

EE3109 Applied Electromagnetics
2024/25 Semester B
Weeks 12-13: Tutorial Solution

1. A 1 m long dipole is excited by a 1 MHz current with an amplitude of 24 A. What is the average power density radiated by the dipole at a distance of 5 km in a direction that is 45° from the dipole axis?

Solution:

At 1 MHz, $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1 \times 10^6} = 300$ m. The electrical length of the dipole is $l_e = 1 \text{ m} = \frac{1}{300} \lambda$. It can be considered a Hertzian dipole.

The maximum power density is

$$S_0 = \frac{15\pi I_0^2}{R^2} \left(\frac{l}{\lambda}\right)^2 = \frac{15\pi (24)^2}{(5 \times 10^3)^2} \left(\frac{1}{300}\right)^2 = 12.06 \text{ [nW/m}^2\text{]}$$

The power density at $\theta = 45^\circ$ is

$$S = S_0 \sin^2 \theta = S_0 \sin^2 45^\circ = \frac{S_0}{2} = 6.032 \text{ [nW/m}^2\text{]}$$

2. For an antenna whose normalized radiation intensity is given by

$$F(\theta, \phi) = \begin{cases} 1, & \text{for } 0^\circ < \theta < 60^\circ \text{ and } 0 \leq \phi < 2\pi \\ 0, & \text{elsewhere} \end{cases}$$

Determine the following:

- (a) The direction of maximum radiation.
- (b) Directivity.
- (c) Beam solid angle.
- (d) Half-power beamwidth in the x-z plane.

Solution:

- (a) The radiation is at the maximum level for the whole range that it is non-zero, i.e. $0 < \theta < 60^\circ$.

- (c) The antenna pattern solid angle is

$$\Omega_p = \iint_{4\pi} F d\Omega = \int_0^{2\pi} \int_0^{60^\circ} \sin \theta d\theta d\phi = 2\pi [\cos 0^\circ - \cos 60^\circ] = \pi$$

- (b) The directivity is

$$D = \frac{4\pi}{\Omega_p} = 4$$

- (d) The +x side of the xz-plane is $\{0 \leq \theta \leq 180^\circ, \phi = 0^\circ\}$; the -x side of the xz-plane is $\{0 \leq \theta \leq 180^\circ, \phi = 180^\circ\}$. The radiated beam is at full strength for $\{0 \leq \theta \leq 60^\circ, \phi = 0^\circ\}$ and $\{0 \leq \theta \leq 60^\circ, \phi = 180^\circ\}$. But the latter is equivalent to $\{-60 \leq \theta \leq 0^\circ, \phi = 0^\circ\}$, which is connected to the first section (in the blue $\{\cdot\}$). Hence the overall beam is at full strength for $\{-60^\circ \leq \theta \leq 60^\circ, \phi = 0^\circ\}$, so the half-power beamwidth in the x-z plane is $60^\circ - (-60^\circ) = 120^\circ$

3. Repeat Q3 for an antenna with

$$F(\theta, \phi) = \begin{cases} \sin^2 \theta \cos^2 \phi & \text{for } 0 < \theta < \pi \text{ and } -\pi/2 \leq \phi \leq \pi/2 \\ 0, & \text{elsewhere} \end{cases}$$

Solution:

(a) The radiation is at the maximum level for $(\theta, \phi) = (\frac{\pi}{2}, 0)$, as that is where $F(\theta, \phi) = 1$.

(c) The antenna pattern solid angle is

$$\begin{aligned} \Omega_p &= \iint_{4\pi} F d\Omega = \int_{-\pi/2}^{\pi/2} \int_0^\pi (\sin^2 \theta \cos^2 \phi) \sin \theta d\theta d\phi = \int_{-\pi/2}^{\pi/2} \cos^2 \phi d\phi \int_0^\pi \sin^3 \theta d\theta \\ &= \left(\frac{\pi}{2}\right) \left[\frac{\cos 3\theta}{12} - \frac{3}{4} \cos \theta \right]_0^\pi = \left(\frac{\pi}{2}\right) \left[\left(-\frac{1}{12} + \frac{3}{4}\right) - \left(\frac{1}{12} - \frac{3}{4}\right) \right] = \left(\frac{\pi}{2}\right) \left(\frac{4}{3}\right) = \frac{2\pi}{3} \end{aligned}$$

(b) The directivity is

$$D = \frac{4\pi}{\Omega_p} = 6$$

(d) On the xz-plane θ varies on either side of 90° while $\phi = 0^\circ$.

$$\frac{1}{2} = F(\theta, \phi = 0^\circ) = \sin^2 \theta \Rightarrow \sin \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \pm 45^\circ$$

So the half-power beamwidth in the x-z plane is $45^\circ - (-45^\circ) = 90^\circ$

4. A 2 m long center-fed dipole antenna operates in the AM broadcast band at 1 MHz. The dipole is made of copper wire with a radius of 1 mm.

(a) Determine the radiation efficiency of the antenna.

(b) What is the antenna gain in decibels?

(c) What antenna current is required so that the antenna will radiate 80W, and how much power will the generator have to supply to the antenna?

Solution:

At 1 MHz, $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1 \times 10^6} = 300$ m. The electrical length of the dipole is $l_e = 2 \text{ m} = \frac{1}{150} \lambda$. It can be considered a Hertzian dipole.

$$\begin{aligned} \text{(a) } R_{rad} &= 80\pi^2 \left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \left(\frac{1}{150}\right)^2 = 35.09 \text{ m}\Omega; \quad R_{loss} = \frac{1}{2\pi a} \sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{1}{2\pi(1 \times 10^{-3})} \sqrt{\frac{\pi(1 \times 10^6)(4\pi \times 10^{-7})}{(5.8 \times 10^7)}} \\ &= 41.52 \text{ m}\Omega. \end{aligned}$$

$$\xi = \frac{R_{rad}}{R_{rad} + R_{loss}} \cong 0.4580$$

$$\text{(b) } G = \xi D \cong (0.4580)(1.5) = 0.6871; \quad G_{dB} = 10 \log(0.6871) = -1.63 \text{ dB}$$

(c) The current on the dipole and the power that needs to be supplied are:

$$P_{rad} = 40\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2 \Rightarrow I_0 = \sqrt{\frac{P_{rad}}{40\pi^2 \left(\frac{l}{\lambda}\right)^2}} = \sqrt{\frac{80}{40\pi^2 \left(\frac{1}{150}\right)^2}} = 67.52 \text{ A}$$

$$P_{supply} = P_{rad} + P_{loss} = \frac{P_{rad}}{\xi} = \frac{80}{0.4580} = 174.7 \text{ W}$$