

Tutorial 9
Sensitivity Analysis**Question 1**

A furniture company makes sofa, tables, and chairs. Each piece of furniture requires a fixed amount of wood and metal. In addition, the production of each piece of furniture requires a fixed amount of labour hours from the carpentry department and the finishing department.

The company has a limited amount of the four resources (wood, metal, carpentry labour, and finishing labour), as indicated by the table below:

	Sofa	Table	Chair	Resources available
<i>Selling price</i>	\$60	\$30	\$25	-
Wood (lbs)	8	6	1	88
Metal (lbs)	5	1	0	48
Carpentry (hrs)	3	2	1	28
Finishing (hrs)	4	3	2	42

Assume that demand for all three products exceeds our production capacities. The company would like to decide on a production plan that maximizes its revenue.

Let X_1 , X_2 , and X_3 be the production quantities of sofa, table, and chair, respectively. The goal is to maximize the production revenue.

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The sensitivity report of the spreadsheet result is attached:

Variable Cells

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
X_1	7	0	60	15	10
X_2	0	-12.5	30	12.5	1E+30
X_3	7	0	25	5	5

Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Wood	63	0	88	1E+30	25
Metal	35	0	48	1E+30	13
Carpentry	28	10	28	2.6	7
Finishing	42	7.5	42	14	4.67

- (a) Using the decision variables X_1, X_2, X_3 as defined above, write a LINEAR program to determine the optimal production plan.

Use the above sensitivity report to answer the following questions without using Excel.

- (b) What is the optimal production plan (i.e. solution) and the optimal revenue?
- (c) Explain whether the optimal solution is unique.
- (d) Suppose you can change the selling price of tables, but all other parameters are fixed. What is the minimum selling price per table in order for the optimal production plan to include a positive number of tables?
- (e) Suppose the selling price of a chair increases to \$30, and the amount of wood available increases to 100. Show how you can compute the new optimal revenue using the sensitivity report, or explain why you cannot determine the new optimal revenue.
- (f) Suppose the selling price of a sofa increases to \$70. Show how you can compute the new optimal revenue using the sensitivity report, or explain why you cannot determine the new optimal revenue.
- (g) Suppose the company reassigned some workers. As a result, the company has 49 finishing hours, but only 25 carpentry hours. Show how you can compute the new optimal revenue using the sensitivity report, or explain why you cannot determine the new optimal revenue.

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Solutions:

- (a) The model is given as
- $$\begin{aligned} \max \quad & 60 X_1 + 30 X_2 + 25 X_3 \\ \text{s.t.} \quad & 8 X_1 + 6 X_2 + X_3 \leq 88 \\ & 5 X_1 + X_2 \leq 48 \\ & 3 X_1 + 2 X_2 + X_3 \leq 28 \\ & 4 X_1 + 3 X_2 + 2 X_3 \leq 42 \\ & X_1, X_2, X_3 \geq 0 \end{aligned}$$
- (b) Optimal solution is to produce 7 sofas and 7 chairs, for an optimal revenue of $7*60+7*25 = \$595$
- (c) The optimal solution is unique because none of the allowable increases and allowable decreases in the first table are zero
- (d) The unit revenue of the table should increase by at least 12.5 to $(30-(-12.5)) = 42.5$.
- (e) Can be predicted from the sensitivity report because the change in selling price of a chair (X_3) (from \$25 increased to \$30) is within the allowable increase (\$5) in the variable cells table. Hence, the optimal production plan does not change. On the other side, the simultaneous change (increase to 100, by 12 lbs) is on an abundant resource "Wood" which will not have any impact as it is already in excess in the optimal solution (63 lbs).
- *As the constraint (Wood) is non-binding (shadow price = 0), sensitivity report can be used to predict. However, if the constraint is binding (shadow price $\neq 0$), sensitivity report could not be used.
- (f) The relative increase of $70-60 = 10$ is within the allowable increase of 15. The optimal solution remains unchanged and the new revenue is $\$70*7+\$25*7 = \$665$.
- (g) The relative change is $(49-42)/14 + (28-25)/7 < 100\%$. The 100% rule says that we can still use the shadow prices. The new revenue is $(\$60*7+\$25*7) + \$10*(25-28) + \$7.50*(49-42) = \$595 - 30 + 52.50 = \$617.50$.

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Question 2

The Battery Park Stable feeds and houses the horses used to pull tourist-filled carriages through the streets of Charleston's historic waterfront area. The stable owner, an ex-racehorse trainer, recognizes the need to set a nutritional diet for the horses in his care. At the same time, he would like to keep the overall daily cost of feed to a minimum.

The feed mixes available for the horses' diet are an oat product, a highly enriched grain, and a mineral product. Each of these mixes contains a certain amount of five ingredients needed daily to keep the average horse healthy. The table below shows these minimum requirements, units of each ingredient per pound of feed mix, and costs for the three mixes.

In addition, the stable owner is aware that an overfed horse is a sluggish worker. Consequently, he determines that 6 pounds of feed per day are the most that any horse needs to function properly. Formulate this problem and solve for the optimal daily mix of the three feeds.

Diet Requirement (Ingredients)	Feed Mix			
	Oat Product (Units / lb)	Enriched Grain (Units / lb)	Mineral Product (Units / lb)	Minimum Daily Requirement (Units)
A	2	3	1	6
B	0.5	1	0.5	2
C	3	5	6	9
D	1	1.5	2	8
E	0.5	0.5	1.5	5
Cost / lb	\$0.09	\$0.14	\$0.17	

Battery Park Stable

	A	G	M	[Optimal solution of each decision variable]		
	Oat	Grain	Mineral			
Number of Pounds	1.333	0.000	3.333			
Cost	\$0.09	\$0.14	\$0.17	0.6867	[Objective]	
Constraints:						
Ingredient A	2	3	1	6.0	>=	6
Ingredient B	0.5	1	0.5	2.3	>=	2
Ingredient C	3	5	6	24.0	>=	9
Ingredient D	1	1.5	2	8.0	>=	8
Ingredient E	0.5	0.5	1.5	5.7	>=	5
Max feed	1	1	1	4.7	<=	6
	LHS	Sign	RHS			

CB2203 Data-Driven Business Modeling**Microsoft Excel 9.0 Answer Report****Worksheet: [P4-16.xls]P3-4**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$E\$6	Cost	\$0.00	\$0.69

Adjustable Cells

Cell	Name	Original Value	Final Value
\$B\$5	Number of Pounds Oat	0.000	1.333
\$C\$5	Number of Pounds Grain	0.000	0.000
\$D\$5	Number of Pounds Mineral	0.000	3.333

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$E\$13	Max feed	4.67	\$E\$13<=\$G\$13	Not Binding	1.333
\$E\$8	Ingredient A	6.00	\$E\$8>=\$G\$8	Binding	0.000
\$E\$9	Ingredient B	2.33	\$E\$9>=\$G\$9	Not Binding	0.333
\$E\$10	Ingredient C	24.00	\$E\$10>=\$G\$10	Not Binding	15.000
\$E\$11	Ingredient D	8.00	\$E\$11>=\$G\$11	Binding	0.000
\$E\$12	Ingredient E	5.67	\$E\$12>=\$G\$12	Not Binding	0.667

Microsoft Excel 9.0 Sensitivity Report**Worksheet: [P4-16.xls]P3-4**

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$5	Number of Pounds Oat	1.33	0.000	0.09	0.003	0.005
\$C\$5	Number of Pounds Grain	0.00	0.005	0.14	1E+30	0.005
\$D\$5	Number of Pounds Mineral	3.33	0.000	0.17	0.010	0.125

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$13	Max feed	4.67	0.000	6.00	1E+30	1.333
\$E\$8	Ingredient A	6.00	0.003	6.00	4.000	2.000
\$E\$9	Ingredient B	2.33	0.000	2.00	0.333	1E+30
\$E\$10	Ingredient C	24.00	0.000	9.00	15.000	1E+30
\$E\$11	Ingredient D	8.00	0.083	8.00	4.000	0.800
\$E\$12	Ingredient E	5.67	0.000	5.00	0.667	1E+30

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Use the Sensitivity Report for this LP model to answer the following questions. Each question is independent of the others.

- (a) If the price of grain decreases by \$0.01 per pound, will the optimal solution change?
- (b) Which constraints are binding? Interpret the shadow price for the binding constraints.
- (c) What would happen to the total cost if the price of mineral decreased by 20% from its current value?
- (d) For what price range of oats is the current solution optimal?

Solutions:

- (a) The decrease of \$0.01 per pound is outside the allowable decrease (0.005). Therefore the optimal solution will change. We need to resolve the LP model to determine the new cost.
- (b)
 - o The constraints prescribing the minimum daily requirement for ingredient A and ingredient D are binding (i.e. $2A + 3G + 1M \geq 6$ & $1A + 1.5G + 2M \geq 8$)
 - o For each additional unit of ingredient A required in the mix, the cost will increase by \$0.003.
 - o For each additional unit of ingredient D required in the mix, the cost will increase by \$0.083.
- (c) A 20% decrease in the cost of mineral implies the cost will decrease by \$0.034 ($=0.17 \times 0.2$), which is less than the allowable decrease (0.125). The optimal solutions remain unchanged. The revised cost will be decrease by 0.11322 ($=0.034 \times 3.33$) to \$0.57348 ($=0.6867 - 0.11322$).
- (d) The price of oats can fluctuate between \$0.085 ($0.09 - 0.005$) and \$0.093 ($0.09 + 0.003$) per pound for the current solution to remain optimal.