

EE3101 Communication Engineering

Tutorial 7

- (1) Assume that the P_B for a BPSK system with ideal coherent detection is 10^{-8} . What is the P_B value if (a) the transmission power is reduced by half, (b) the bit rate is doubled but with the same transmission power ?

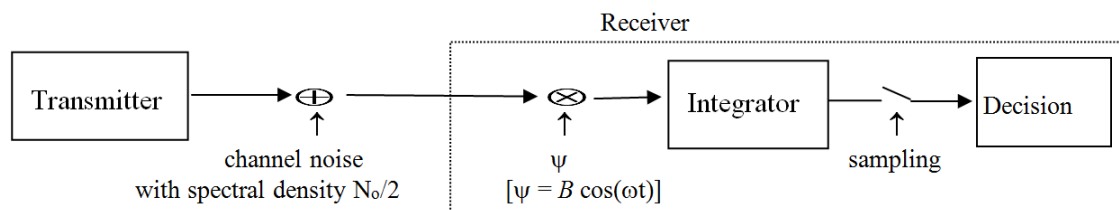
{ Note: $Q(x) \approx \frac{1}{x\sqrt{2\pi}} \exp[-\frac{x^2}{2}]$, and $Q(x) = 10^{-8}$ for $x=5.612$ }

- (2) Consider the following BPSK transmission systems where an additional carrier component is included in the transmitted signal (like DSB-LC), such that:

$$s(t) = A \sin(\omega_c t) \pm B \cos(\omega_c t)$$

where $0 \leq t < T_b$ and T_b is the bit duration.

The first term represents a carrier component for the purpose of coherent detection at the receiver, and the plus and negative signs correspond to binary “1” and “0” respectively. An Additive White Gaussian Noise Channel (AWGN) with two-sided power spectral density of $N_0/2$ is used. Assume the probability of error for this particular setup is 10^{-5} and the bit rate is 1Mb/s.



- (a) What is the minimum Nyquist bandwidth in the passband without ISI ?
- (b) Show that probability of bit error is: $P_B = Q(\sqrt{\frac{B^2 T}{N_0}})$
- (c) Assume 15% of the transmitted signal power is allocated to the additional carrier, evaluate the probability of bit error for a conventional BPSK system (i.e. without the additional carrier) with the same overall transmission power.

[Note: $Q(4.26) \approx 1 \times 10^{-5}$, $Q(x) \approx \frac{1}{x\sqrt{2\pi}} \exp(-\frac{x^2}{2})$]

- (3) For a 2 level ASK system, suppose that the average signal level at the decision device input is d and $d + x$ for binary “0” and “1” respectively, and White Gaussian Noise Channel with double-sided power spectral density $N_0/2$ is used. Assume that the data is random with equal probability, show that the optimal threshold value for detection, i.e. with minimum P_B , is $d + x/2$.
- (4) Consider a BPSK system with coherent detection, assume that the probability of binary “1” and “0” is P_1 and P_0 respectively, and White Gaussian Noise Channel with double-sided power spectral density $N_0/2$ is used. The signals for binary “1” and “0” are $E\sqrt{\frac{2}{T}} \cos(\omega_c t)$ and $-E\sqrt{\frac{2}{T}} \cos(\omega_c t)$ respectively. The reference signal for coherent detection is $\sqrt{\frac{2}{T}} \cos(\omega_c t)$.

(i) Show that the overall average bit error probability P_B is:

$$P_0 Q\left[\sqrt{\frac{2E_b}{N_0}}\left(1 + \frac{y}{\sqrt{E_b}}\right)\right] + P_1 Q\left[\sqrt{\frac{2E_b}{N_0}}\left(1 - \frac{y}{\sqrt{E_b}}\right)\right]$$

where E_b is the average bit energy and y is the threshold value

(ii) Derive the optimum threshold with a minimum P_B .

$$\left\{ \begin{array}{l} \text{Note: } Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) du, \quad \Rightarrow \quad \frac{dQ(x)}{dx} = -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \\ \Rightarrow \quad \frac{dQ(z)}{dy} = \frac{dQ(z)}{dz} \frac{dz}{dy} \end{array} \right\}$$