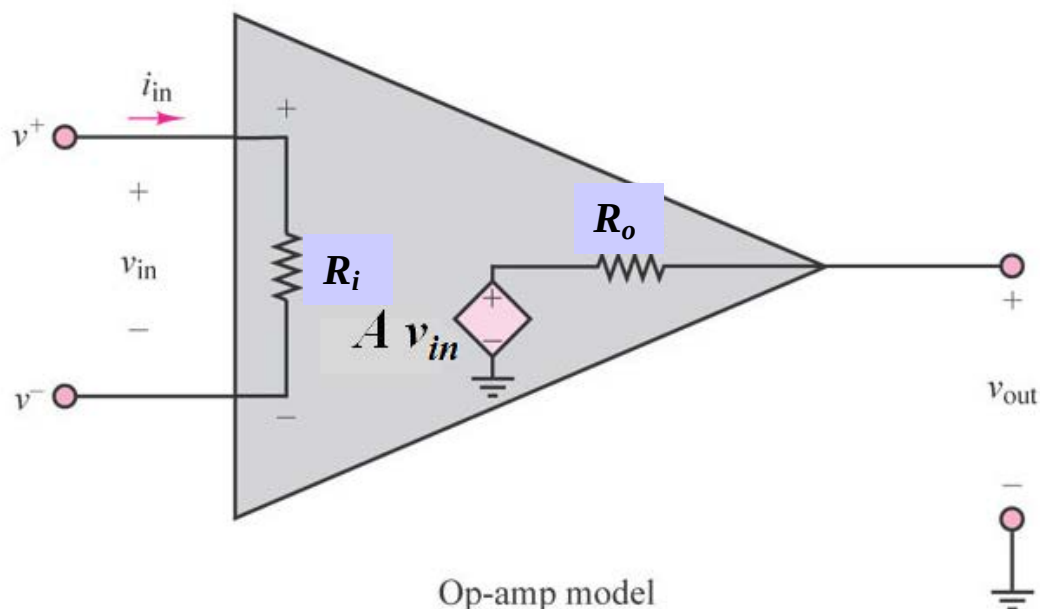


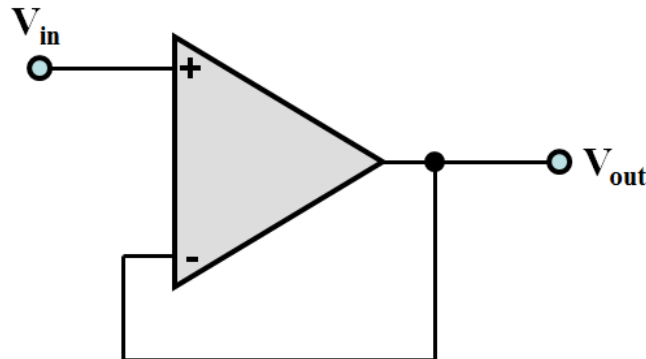
Figure 2: Op amp model

In the next section, we will analyze a number of circuits based on op amps. You will find there are a very great number of different combinations (Hint: memorizing all the possible combinations is futile).

II. Op amp circuits

Circuit 1: Source follower

Closed loop gain (V_{out}/V_{in})



- (1) Closed-loop gain (G): $G = V_{out}/V_S = 1$
- (2) Circuit input resistance (R_{in}): $R_{in} \rightarrow \infty$

V_{out} is connected to V_-

$$\Rightarrow V_{out} = V_-$$

$V_+ = V_-$ (Infinite loop gain with negative feedback)

Negative feedback: connect output to inverting input.

Feed some/all of the output back to the input to reduce the difference between the inputs. If A is infinite, then this difference becomes zero!

$$\Rightarrow V_{out} = V_+$$

$$V_{in} = V_+$$

$$\Rightarrow V_{out} = V_{in}$$

The open loop gain (A) is specific to the op amp while the closed loop gain (G) refers to the ratio of output to input when the circuit is in negative feedback.

Input resistance

Input resistance is defined by ratio of the input voltage over the input current. In this case, the input current is the input current going into V^+ of the op amp, which is zero (ideal op amp).

$\Rightarrow R_{in} = R_i$, which is therefore infinite in this case.

Key features to highlight

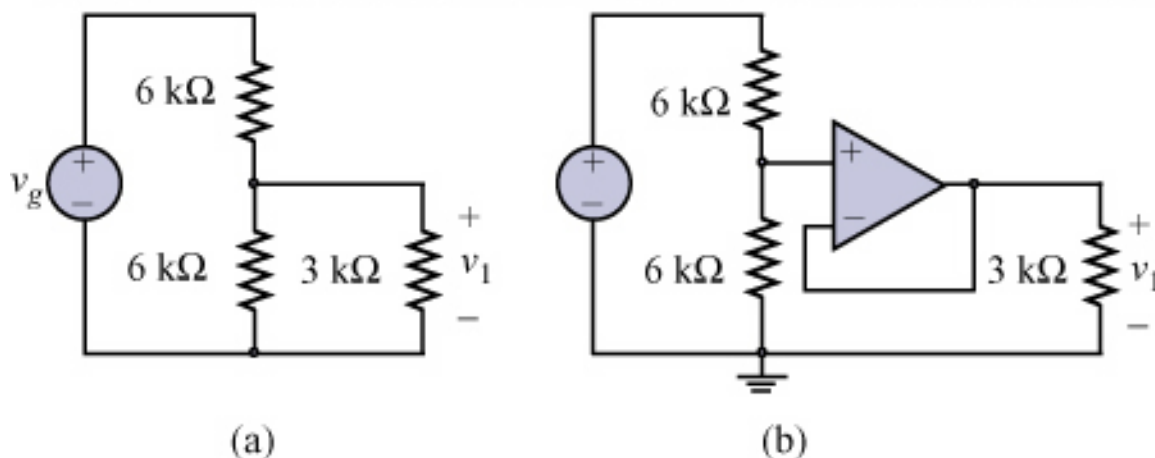
Large input resistance

Small output resistance

Gain of close to one

Application: Buffer between in a large source resistance and small load resistance

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Find the v_1 for the above 2 circuits

3k Ω resistor represents an output load

Without the 3k Ω load, the open circuit voltage across the 6k Ω resistor = $v_g/2$ (This is the ideal case with the load is actually infinite – i.e. we get an open circuit).

For circuit on left when 3k Ω is connected:

6k Ω and 3k Ω are in parallel = 2k Ω

So $v_1 = [2/(2+6)]*v_g = v_g/4$

Note that the output voltage has now dropped by half due to the 3k Ω load compared to the no load case.

For the circuit on the right with the buffer:

Voltage across 6k Ω = $v_g/2$ (2 resistors are in series since no current is flowing into the op amp inputs)

Since the gain is 1, $v_1 = v_g/2$

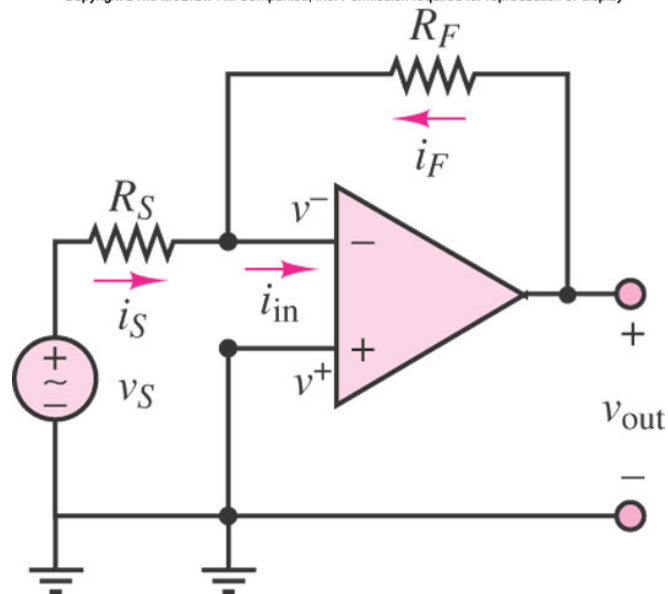
With the source follower, the 3k Ω load has no effect on the circuit on the left hand side.

In other words, the source follower works as a buffer. It isolates the input from the output load as the output resistance of the op amp is zero.

Circuit 2: Inverting amplifier

Derive the gain

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- (3) Closed-loop gain (G): $G = \frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$
- (4) Circuit input resistance (R_{in}): $R_{in} = R_S$

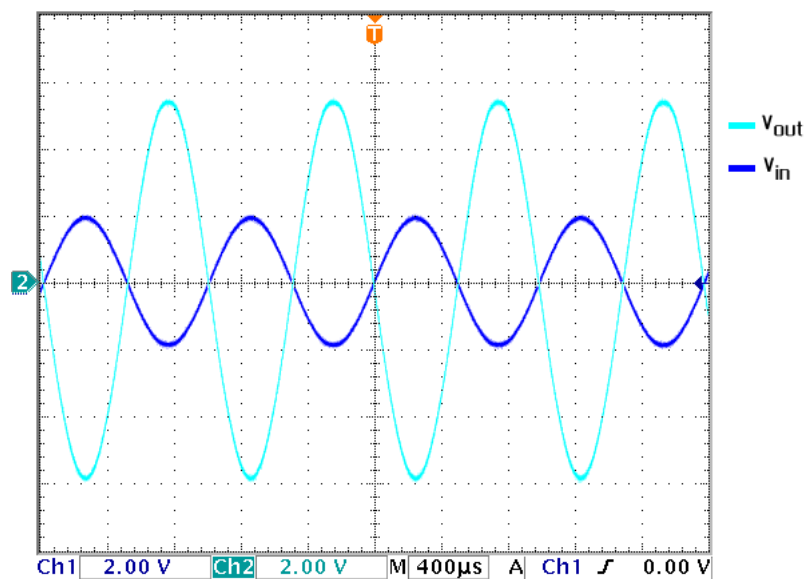
Apply KCL at inverting input (V-):

Note that $v^- = v^+ = 0$:

$$\frac{v_s - 0}{R_S} + \frac{v_{out} - 0}{R_F} = 0$$

$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

What does the negative sign mean?



Comparison of input to output for inverting amp

Note that the sign is an indication of whether the closed loop gain is inverting or not. The value of $|G|$ indicates how large the amplification is.

Input resistance

The input resistance is the ratio of the input voltage over the input current.

$$I_{in} = (V_S - 0)/R_S$$

$$\Rightarrow V_S/I_{in} = R_S$$

$$\Rightarrow \text{Input resistance} = R_S$$

If the source had an internal resistance of R_a , the output would be reduced by:

$$\frac{R_S}{R_S + R_a}$$
$$V_{out} = \left(\frac{R_S}{R_S + R_a} \right) \left(-\frac{R_F}{R_S} V_S \right)$$

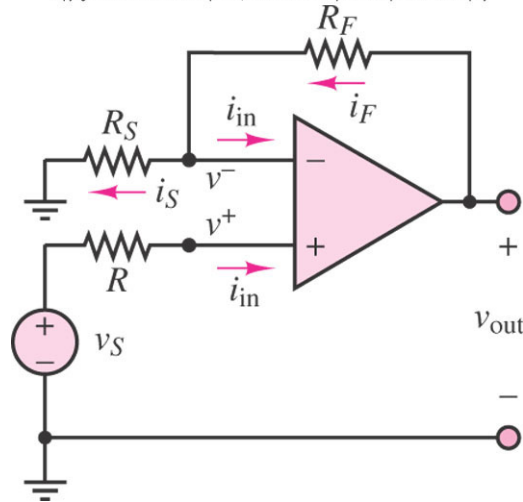
Verify: Replace R_S with $R_S + R_a$

$$V_{out} = -\frac{R_F}{R_S + R_a} V_S$$

Every real source will have its own source resistance (denoted by R_a). The above result highlights the point that the value of R_S relative to R_a affects the gain of the circuit in this case of the inverting amplifier described here.

Circuit 3: Non-inverting amplifier

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- (1) Circuit input resistance (R_{in}): $R_{in} \rightarrow \infty$
- (2) Closed-loop gain (G): $G = \frac{v_{out}}{v_S} = 1 + \frac{R_F}{R_S}$

Closed-loop gain (G)

Note that $i_{in} = 0$ since it is going into the op amp input

$$\Rightarrow v_S = v^+ = v^- \quad (1)$$

Note there is also no current running into inverting input of op amp as well ($i_{in} = 0$ for either inputs)

$\Rightarrow R_S$ and R_F are thus in series

Apply voltage divider rule:

$$v^- = \frac{R_S}{R_S + R_F} v_{out} \quad (2)$$

Substitute (1) into (2):

$$v_s = \frac{R_S}{R_S + R_F} v_{out} \quad (3)$$

After rearranging (3):

$$G = \frac{v_{out}}{v_s} = 1 + \frac{R_F}{R_S}$$

Circuit input resistance (R_{in})

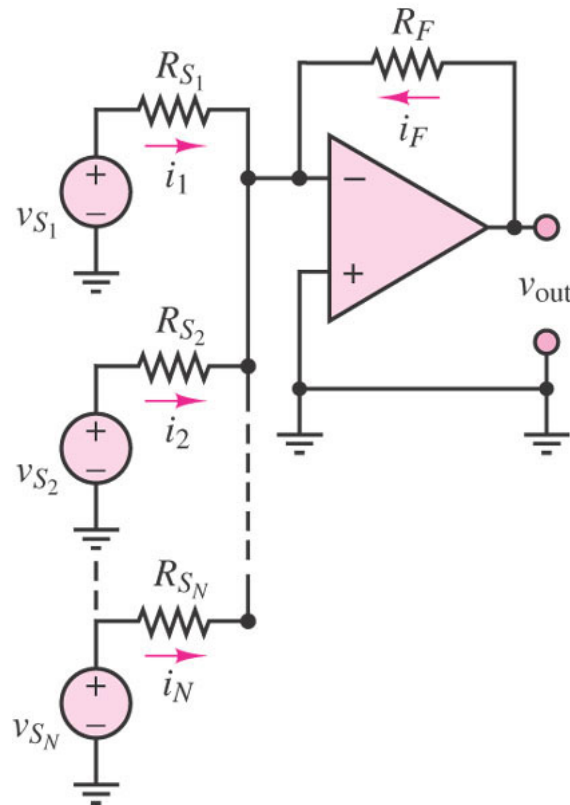
As mentioned already, $i_{in} = 0$ since this sees the infinite R_i of the op amp. Hence if $i_{in} = 0$, then:

$$R_{in} = \frac{v_s}{i_{in}} \rightarrow \infty$$

Note the expression for R_{in} above does not depend on R in the circuit. Hence any resistance at the input has no effect on the output.

Circuit 4: Summing amplifier

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$$v_{out} = - \left(\frac{R_F}{R_{S1}} v_{S1} + \frac{R_F}{R_{S2}} v_{S2} + \dots + \frac{R_F}{R_{SN}} v_{SN} \right)$$

This circuit is very similar to the inverting amplifier.

Note that since $v^+ = 0$, therefore $v^- = 0$.

Therefore:

$$i_1 = \frac{v_{S1}}{R_{S1}}, i_2 = \frac{v_{S2}}{R_{S2}}, \dots, i_N = \frac{v_{SN}}{R_{SN}}$$

Applying KCL at v^- :

$$i_F + i_1 + i_2 + \dots + i_N = 0$$

Hence by nodal voltage analysis:

$$\frac{v_{out}}{R_F} + \frac{R_F}{R_{S1}} v_{S1} + \frac{R_F}{R_{S2}} v_{S2} + \dots + \frac{R_F}{R_{SN}} v_{SN} = 0$$

Rearrange the above equation, we obtain:

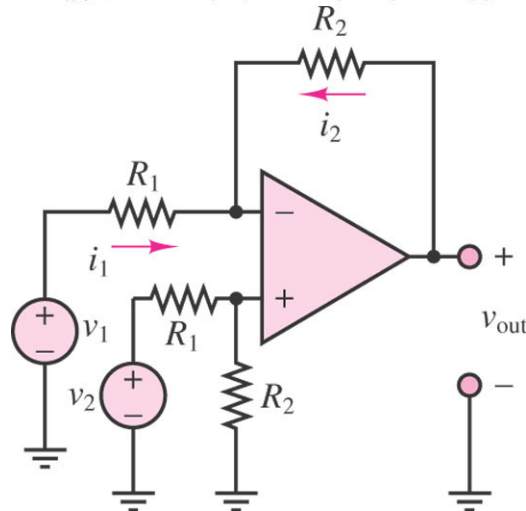
$$v_{out} = - \left(\frac{R_F}{R_{S1}} v_{S1} + \frac{R_F}{R_{S2}} v_{S2} + \dots + \frac{R_F}{R_{SN}} v_{SN} \right)$$

Note that as the name suggests, the function of this amplifier circuit is to amplify each input voltage and add them all up at the output. This sum however is inverted relative to the input as indicated by the negative sign.

Circuit 5: Differential amplifier

This circuit is very useful for building instruments that make fine measurements. You will have the chance to build your own temperature sensing instrument in Lab 3.

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Apply KCL at inverting terminal:

$$i_1 + i_2 = 0$$

$$\frac{v_1 - v^-}{R_1} + \frac{v_{out} - v^-}{R_2} = 0$$

$$\left(\frac{v_1}{R_1} + \frac{v_{out}}{R_2} \right) / \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = v^-$$

$$\left(\frac{R_2}{R_1 + R_2} \right) v_1 + \left(\frac{R_1}{R_1 + R_2} \right) v_{out} = v^- \quad (1)$$

At the non-inverting terminal, no current goes into the op amp so R_1 and R_2 connected to v^+ are in series and we can thus use voltage divider rule:

$$v^+ = \left(\frac{R_2}{R_1 + R_2} \right) v_2 \quad (2)$$

Now since $v^- = v^+$ due to infinite open loop gain (A) when in negative feedback, by combining (1) and (2):

$$\left(\frac{R_2}{R_1 + R_2} \right) v_2 = \left(\frac{R_2}{R_1 + R_2} \right) v_1 + \left(\frac{R_1}{R_1 + R_2} \right) v_{out}$$

$$\left(\frac{R_1}{R_1 + R_2} \right) v_{out} = \left(\frac{R_2}{R_1 + R_2} \right) (v_2 - v_1)$$

$$v_{out} = \left(\frac{R_2}{R_1} \right) (v_2 - v_1)$$

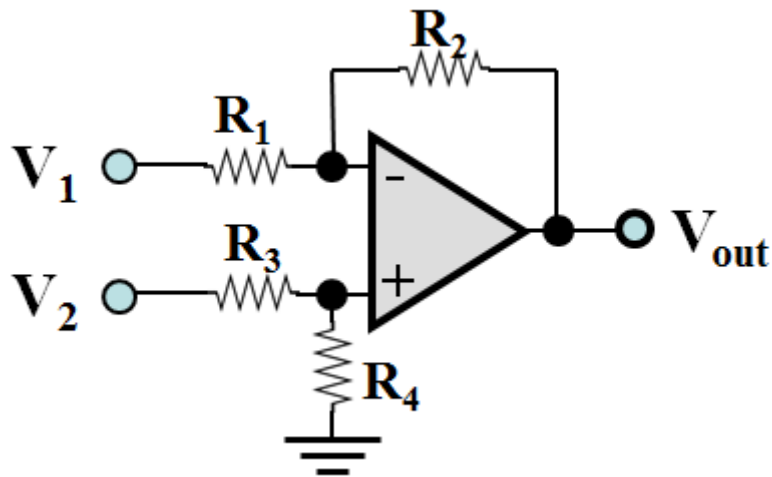
The gain expression takes the form:

$$V_{out} = G V_{diff}, \text{ where } V_{diff} = V_2 - V_1$$

Note that the function of this opamp circuit is to take the difference between two input voltages and amplify them at the output.

But what if there is some **difference** between the values of the resistors such that all the resistor values are now different from each other?

Repeating the same procedure above, find the V_{out} in terms of V_1 and V_2 for the circuit below:



Apply KCL at v^- :

$$\frac{v_1 - v^-}{R_1} + \frac{v_{out} - v^-}{R_2} = 0$$

$$\left(\frac{R_2}{R_1 + R_2} \right) v_1 + \left(\frac{R_1}{R_1 + R_2} \right) v_{out} = v^- \quad (3)$$

Now at v^+ where R_3 and R_4 are in series and we can use voltage divider rule:

$$v^+ = \left(\frac{R_4}{R_3 + R_4} \right) v_2 \quad (4)$$

Using the fact that $v^+ = v^-$ due to the infinite open loop gain of the op amp in negative feedback, we can equate (3) and (4):

$$\begin{aligned}\left(\frac{R_4}{R_3 + R_4}\right)v_2 &= \left(\frac{R_2}{R_1 + R_2}\right)v_1 + \left(\frac{R_1}{R_1 + R_2}\right)v_{out} \\ \left(\frac{R_1}{R_1 + R_2}\right)v_{out} &= \left(\frac{R_4}{R_3 + R_4}\right)v_2 - \left(\frac{R_2}{R_1 + R_2}\right)v_1 \\ v_{out} &= \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)v_2 - \frac{R_2}{R_1}v_1\end{aligned}$$

The gain expression takes the form:

$$V_{out} = G_1 V_2 - G_2 V_1$$

$$G_1 = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)$$

$$G_2 = \frac{R_2}{R_1}v_1$$

If we group the inputs into differential and common, we then get:

$$V_{out} = G_{dm} V_{dm} + G_{cm} V_{cm}$$

Note here that:

- (1) $V_{dm} = (V_2 - V_1)/2$, where $V_2 = -V_1$
- (2) $V_{cm} = V_2 = V_1$

G_{cm} is known as the common mode gain, while G_{dm} is the differential mode gain.

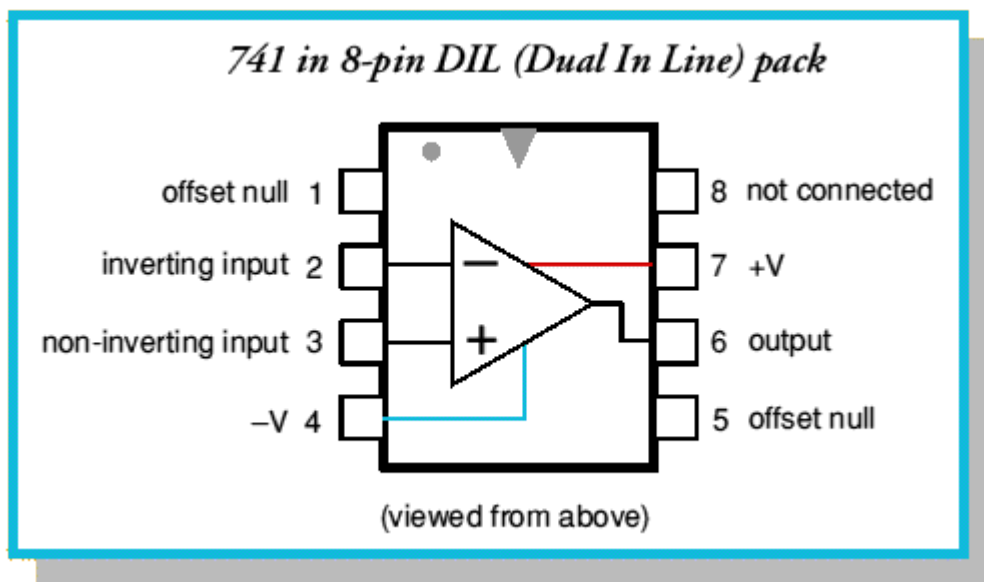
In the ideal case when resistors R_1 and R_3 as well as R_2 and R_4 are matched to each other perfectly, then G_{cm} will be zero. The amplifier will only amplify difference in voltage between the two inputs. If the inputs were exactly the same, then the output would be zero. This is the ideal case.

But we see that in the real practical case when there is some difference in resistor values, the output will not be zero even when the two input are identical. This has implications for the differential amplifier:

Whatever internal resistance of the sources will cause a difference to appear between R_1 and R_3 .

III. Op amps in real life

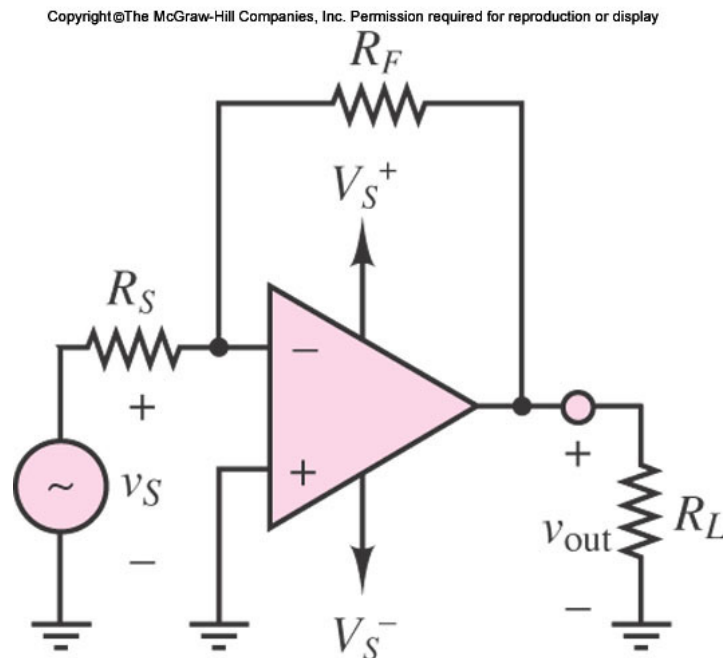
Op amps are actually electronic chips that are kept in a package. The various terminals of the op amp are then connected to different pins on the package. The figure below shows one example:



One may see that the package has 8 pins, of which only 7 are used (1 pin has no connection – pin 8). To build the circuits mentioned so far, we only need to consider 5 of the pins.

The pins -V and +V each refer to the negative and the positive supply. For the op amp to work, power needs to be supplied to the -V and +V pins.

The following figure then shows an inverting amplifier with the power supply connections included now. You might realize that we did not consider the power supply pins when deriving expressions for v_{out} .



You find that the gain expression for the circuit is still:

$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

In practice, we need to supply power otherwise the op amp will not work and will not get these expressions of V_{out} .

So what then is the effect of the power supply pins apart from giving power to the op amp for it work?

IV. Saturation

From the expression of V_{out}/V_{in} , you cannot see whether there is a limit on the maximum V_{out} . In a real op amp, V_{out} cannot exceed either of the supply voltages.

Hence if V_{s+} is 15V and V_{s-} is -15V, then V_{out} cannot be higher than 15V. It also cannot be lower than -15V.

Example on saturation

You have built an inverting op amp with gain of 10 and you use power supply of 15V to the op amp.

Case 1: $V_s = 2V$

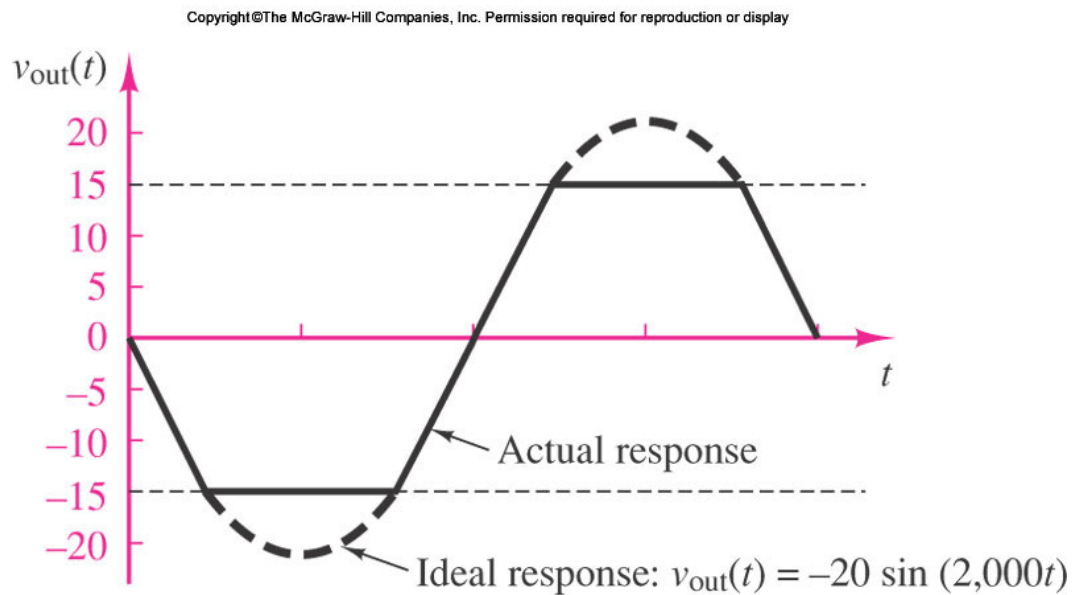
If we multiply 2V by -10, we should be -20V. But this is lower than the positive supply. So V_{out} will be -15V. In the labs, you will find that the minimum V_{out} does not even reach V_{s-} .

Case 2: $V_s = -2V$

$V_{out} = 15V$. In the labs, you will find that the maximum V_{out} does not even reach V_{s+} .

Case 3: $V_s = 2 \sin(2000t) \text{ V}$

When V_{out} reaches V_{s+} or V_{s-} , it gets chopped off.



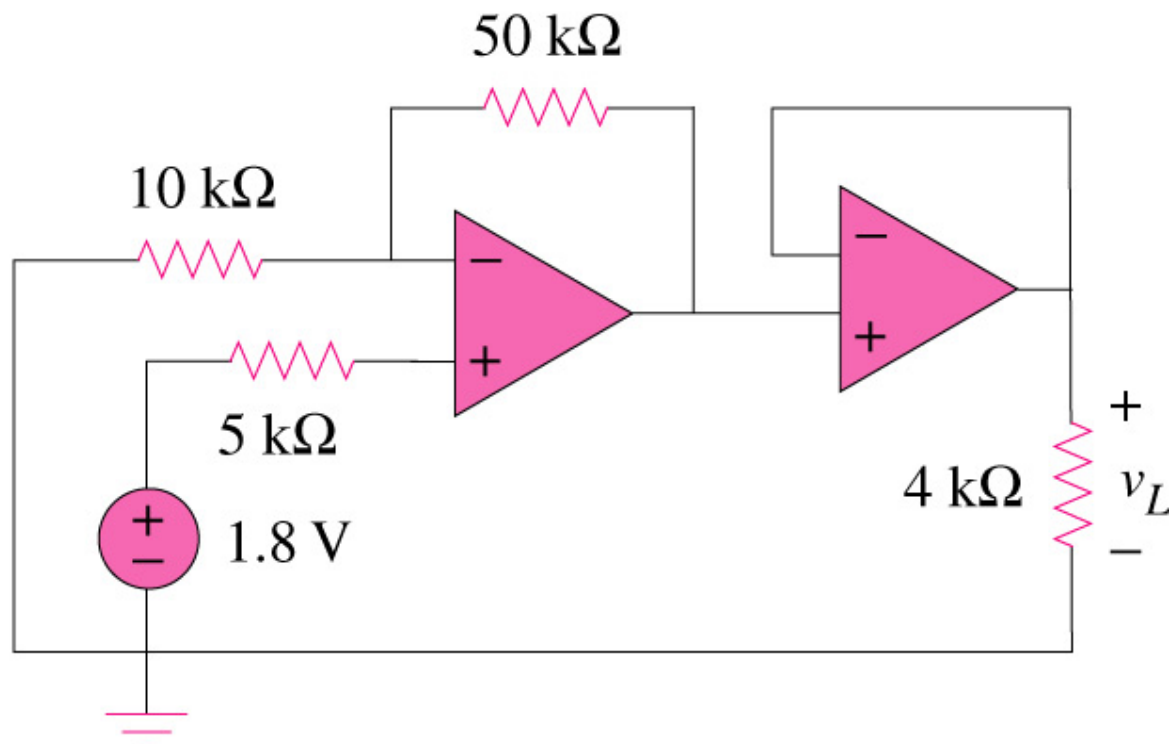
Op amps are capable of amplifying signals within the range of their supply voltages.

V. Cascaded amplifiers

The configurations above represent the most common ones used for amplifier circuits. We can then combine them, taking the output of one amplifier stage and then feeding it to the input of a second stage. This is known as cascading. The following are just a few examples.

Example on cascaded amplifiers 1

Identify the types of amplifiers in the circuit below.



First stage: Non-inverting

Second stage: Follower

Non-inverting amplifier followed by a follower

Gain of the first stage:

$$\text{Gain} = 1 + 50/10 = 6$$

Gain of the second stage:

$$\text{Gain} = 1$$

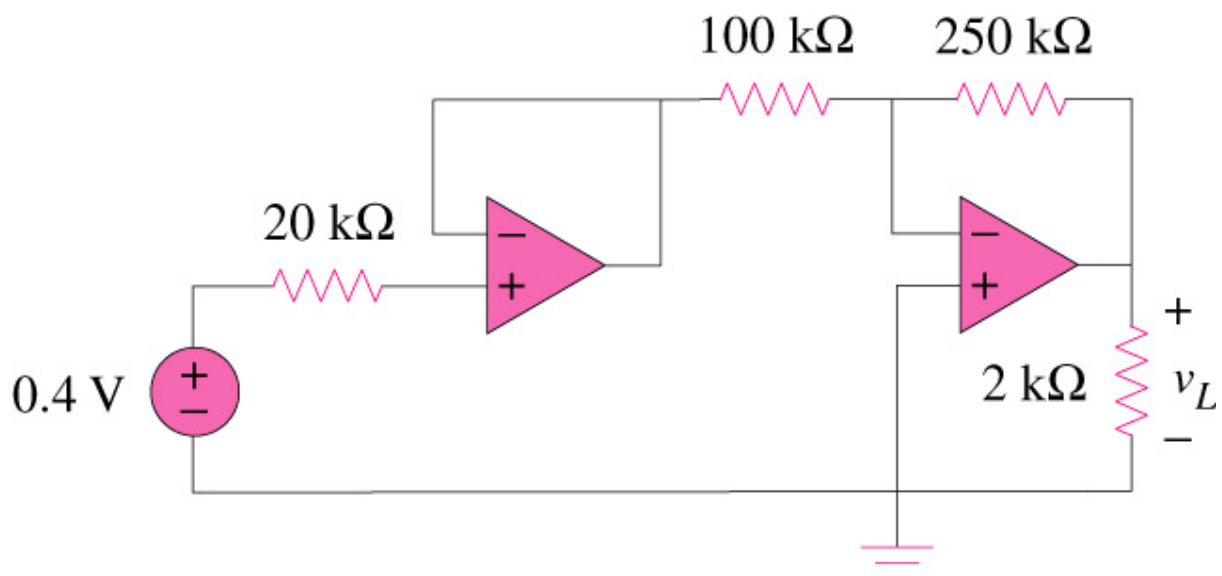
Multiply the 2 stages to find the total gain:

$$V_L = (1.8) * (6) * (1) = 10.8V$$

$$V_L = 10.8V$$

Example on cascaded amplifiers 2

Identify the types of amplifiers in the circuit below.



First stage: Follower

Second stage: Inverting

Inverting amplifier with follower at the input

Gain of the first stage:

$$\text{Gain} = 1$$

Gain of the second stage:

$$\text{Gain} = -250/100 = -2.5$$

Multiply the 2 stages to find the total gain:

$$V_L = 0.4 * 1 * (-2.5) = -1V$$

$$V_L = -1V$$

What if the follower was not used, so that the source of 0.4V and 20k Ω were connected directly to the inverting amplifier?

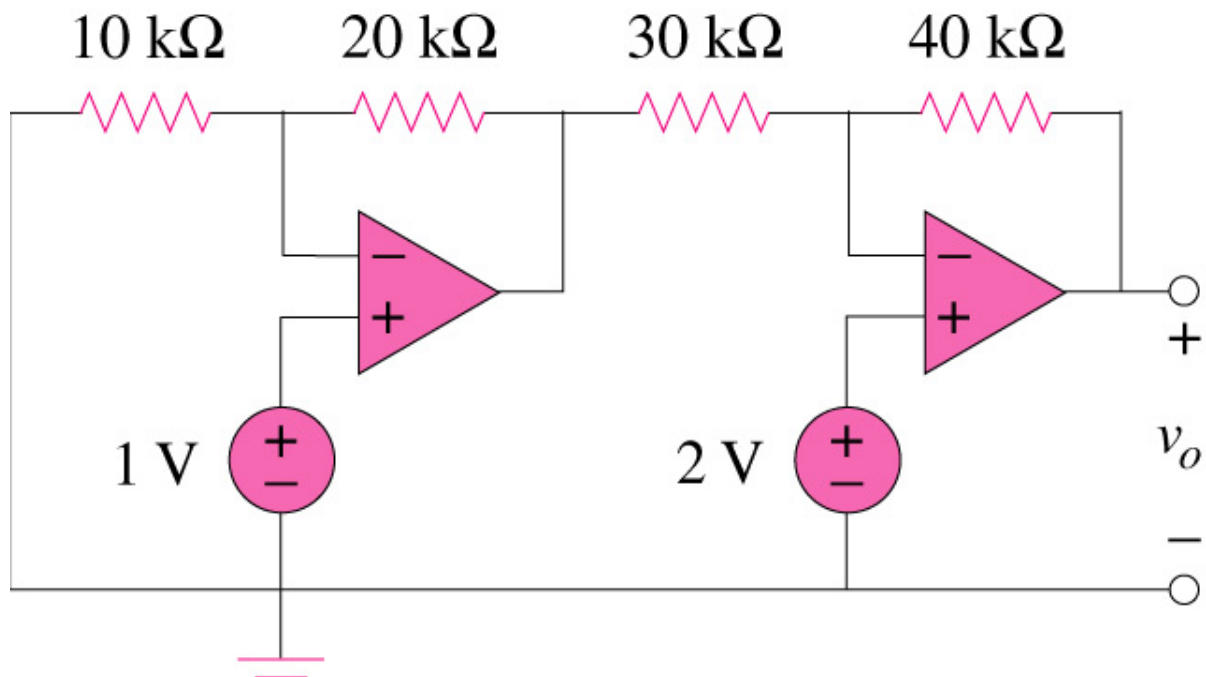
Re-calculate the gain expression for the second stage with 20k Ω included in series with 100k Ω :

$$\begin{aligned} V_L &= -\frac{250}{(100 + 20)} \times 0.4 \\ &= -0.83V \end{aligned}$$

The source follower isolates the 20k Ω resistor from the limited input resistance of the inverting amplifier (here it is 100k Ω).

Example on cascaded amplifiers 3

Identify the types of amplifiers in the circuit below.



First stage: Non-inverting

Second stage: Inverting **with v^+ terminal at 2V**

Gain of 1st stage:

$$\text{Gain} = (20/10) + 1 = 3V$$

$$\Rightarrow G_1 = 3$$

$$\Rightarrow \text{Output of first stage} = 3V$$

Gain of 2nd stage:

$$\text{Inverting amplifier with gain} = -40/30 = -4/3$$

$$\Rightarrow G_2 = -4/3 \text{ with an offset of } 2V$$

Given the 2V offset at V_+ , the input of the second stage can be readjusted to $3V - 2V = 1V$

Final output voltage, $v_o = (1) * (-4/3) + 2 = (2/3)V$

Alternatively,

Apply KCL at v^- of second amplifier:

$$\frac{3V - 2V}{30k} + \frac{V_{out} - 2V}{40k} = 0$$

Bear in mind that v^- has a voltage of 2V.

Summary

- 1) Remember the 3 characteristics of an ideal op amp
- 2) The output voltage is limited by the power supply
- 3) Op amps can be cascaded and combined