

Tutorial 5

Electric Potential

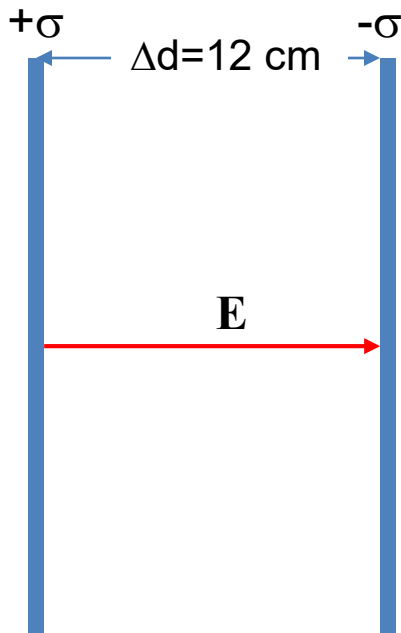
Practice Questions: Electric Potential

Chapter 24

Questions: *4,***10,**15,**22,*24,**45,**54

*4

- 4 Two large, parallel, conducting plates are 12 cm apart and have charges of equal magnitude and opposite sign on their facing surfaces. An electrostatic force of $3.9 \times 10^{-15} \text{ N}$ acts on an electron placed anywhere between the two plates. (Neglect fringing.) (a) Find the electric field at the position of the electron. (b) What is the potential difference between the plates?



(a) E is uniform between the two plates

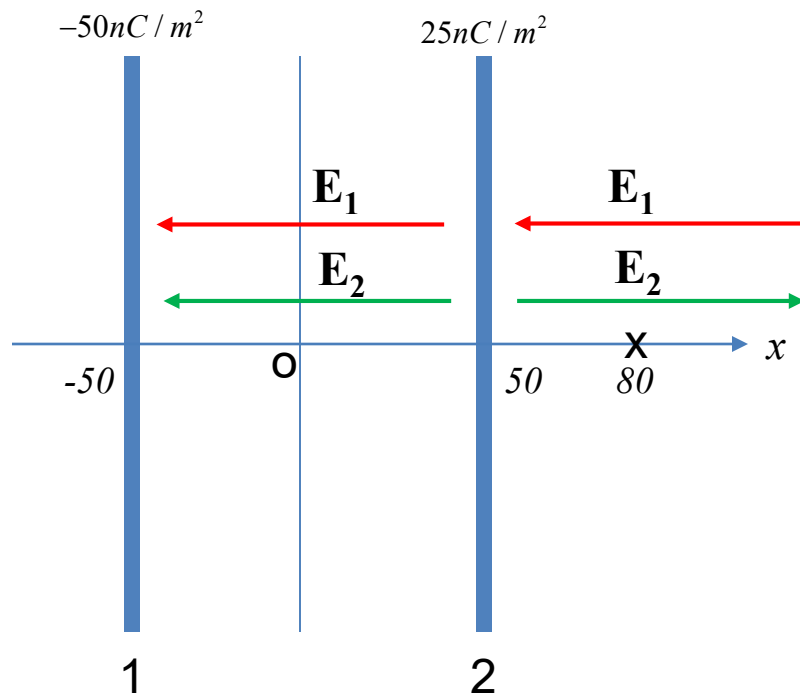
$$E = F/e = (3.9 \times 10^{-15} \text{ N}) / (1.60 \times 10^{-19} \text{ C}) \\ = 2.44 \times 10^4 \text{ N/C} = 2.44 \times 10^4 \text{ V/m.}$$

(b) Since E is uniform, potential difference is just $E\Delta d$

$$\Delta V = E\Delta d \\ = (2.4 \times 10^4 \text{ N/C})(0.12 \text{ m}) \\ = 2.9 \times 10^3 \text{ V.}$$

***10

•••10 Two uniformly charged, infinite, nonconducting planes are parallel to a yz plane and positioned at $x = -50$ cm and $x = +50$ cm. The charge densities on the planes are -50 nC/m² and $+25$ nC/m², respectively. What is the magnitude of the potential difference between the origin and the point on the x axis at $x = +80$ cm? (Hint: Use Gauss' law.)



In the “inside” region between the plates, the individual fields (given by Eq. 24-13) are in the same $-x$ direction

$$\vec{E}_{\text{in}} = -\left(\frac{50 \times 10^{-9} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} + \frac{25 \times 10^{-9} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \right) \hat{i} \\ = -(4.2 \times 10^3 \text{ N/C}) \hat{i}$$

In the “outside” region where $x > 0.5$ m, the individual fields point in opposite directions:

$$\vec{E}_{\text{out}} = -\frac{50 \times 10^{-9} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \hat{i} + \frac{25 \times 10^{-9} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \hat{i} \\ = -(1.4 \times 10^3 \text{ N/C}) \hat{i}.$$

Therefore

$$\Delta V = -\int_0^{0.8} \vec{E} \cdot d\vec{s} = \int_0^{0.5} |\vec{E}_{\text{in}}| dx + \int_{0.5}^{0.8} |\vec{E}_{\text{out}}| dx \\ = (4.2 \times 10^3)(0.5) + (1.4 \times 10^3)(0.3) \\ = 2.5 \times 10^3 \text{ V}.$$

**15

15 SSM ILW A spherical drop of water carrying a charge of 30 pC has a potential of 500 V at its surface (with $V = 0$ at infinity). (a) What is the radius of the drop? (b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop?

(a) The electric field E for $r > R$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

The potential on the sphere surface

$$\begin{aligned} V &= -\int_{\infty}^R \vec{E} \cdot d\hat{r} = -\frac{q}{4\pi\epsilon_0} \int_{\infty}^R \frac{1}{r^2} dr \\ &= -\frac{q}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_{\infty}^R = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{\infty} \right] = \frac{q}{4\pi\epsilon_0 R} \\ R &= \frac{q}{4\pi\epsilon_0 V} \\ &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(30 \times 10^{-12} \text{ C})}{500 \text{ V}} \\ &= 5.4 \times 10^{-4} \text{ m}. \end{aligned}$$

(b) After the drops combine the total volume is twice the volume of an original drop, so the new volume v' is

$$v' = \frac{4}{3} \pi (R')^3 = 2 \times \frac{4}{3} \pi R^3$$

$$R' = 2^{1/3} R$$

The charge is twice the charge of original drop: $q' = 2q$. Thus,

$$\begin{aligned} V' &= \frac{1}{4\pi\epsilon_0} \frac{q'}{R'} \\ &= \frac{1}{4\pi\epsilon_0} \frac{2q}{2^{1/3} R} \\ &= 2^{2/3} V = 2^{2/3} (500 \text{ V}) \approx 790 \text{ V}. \end{aligned}$$

**22

••22 In Fig. 24-36a, a particle of elementary charge $+e$ is initially at coordinate $z = 20$ nm on the dipole axis (here a z axis) through an electric dipole, on the positive side of the dipole. (The origin of z is at the center of the dipole.) The particle is then moved along a circular path around the dipole center until it is at coordinate $z = -20$ nm, on the negative side of the dipole axis. Figure 24-36b gives the work W_a done by the force moving the particle versus the angle θ that locates the particle relative to the positive direction of the z axis. The scale of the vertical axis is set by $W_{as} = 4.0 \times 10^{-30}$ J. What is the magnitude of the dipole moment?

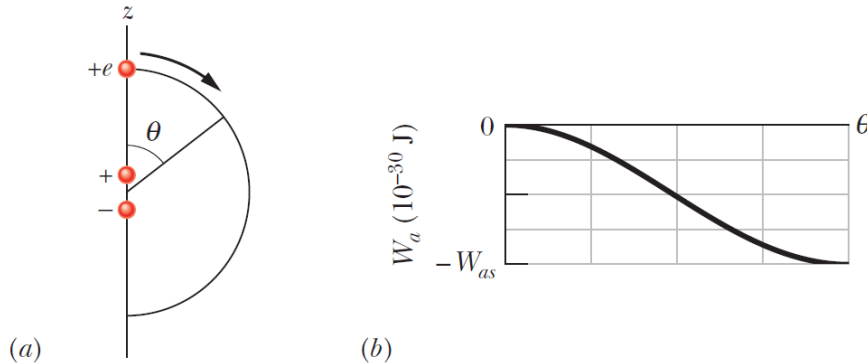


Fig. 24-36 Problem 22.

From slide 25 of the lecture note, the potential due to an electric dipole is

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2} \quad (\text{electric dipole}),$$

From slide 14 of the lecture note, we found the relationship between work and potential

$$\Delta V = -\frac{W}{q} \quad \text{Potential difference definition}$$

$$W_a = q\Delta V = e \left(\frac{p \cos \theta}{4\pi\epsilon_0 r^2} - \frac{p \cos \theta_i}{4\pi\epsilon_0 r^2} \right) = \frac{ep \cos \theta}{4\pi\epsilon_0 r^2} (\cos \theta - 1)$$

W_a is the work done by the external force, which is equal to the negative of the work done by the system.

With $r = 20 \times 10^{-9}$ m. For $\theta = 180^\circ$ the graph indicates $W_a = -2.0 \times 10^{-30}$ J, from which we can determine p .

The magnitude of the dipole moment is therefore

$$p = 2.8 \times 10^{-37} \text{ C}\cdot\text{m}.$$

*24

•24 In Fig. 24-38, a plastic rod having a uniformly distributed charge $Q = -25.6 \text{ pC}$ has been bent into a circular arc of radius $R = 3.71 \text{ cm}$ and central angle $\phi = 120^\circ$. With $V = 0$ at infinity, what is the electric potential at P , the center of curvature of the rod?

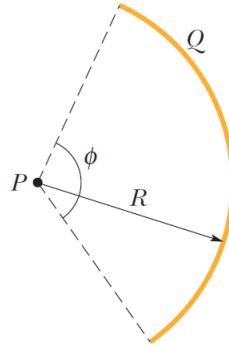


Fig. 24-38
Problem 24.

The potential is

$$\begin{aligned} V_P &= \frac{1}{4\pi\epsilon_0} \int_{\text{rod}} \frac{dq}{R} = \frac{1}{4\pi\epsilon_0 R} \int_{\text{rod}} dq \\ &= \frac{Q}{4\pi\epsilon_0 R} = -\frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(-25.6 \times 10^{-12} \text{ C})}{3.71 \times 10^{-2} \text{ m}} \\ &= -6.21 \text{ V}. \end{aligned}$$

We note that the result is exactly what one would expect for a point-charge Q at a distance R . This “coincidence” is due, in part, to the fact that V is a scalar quantity.

**45

••45 ILW A particle of charge q is fixed at point P , and a second particle of mass m and the same charge q is initially held a distance r_1 from P . The second particle is then released. Determine its speed when it is a distance r_2 from P . Let $q = 3.1 \mu\text{C}$, $m = 20 \text{ mg}$, $r_1 = 0.90 \text{ mm}$, and $r_2 = 2.5 \text{ mm}$.

The initial potential energy is $U_i = q^2/4\pi\epsilon_0 r_1$ and the initial kinetic energy is $K_i = 0$, the final potential energy is $U_f = q^2/4\pi\epsilon_0 r_2$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$ where v is the final speed of the particle. Conservation of energy yields

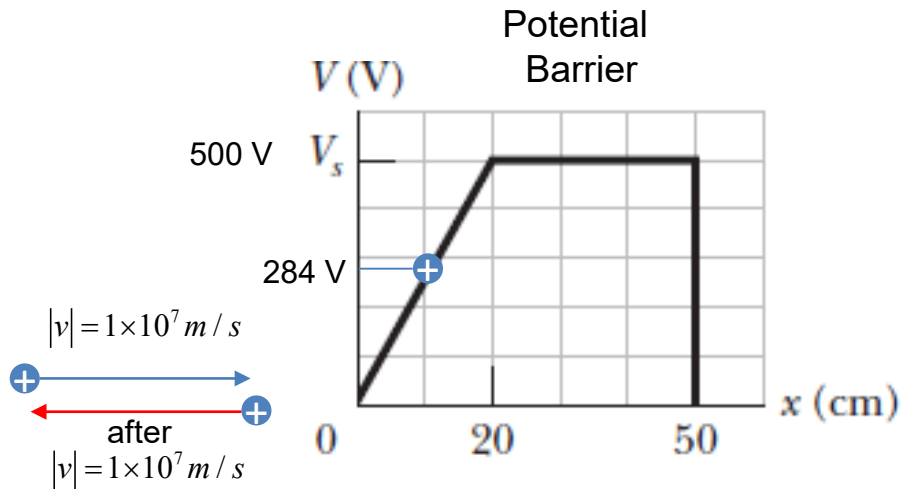
$$\frac{q^2}{4\pi\epsilon_0 r_1} = \frac{q^2}{4\pi\epsilon_0 r_2} + \frac{1}{2}mv^2.$$

The solution for v is

$$\begin{aligned} v &= \sqrt{\frac{2q^2}{4\pi\epsilon_0 m} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)} \\ &= \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2)(3.1 \times 10^{-6} \text{ C})^2}{20 \times 10^{-6} \text{ kg}} \left(\frac{1}{0.90 \times 10^{-3} \text{ m}} - \frac{1}{2.5 \times 10^{-3} \text{ m}} \right)} \\ &= 2.5 \times 10^3 \text{ m/s.} \end{aligned}$$

**54

****54** A positron (charge $+e$, mass equal to the electron mass) is moving at 1.0×10^7 m/s in the positive direction of an x axis when, at $x = 0$, it encounters an electric field directed along the x axis. The electric potential V associated with the field is given in Fig. 24-52. The scale of the vertical axis is set by $V_s = 500.0$ V. (a) Does the positron emerge from the field at $x = 0$ (which means its motion is reversed) or at $x = 0.50$ m (which means its motion is not reversed)? (b) What is its speed when it emerges?



$$\begin{aligned}
 K.E. &= \frac{1}{2}mv^2 = \frac{1}{2}(9.1 \times 10^{-31})(1.0 \times 10^7)^2 \\
 &= 4.6 \times 10^{-17} \text{ J} \\
 &= \frac{4.6 \times 10^{-17} \text{ J}}{1.6 \times 10^{-19}} = 284 \text{ eV}
 \end{aligned}$$

We define electron-volt (eV) as the energy equal to the work required to move a single elementary charge e , such as that of the electron or the proton, through a potential difference of exactly one volt. The magnitude of this work is $q\Delta V$.

$$\begin{aligned}
 1 \text{ eV} &= e(1 \text{ V}) \\
 &= (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.60 \times 10^{-19} \text{ J.}
 \end{aligned}$$

The potential barrier has a height of 500 V, the positron needs to have sufficient energy of at least 500 eV to overcome this barrier.

At 284 eV the positron does not have enough energy to get over this barrier, therefore it will emerge from the field at $x=0$.

The final velocity is when it emerge from the field at $x=0$ having the same magnitude as the initial velocity, but in the opposite direction.