

1. Evaluate  $\log_5 100 - \log_5 4 + \log_6 12 + \log_6 18 - 7^{\log_7 8} + \frac{\log_8 9}{\log_8 3}$ .

2. The functions  $F(x)$  and  $G(x)$  are defined by

$$F(x) = e^{2x}, \quad x \in \mathbf{R},$$

$$G(x) = \log_e(1+x), \quad x \geq 0.$$

(a) For each of  $F(x)$  and  $G(x)$ , state the range of the function.

(b) Find and simplify

(i)  $F^{-1}(x)$ ,

(ii)  $G^{-1}(x)$ ,

(iii)  $(F \circ G)(x)$ .

3. The functions  $F$  and  $G$  are defined as follows:

$$F(x) = \frac{1}{2}(e^x + e^{-x}), \quad \text{for } x \in [0, \infty),$$

$$G(x) = \frac{1}{2}(e^x + e^{-x}), \quad \text{for } x \in (-\infty, 0].$$

(a) Give the ranges of  $F$  and  $G$ .

(b) Find the inverse functions  $F^{-1}(x)$  and  $G^{-1}(x)$  in a form similar to the above definitions, giving your answers as natural logarithms.

4. The functions  $F$  and  $G$  are defined by

$$F(x) = \frac{1}{2}(2^x - 2^{-x}),$$

$$G(x) = \log_e \left( \frac{1+x}{1-x} \right).$$

(a) Give the largest possible domains of  $F(x)$  and  $G(x)$ .

(b) Show that  $F^{-1}(x) = \log_2 \left( x + \sqrt{x^2 + 1} \right)$

5. Solve the equation  $2 \log_{10} x = 1 + \log_{10} \left( \frac{2(2x+5)}{5} \right)$ .

6. Solve the equation  $10^x + 10^{-x} - 4 = 0$ .

7. Given that  $\cosh x = \frac{1}{2}(e^x + e^{-x})$  and  $\sinh x = \frac{1}{2}(e^x - e^{-x})$ , prove that  $\sinh 2x = 2 \sinh x \cosh x$ . Hence, or otherwise, solve the equation  $\sinh 2x = \sinh x$ , expressing your answer as natural logarithm.

8. (a) Given that  $\cosh x = \frac{1}{2}(e^x + e^{-x})$  and  $\sinh x = \frac{1}{2}(e^x - e^{-x})$ , prove that
- (i)  $\cosh^2 x - \sinh^2 x = 1$ ,
  - (ii) the inverse of  $\sinh x$  is  $\sinh^{-1} x = \log_e \left( x + \sqrt{1 + x^2} \right)$ .
- (b) Solve the equation  $\cosh^2 x - 2\sinh x = 0$ , giving your answer as natural logarithms.

**Answers:**

1.  $-1$
2. (a)  $\text{Ran}(F) = (0, \infty)$ ,  $\text{Ran}(G) = [0, \infty)$ 
  - (b) (i)  $F^{-1}(x) = \frac{1}{2} \log_e x$
  - (ii)  $G^{-1}(x) = e^x - 1$
  - (iii)  $(F \circ G)(x) = (1 + x)^2$
3. (a)  $\text{Ran}(F) = [1, \infty)$ ,  $\text{Ran}(G) = [1, \infty)$ 
  - (b)  $F^{-1}(x) = \log_e \left( x + \sqrt{x^2 - 1} \right)$
  - $G^{-1}(x) = \log_e \left( x - \sqrt{x^2 - 1} \right) = -\log_e \left( x + \sqrt{x^2 - 1} \right)$
4. (a)  $\text{Dom}(F) = \mathbf{R}$ ,  $\text{Dom}(G) = (-1, 1)$
5.  $x = 10$  ( $x = -2$  is rejected)
6.  $x = \log_{10}(2 - \sqrt{3})$  or  $x = \log_{10}(2 + \sqrt{3})$
7.  $x = \log_e 1 (= 0)$
8. (b)  $x = \log_e(1 + \sqrt{2})$