

Practice Problem Set 3 Solutions

Question 1: Suppose the demand function is

$$q^D(p) = Kp^{-c},$$

where K and c are positive constants. What is the elasticity of demand? When is demand elastic?

The elasticity of demand is

$$\varepsilon(p) = -\frac{q'p}{q} = -\frac{(-cKp^{-c-1})p}{Kp^{-c}} = c$$

The demand is elastic if $c > 1$.

Question 2: Consider a monopoly with cost function

$$C(q) = 100 + 80q - 50q^2 + 0.5q^3,$$

and the demand set for its product is

$$D = \{(q, p) | 2p + q^2 - 20q = 100\}.$$

Find the level of production that maximizes the firm's profit if the upper limit on its output is

(a) 30

(b) 50

The inverse demand function is

$$p^D(q) = 50 + 10q - 0.5q^2$$

The profit function is

$$\begin{aligned}\Pi(q) &= qp^D(q) - C(q) = q(50 + 10q - 0.5q^2) - (100 + 80q - 50q^2 + 0.5q^3) \\ &= -q^3 + 60q^2 - 30q - 100\end{aligned}$$

Setting $\Pi'(q) = 0$, we have

$$-3q^2 + 120q - 30 = -3(q^2 - 40q + 10) = 0$$

Then, the critical points are

$$q = \frac{-(-40) \pm \sqrt{(-40)^2 - (4)(1)(10)}}{2} = 20 \pm \sqrt{390}$$

Therefore, $q = 0.2516$ or $q = 39.7484$. The second derivative of Π is $\Pi''(q) = -6q + 120$.

$$\Pi''(0.2516) > 0 \quad \quad \Pi''(39.7484) < 0$$

So, we have a local minimum at $q = 0.2516$ and a local maximum at $q = 39.7484$. We have

$$\Pi(0.2516) = -103.766 \quad \quad \Pi(39.7484) = 30703.8$$

Also, note that

$$\Pi(0) = -100, \quad \Pi(30) = 26000, \quad \Pi(50) = 23400$$

Thus, profits are maximized at $q = 30$ if the upper limit on the output is 30. However, profits are maximized at $q = 39.7484$ if the upper output limit is 50.

Question 3: Suppose that there is an efficient, perfectly competitive firm that cannot produce more than 6 units of its product each week. The cost function is

$$C(q) = 100 + 20q - 6q^2 + q^3$$

Determine

- (a) the fixed cost

The fixed cost is $C(0) = 100$.

- (b) the profit function

If the firm is in production, then $p = C'(q)$ and the profit function is given by

$$\begin{aligned} \Pi(q) &= qC'(q) - C(q) = q(20 - 12q + 3q^2) - (100 + 20q - 6q^2 + q^3) \\ &= 2q^3 - 6q^2 - 100 \end{aligned}$$

- (c) the startup point

The startup point is the value q_s such that

$$\Pi(q_s) = \Pi(0) = -C(0)$$

$$\implies 2q^3 - 6q^2 - 100 = -100$$

$$\implies q^2(q - 3) = 0$$

For $q \neq 0$, $q_s = 3$.

- (d) the equation that needs to be satisfied at the breakeven point (*note that the question does not ask you to exactly find the breakeven point because it requires solving a cubic equation, and that is beyond the scope of this course.*)

The breakeven point satisfies

$$\Pi(q_b) = 0$$

$$\implies 2q^3 - 6q^2 - 100 = 0$$