

Name: _____ Student ID: _____ Seat No.: _____

MA1502 Algebra
Semester A, 2019-20
Test

Time allowed: 90 minutes

Instructions to candidates:

This paper has NINE pages (including this cover page).

This paper has SEVEN questions.

The questions will begin on page 2.

Write your answers in the space provided under each question.

Answer ALL questions. Show ALL steps in order to get full marks.

Question	Marks
1	
2	
3	
4	
5	
6	
7	
Total mark:	

NOT TO BE TAKEN AWAY

Question 1

Find the largest possible domain and largest possible range of the function

$$f(x) = \frac{10 - 3x - x^2}{|x - 2|}.$$

[12 marks]

Question 2

Consider the function

$$g(x) = 1 + \sqrt{x - 3} \quad \text{for } x \geq 4.$$

(a) Determine whether it is a one-to-one function.

[5 marks]

(b) Find $g^{-1}(x)$, the inverse function of $g(x)$.

[5 marks]

(c) Find the largest possible domain and largest possible range of $g^{-1}(x)$.

[6 marks]

(Please turn over for Question 3)

Question 3

Given that $\sin A = -\frac{3}{5}$ and $\cos B = -\frac{12}{13}$, where $-90^\circ < A < 0^\circ$ and $180^\circ < B < 270^\circ$. Without using a calculator, find the values of

(a) $\cot(A + B)$ [9 marks]

(c) $\cos\left(\frac{B}{2}\right)$ [5 marks]

(Please turn over for Question 4)

Question 4

(a) Prove the following identity

$$\frac{\cos x - \sin x}{\cos x + \sin x} = \frac{\cos 2x}{1 + \sin 2x}.$$

[6 marks]

(b) Find the general solution (in radians) of the following equation

$$\sin 6x + \cos 3x = 0.$$

[10 marks]

(Please turn over for Question 5)

Question 5

Solve the inequality $|x - 5| > 3 + |4x + 1|$.

[10 marks]

(Please turn over for Question 6)

Question 6

(a) Show that $2x + 1$ is a factor of $q(x) = 2x^3 + 5x^2 + 12x + 5$.

[2 marks]

(b) Express $\frac{10x^2+9x+19}{2x^3+5x^2+12x+5}$ into partial fractions.

[10 marks]

Question 7

Consider the following system of linear equations, where p and q are real numbers.

$$\begin{cases} x + 2y + 2z - w = 3 \\ 2x + 5y + 3z + w = 4 \\ 3x + 8y + 4z + pw = q \end{cases}$$

- (a) Use Gaussian elimination to transform the augmented matrix of the above system into row echelon form. [6 marks]

- (b) State all possible values of p and q for which the above system of linear equations has

(i) no solution; [2 marks]

(ii) infinitely many solutions with one free variable; [2 marks]

(iii) infinitely many solutions with two free variables. [2 marks]

(Please turn over for Question 7(c))

(c) Solve the system in each case if it is consistent.

[8 marks]